Comparative Advantage and Moonlighting*

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Abstract

The proportion of multiple jobholders (moonlighters) is negatively correlated with productivity (wages) in cross-sectional and time series data, but positively correlated with education. We develop a model of the labor market to understand these seemingly contradictory facts. An income effect explains the negative correlation with productivity while a comparative advantage of skilled workers explains the positive correlation with education. We provide empirical evidence of the comparative advantage in CPS data. We calibrate the model to 1994 data on multiple jobholdings, and assess its ability to reproduce the 2017 data. There are three exogenous driving forces: productivity, number of children and the proportion of skilled workers. The model accounts for 68.7% of the moonlithing trend for college-educated workers, and overpredicts it by 33.7 percent for high school-educated workers. Counterfactual experiments reveal the contribution of each exogenous variable.

Keywords: Macroeconomics, labor supply, multiple jobholders, productivity, full-time job, part-time job, comparative advantage, income effect.

JEL classification: E1, J2, J22, J24, O4

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1 Introduction

Moonlighting, that is when a worker simultaneously holds more than one job, represents an important mechanism for workers to adjust their labor supply. Our goal is to explain two seemingly contradictory facts regarding moonlighting in the past 25 years. First, conditional on education the most productive workers are the least likely to hold multiple jobs; Second, the most educated workers are the most likely to hold multiple jobs, even though they are the most productive. We document these facts in Section 2. Even though this margin of labor supply adjustment has not been studied as much as other extensive margins (e.g. school, home production, retirement), moonlighting represents a non-negligible segment of the labor market. Since 1994 the proportion of multiple jobholders is of the same order of magnitude as the unemployment rate.[1] Moreover, moonlighting provides an increase in income comparable with the average unemployment benefit.[2]

We develop a static equilibrium model of the labor market with the following features. Workers are heterogeneous in preferences for leisure and skill. They are either unskilled (i.e. high school diploma or less) or skilled (i.e. college-educated), and the skill distribution is exogenous. Workers are exogenously endowed with children who impose a goods cost. The number of children is function of the worker's skill. Workers can choose between a total of five combinations of working one or two jobs, full-time or part-time. We focus only on extensive margin adjustments, abstracting from the intensive margin. Labor demand is modeled via a production technology where the inputs of workers are imperfect substitutes across education (high school v. college) and job type (part- v. full-time).

Our model explains the data via two distinct mechanisms. First, the negative correlation between productivity and the prevalence of multiple jobholders, conditional on education, is rationalized by an income effect: when productivity increases workers seek more leisure time. In our model this is can be achieved by working only one job instead of two. Second, the higher rate of multiple jobholding among college-educated is rationalized by a comparative

[1] The Current Population Survey started reporting the number of workers with more than one job since 1994.

[2] In 2017, for example, the average moonlighter gained around \$330 per week from the second job. By comparison, in 2017 the average unemployment benefit payment was \$333 per week. Thus, the multiple-jobs segment of the U.S. economy involves total payments around the same size as the unemployment insurance system. (The average unemployment benefit for 2017 was calculated using administrative data from the U.S. Department of Labor's Benefit Accuracy Measurement (BAM) program.)

advantage effect. Given the college wage premium, the proportion of multiple jobholders is higher for college-educated workers whenever the part-time-to-full-time wage ratio is higher for college-educated workers than for high school-educated workers; thus, college-educated workers have a comparative advantage in part time jobs. The logic behind this is as follows: consider two workers with identical preferences for leisure, but one is high school-educated and one is college-educated. Suppose that both work one full time job, and are considering taking a second, part-time job. The cost is identical for both of them, lost leisure time, but the benefit differs. The college premium implies that the marginal utility of consumption is higher for a high school-educated worker than for a college-educated worker. In order to induce the college-educated worker to take on the second job, but not the high school-educated worker, it must be that the increase in income resulting from the part-time job is larger for the college-educated than for the high school-educated.

Empirically, we proceed as follows. We first calibrate the model to U.S. data in 1994. We choose parameters to match the proportion of multiple jobholders by education, the proportion of workers in two part-time jobs, in two full-time jobs and workers holding both a full-time and a part-time job. Finally, we target the college premium and the average cost of raising children. Second, we compute an equilibrium of the model corresponding to 2017. We assume three differences between 1994 and 2017: (i) productivity increases exogenously in a way consistent with the observed change in the college premium and overall income growth; (ii) the proportion of skilled workers changes in a way consistent with growing college attainment; and (iii) the number of children for skilled and unskilled workers changes as in the U.S. data.

The model replicates the ordering of the prevalence of multiple jobholders by skill group well, in both the initial and final equilibrium. Over time the model accounts for 68.7 percent of the drop in the proportion of multiple jobholders among skilled workers, and overpredicts the decline for unskilled workers by 33.7 percent. We conduct counterfactual experiments revealing the relative contributions of the three aforementioned differences between the 1994 and 2017 economies. Finally, we report empirical evidence of the comparative advantage of skilled workers in holding multiple jobs.

First, productivity alone accounts for 52.8 and 23.9 percent of the drop in the proportion of multiple jobholders for skilled and unskilled, respectively. Second, the rise in educational attainment alone has a large effect on the proportion of multiple jobholders, but the direction of the effect differs across skills. This is because when the proportion of skilled workers increases, the skilled wage decreases (all else equal) while the unskilled wage increases. Thus, in this experiment, the proportion of multiple jobholders increases for skilled workers and decreases for unskilled workers. Finally, the number of children, which increased for skilled workers and decreased for the unskilled also has a significant albeit smaller effect. For unskilled workers, the decline in the number of children alone accounts for 10 percent of the decline of the proportion of multiple jobholders.

Our paper contributes to the macroeconomic literature on long-run trends and/or country differences in labor markets.[3] A common theme in this literature is the emphasis on some form of extensive margin of labor supply either between home and the market (e.g. Greenwood et al., 2005; Ngai and Pissarides, 2008; Kopecky, 2011; Aguiar et al., 2017); or between schooling, leisure and the market (e.g. Restuccia and Vandenbroucke, 2014); or between sectors (e.g. Rogerson, 2008). We complement this literature by emphasizing another margin of labor supply, i.e., the number of jobs, and by focusing simultaneously on the the long-run and the cross-sectional behavior along this margin. Other authors have written about multiple jobholding, but with different focuses than ours: An early model can be found in Shishko and Rostker (1976). Kimmel and Smith Conway (2001) documents who moonlights and why. Lalé (2019) uses a search model to understand why workers adjust their labor supply via the number of jobs more than via hours.

2 Data

In this section we describe the data and establish several important empirical facts regarding multiple jobholders. The Current Population Survey (CPS) is our primary data set. Specifically we use the Outgoing Rotations Group (ORG). This particular extract of the CPS follows individuals for four months after they enter the survey, they are ignored for eight months, and then interviewed for four more months. The primary advantage of this data set is the availability of earnings information, which is gathered during months four and eight for each individual. In these months individuals are asked questions regarding hours worked and earnings, both overall and in their "usual" job. Multiple jobholders are defined as those workers who had two or more jobs in the reference week of the CPS survey. Data on multiple jobholders are available starting in 1994. While the definition of multiple jobholders includes

[3] Hirsch et al. (2016) concludes that the rate of multiple jobholders is mostly acyclical. Thus, our paper does not contribute to the literature on the business cycle behavior of hours worked.

two or more jobs, in our sample less than 1 percent of all multiple jobholders had more than two jobs; therefore, hereafter we take multiple jobs to refer to an individual working two jobs.[4]

2.1 Overview

Figure 1 displays the two key facts we seek to explain in this paper. This figure plots the percentage of employed individuals working more than one job over time by education, comparing workers with a high school diploma or less to workers with at least some college education. First, note that regardless of education, the percentage of multiple jobholders has steadily declined since 1998. Second, multiple jobholding is positively associated with education level, with the Some-college group having a higher percentage of workers in multiple jobs.[5] This pattern holds regardless of how coarsely we define the education groups. For example, consider the maximum number of education groups available in the data. In 2015, among workers that did not graduate from high school, 2.1 percent had more than one job. For workers with a high school degree, 3.4 percent were multiple jobholders. For high school graduates that received up to 4 years of college education, the same percentage was 5.1. Finally, 6.3 percent of workers with an advanced college degree had more than one job. This relative ranking is robust across all years in the data.

These two facts paint contrasting pictures. To see this, consider Figure 2 which plots the percentage of multiple jobholders (left axis) and the average real hourly wage (in 2017 $'s on the right axis). There is a negative correlation between multiple jobholding and productivity; furthermore, we show in Section 2.2 below that there is also a negative correlation between multiple jobholdings and wages in cross-sectional data, conditional on education. If higher productivity reduces the likelihood of working multiple jobs, then high productivity workers should be less likely to work multiple jobs. Since education is positively correlated with productivity, there should then be a negative correlation between the education level and the percentage of multiple jobholders. It is not the case, hence the apparent contradiction.

Figure 3 shows hours worked by type of worker and education. Multiple jobholders tend to work about 10 hours more, each week, indicating that the second job is typically a part-time

[4]See Appendix A, Figure A.1.

[5]Figure A.2 of Appendix A shows that the downward trend in the proportion of multiple jobholders is true for both men and women, albeit it is more pronounced for men.

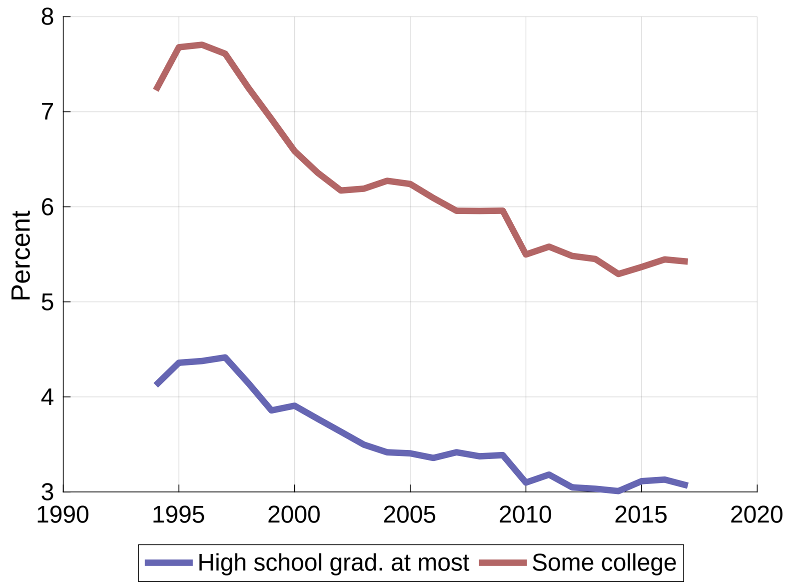


Figure 1: Proportion of employees with two jobs, by education

Source: Current Population Survey.

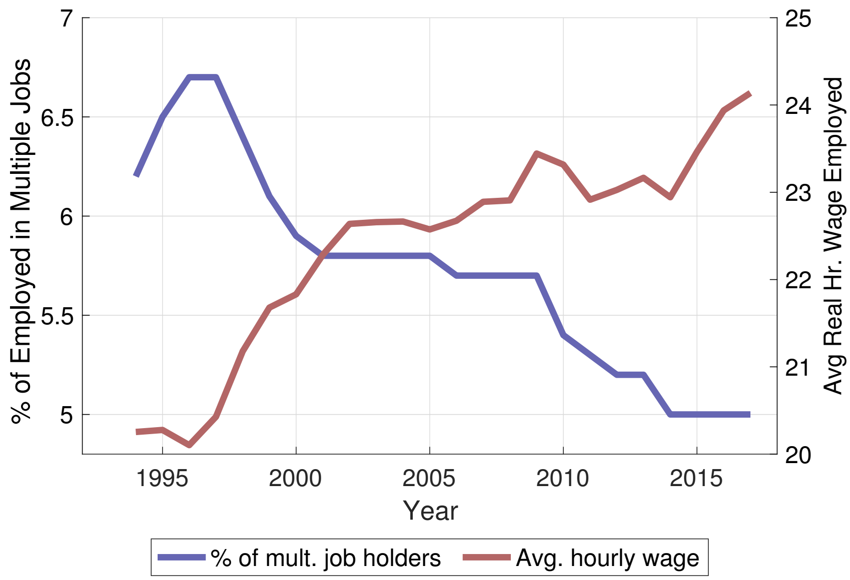


Figure 2: Proportion of employees with multiple jobs (LHS) and Avg. real wages (RHS)

Source: Current Population Survey.

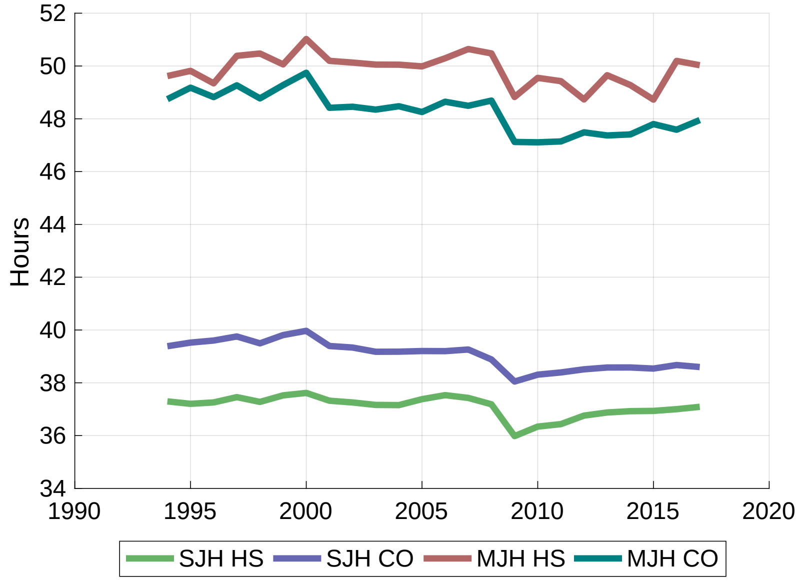


Figure 3: Weekly hours worked

Note: "SJH" means single-job holder and "MJH" means multiple jobholders. "HS" means at most high school graduate and "CO" means some college education.
Source: Current Population Survey.

job. Note that hours worked are stable over time. We use this observation to justify two features of our model in Section 3. First, we do not model the intensive margin of labor supply; second we assume that there are two types of jobs: full time (with fixed hours) and part time (with fixed hours). Out of all multiple jobholders, on average from 1994-2017, 54% worked a full and a part-time job. The next largest category are workers with two part-time jobs, accounting for 24% of multiple jobholders on average. Less than 4% of multiple jobholders have two full-time jobs. Finally, on average only 18% of multiple jobholders have hours that vary on either the primary or the secondary job. These patterns are stable over time (see Figure A.3 in Appendix A).

Figure 4 shows the number of children under 18 for employed individuals between the age of 20 and 55. Since we show, in Section 2.2 below, that the number of children is an important correlate of multiple jobholding, we discuss here the general trend in the number of children for our two education groups. Figure 4 reveals a generally decreasing trend in the number

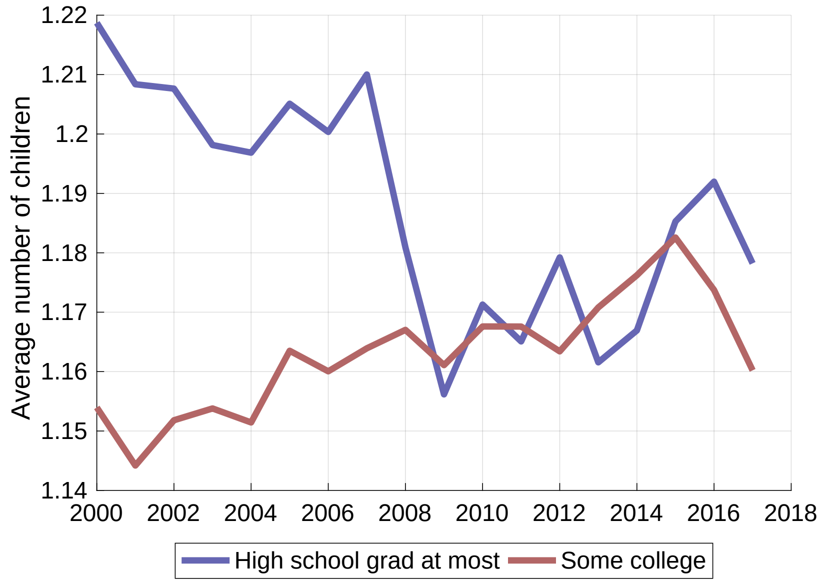


Figure 4: Average number of children under 18 by education

Source: Current Population Survey.

of children for the high school group, and a generally increasing trend for the college group.[6] In the model of Section 3 we endow workers with an exogenous number of children, and, in the quantitative analysis of Section 4, we let this number change in line with the data presented in Figure 4.

2.2 Correlations

In this section we analyze correlates of multiple jobholding using a probit model with a (0/1) indicator variable for multiple jobholding as the dependent variable. Table 1 presents the results. The coefficient estimates are presented as odds ratios, so a coefficient less (greater) than 1 indicates a characteristic that reduces (increases) the likelihood of working multiple jobs. The models are estimated separately for males and females, under several alternative specifications. All specifications include both State and Year fixed effects.

[6]This general pattern is consistent with findings by Bar et al. (2018) who find an increase in fertility among higher income families over the past 25 years.

The specifications in Models 1 and 2 differ in the treatment of children. From 2000-2017, the CPS data contain the ages of children of the adult respondents. Specifically the data contain the number of children under the age of 18 in the household. "Model 1" is a specification using the number of children under the age of 18 as an explanatory variable; "Model 1F" is restricted to females, "Model 1M" is restricted to males (the same notation holds for Models 2 and 3), and "Model 1A" uses all observations but includes an indicator for Female (=1 if female). "Model 2" includes indicator variables for having children of a certain age, e.g. children ages 0-2, 3-5, etc. For these indicator variables the reference group is no children under the age of 18. "Model 3" is the same as Model 2, except that we also add indicator variables for Occupation. We only have a consistent set of Occupation definitions starting in 2003, so we lose some observations in this model

Several interesting patterns emerge in Table 1. First, and across all specifications, education has a positive effect on multiple jobholding (Less than high school is the reference group for the education indicators). To gauge the economic magnitude of the effect of education Figure 5 plots the probability of multiple jobholding predicted by Model 1A as a function of the level of education, all other variable being held constant at their sample mean. Figure 5 suggests that the effect of education is both statistically and economically significant: the probability of multiple jobholding being multiplied by more than three over the education spectrum.

Second, as indicated earlier, the costs associated with children represents a possible reason why workers may decide to work multiple jobs. We note from models 1F and 1M that the presence of children reduces the likelihood of multiple jobholdings for women and increases it for men. This can be interpreted as follows. Children impose both time and good costs on households, although the time cost is borne mostly by the mother. The presence of a child thus induces the father to accept a second job to help with the good cost, while it deters the mother to do so in order to help with the time cost. This is confirmed by Models 2 and 3 where it appears that, as children become older, the mother is less likely to spend time at home and more likely to take on a second job. The importance of the second job may also increase as the children becomes older because of the need to finance their education. In the model of Section 3 we only model the good cost of a child, and abstract from the time cost. We do this to simplify the analysis given the fact that the time cost is age-related and would require a model that distinguishes between gender and age.

Figure 6 plots the probability of multiple jobholding predicted by model 1A as a function

Table 1: Probit Model Multiple Jobholding Dependent Variable

Variables	(1) Model 1F	(2) Model 1M	(3) Model 1A	(4) Model 2F	(5) Model 2M	(6) Model 3F	(7) Model 3M
Education (Less than HS reference group)							
HS	1.179***	1.259***	1.217***	1.182***	1.258***	1.185***	1.260***
	(0.0178)	(0.0175)	(0.0131)	(0.0177)	(0.0174)	(0.0187)	(0.0174)
Some college	1.407***	1.499***	1.453***	1.411***	1.498***	1.407***	1.464***
	(0.0217)	(0.0236)	(0.0171)	(0.0217)	(0.0236)	(0.0235)	(0.0203)
College	1.456***	1.544***	1.501***	1.465***	1.543***	1.432***	1.469***
	(0.0272)	(0.0277)	(0.0210)	(0.0273)	(0.0277)	(0.0303)	(0.0253)
Advanced	1.646***	1.755***	1.703***	1.668***	1.755***	1.598***	1.609***
	(0.0304)	(0.0358)	(0.0280)	(0.0308)	(0.0358)	(0.0311)	(0.0272)
Number of children	0.976***	1.026***	1.004**				
	(0.00227)	(0.00294)	(0.00220)				
Age of Children (No children under 18 reference group)							
Child 0-2				0.820***	1.028***	0.808***	1.023***
				(0.00594)	(0.00751)	(0.00653)	(0.00846)
Child 3-5				0.927***	1.038***	0.923***	1.036***
				(0.00642)	(0.00861)	(0.00690)	(0.00850)
Child 6-13				0.948***	1.020***	0.943***	1.015**
				(0.00625)	(0.00522)	(0.00604)	(0.00594)
Child 14-17				1.068***	1.062***	1.060***	1.066***
				(0.00619)	(0.00831)	(0.00574)	(0.00882)
Female (Y/N)			0.885***				
			(0.00625)				
Real wage, 2017 $'s	0.999***	0.997***	0.998***	0.999***	0.997***	0.999***	0.998***
	(0.000339)	(0.000293)	(0.000286)	(0.000337)	(0.000293)	(0.000337)	(0.000265)
Married	0.829***	1.038***	0.894***	0.835***	1.040***	0.834***	1.052***
	(0.00544)	(0.0129)	(0.00581)	(0.00558)	(0.0130)	(0.00640)	(0.0147)
Constant	0.165***	0.151***	0.176***	0.162***	0.151***	0.148***	0.140***
	(0.00374)	(0.00358)	(0.00325)	(0.00369)	(0.00357)	(0.00399)	(0.00314)
Observations	1,018,409	978,118	1,996,527	1,018,409	978,118	846,628	811,143
State FE	YES	YES	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES	YES	YES
Occupation FE						YES	YES

*** p<0.01, ** p<0.05, * p<0.1
Odds ratios presented. Robust z statistics in parentheses. Standard errors clustered at the state level.

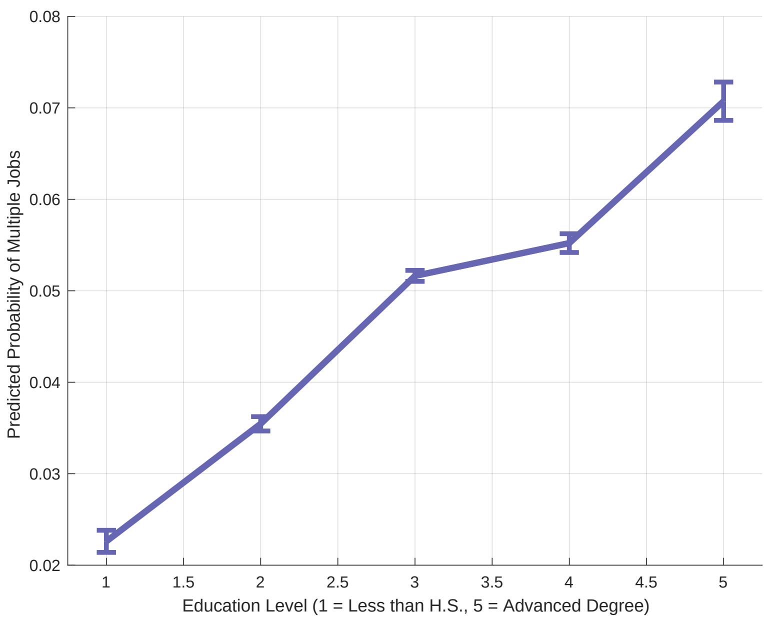


Figure 5: Predicted probabilities as a function of education level

Note: Calculated at the mean of all covariates.

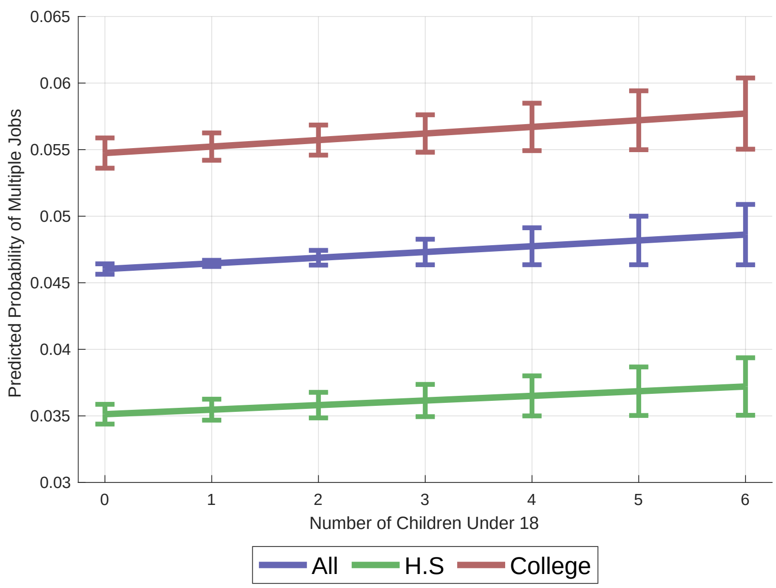


Figure 6: Predicted probabilities as a function of number of children

Note: Calculated at the mean of all covariates.

of the number of children, all other variable being held constant at their sample mean. All else equal an increasing number of children raises the likelihood of multiple jobholding. On Figure 6 this effect appears relatively small. This is due to the fact that in model 1A the worker is of the "average gender," unlike in model 1F or 1M where the gender is fixed. As mentioned above, the coefficients for models 1F and 1M show that the effect of children is positive for men and negative for women. The small effect of children in model 1A is the combination of these two opposing forces. Since, on the one hand, our theoretical model of Section 3 does not distinguish between gender the worker in the theoretical model can be viewed as corresponding to the worker represented in Probit model 1A. On the other hand, the theoretical model features only a good cost which, as we have explained above, is borne by men. From this perspective the worker in the theoretical model would be closer to the worker represented in Probit model 1M. In both cases the effect of children is positive, albeit the effect is stronger in Probit model 1M.

Third, in all models the real wage has a negative effect on multiple jobholding.[7] Thus, conditional on characteristics such as education and family size/composition, higher real wages reduce the incidence of multiple jobholding. We interpret this result as indicating that more productive workers are less likely to be multiple jobholders. Although the coefficients are close to one, the negative effect of real wages is statistically significant. Figure 7 shows this: there is a significant decline in the probability of multiple jobholdings as the real wage on the usual job increases. This is true across education groups and holding all other characteristics fixed.

The statistical models reported in Table 1 also included other demographic characteristics, including marital status, age and race. In all specifications, marriage has a negative effect on female multiple jobholding and a positive effect on male multiple jobholding. For both Males and Females, age has a non-monotonic effect. In all but model 3F, age has a positive effect on multiple jobholding with the peak effect for the 40-49 age group. Black, Hispanic, and Other races are generally less likely to be multiple jobholders relative to White, with the effects slightly stronger for females relative to males. Table 7 in Appendix A reports the full set of parameters in our estimations.

This section has established that (i) multiple jobholders hold two jobs; (ii) conditional on education, increasing real wages decreases multiple jobholding (over time with growth and

[7]Real wages are hourly earnings on the usual job, including overtime, tips, and commissions. The nominal wages are converted to 2017 $'s using the CPI-U-RS (see www.bls.gov)

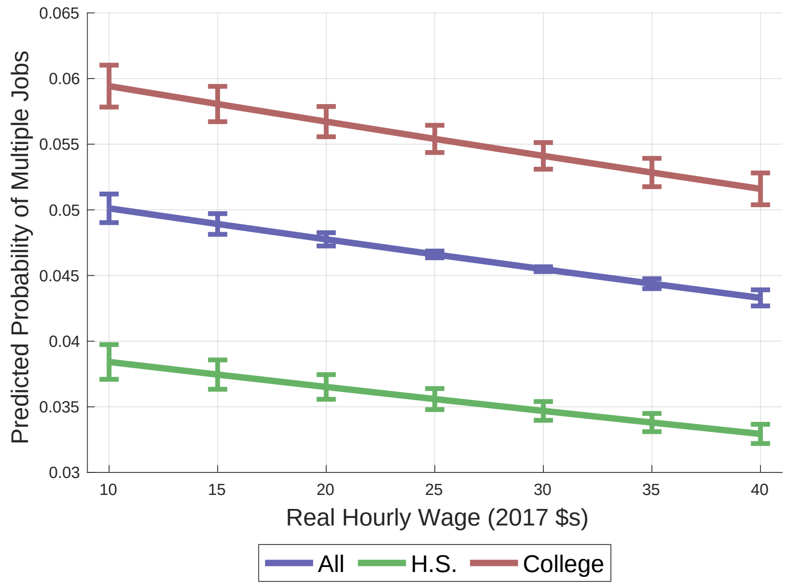


Figure 7: Predicted probabilities as a function of the real wage

Note: Calculated at the mean of all covariates. Bars indicate 95% confidence intervals.

in the cross section); (iii) higher skilled workers (using education as a proxy for skill) are more likely to work multiple jobs; (iv) hours worked are stable over time for single- and multiple jobholders, regardless of their skills; and (v) the number of children for employed people between age 20 and 55 decreased for the least educated and increased for the most educated and is an important determinant of the likelihood that a worker holds multiple jobs. The Probit models presented in Table 1 reveal that the effect of wages, education and children are significant after controlling for sex, marital status, age and race. In the rest of this paper we develop and use an equilibrium model of the labor market to understand the determinants of multiple jobholding. Guided by the results of this Section, we abstract from modeling sex, marital status, age and race to focus on the effect of productivity, education and the number of children.

3 Model

Time lasts for one period. Each worker has 1 unit of time to allocate between work and leisure. There are two types of jobs: full-time and part-time. A full-time job requires a fraction n_F of the time, and a part-time job requires a fraction n_P of the time. There are also two types of workers: skilled and unskilled. The proportion of skilled workers is exogenous and denoted by μ.

Production

A representative firm produces output via a constant-returns-to-scale technology using the services of skilled labor and unskilled labor:

$$Y = F\left(L_S, L_U\right), \tag{1}$$

where L_S (L_U) aggregates labor from full-time and part-time skilled (unskilled) workers:

$$L_x = \left(z_{x,F} L_{x,F}^{\phi_x} + z_{x,P} L_{x,P}^{\phi_x}\right)^{1/\phi_x}, \; x \in \{S, U\}. \tag{2}$$

The parameters $z_{x,F}$ and $z_{x,U}$ are skill- and job-specific technology parameters, and $\phi_x \leq 1$ controls the elasticity of substitution between full-time and part-time labor. The terms $L_{x,F}$

and $L_{x,P}$ denote the total labor input from full-time and part-time workers with skill x, respectively. The firm's optimization problem is

$$\max_{\{L_{x,F},L_{x,P}\}} Y - \sum_{x\in\{S,U\}} w_{x,F}L_{x,F} - \sum_{x\in\{S,U\}} w_{x,P}L_{x,P}. \tag{3}$$

Workers

Workers have preferences defined over consumption and leisure. A typical worker's preferences are represented by the utility function

$$U(c) + \alpha V(\ell)$$

where $\alpha > 0$, and where c and ℓ stand for consumption and leisure, respectively. The functions U and V are increasing, twice-continuously differentiable and concave utility indexes.

Besides skills, workers are also differentiated by the intensity of their taste for leisure, α. In each skill group there is a continuum of individuals indexed by α. The cumulative distribution function for α is denoted $A(\alpha)$ and is identical for skilled and unskilled workers. Skilled and unskilled workers are endowed with k_S and k_U children, respectively. Each child imposes a goods cost denoted by θ.

There are five types of employment a worker can choose from: one full-time job (F), one part time job (P), one full-time and one part-time job (FP), two part-time jobs (PP) and two full-time jobs (FF). Let e indicate a particular employment type: $e \in \{F, P, FP, PP, FF\}$. The value function of a worker α with skill x and employment type e is

$$\begin{aligned} W_{x,e}(\alpha) &= U(c_{x,e}) + \alpha V(\ell_{x,e}) \\ \text{s.t.} \quad & c_{x,e} + \theta k_x = y_{x,e} \\ & c_{x,e}, \ell_{x,e} > 0 \end{aligned}$$

where $c_{x,e}$ indicates consumption, $y_{x,e}$ is income and $\ell_{x,e}$ is leisure time. Table 2 shows income and leisure for all x and e. The worker's labor supply is determined by

$$\max_{e} W_{x,e}(\alpha).$$

e	$y_{x,e}$	$\ell_{x,e}$
F	$w_{x,F}n_F$	$1 - n_F$
P	$w_{x,P}n_P$	$1 - n_P$
FP	$w_{x,F}n_F + w_{x,P}n_P$	$1 - n_F - n_P$
PP	$2w_{x,P}n_P$	$1 - 2n_P$
FF	$2w_{x,F}n_F v$	$1 - 2n_F$

Table 2: Income and leisure for worker with skill x by employment type

3.1 Equilibrium

An equilibrium is a set of prices $\{w_{x,j}\}$ for $(x, j) \in \{S, U\} \times \{F, P\}$ and an allocation of workers to jobs $\{p_{x,e}\}$ for $(x, e) \in \{S, U\} \times \{F, P, FP, PP, FF\}$ such that

1. The Firm optimizes given prices:

$$F_1(L_S, L_U)\frac{\partial L_S}{\partial L_{S,j}} = w_{S,j}, \quad \text{for } j \in \{F, P\}$$

and

$$F_2(L_S, L_U)\frac{\partial L_U}{\partial L_{U,j}} = w_{U,j}, \quad \text{for } j \in \{F, P\}.$$

2. Workers optimize given prices:
The proportion of workers with skill x optimally choosing employment type e is

$$p_{x,e} = \int_{\left\{\alpha : W_{x,e}(\alpha) \geq W_{x,e'}(\alpha) \ \forall \ e' \neq e\right\}} dA(\alpha).$$

3. The labor market clears:

$$\begin{aligned}
L_{S,F} &= \mu(p_{S,F} + p_{S,FP} + 2p_{S,FF})n_F, \\
L_{S,P} &= \mu(p_{S,P} + p_{S,FP} + 2p_{S,PP})n_P, \\
L_{U,F} &= (1-\mu)(p_{U,F} + p_{U,FP} + 2p_{U,FF})n_F, \\
L_{U,P} &= (1-\mu)(p_{U,P} + p_{U,FP} + 2p_{U,PP})n_P.
\end{aligned}$$

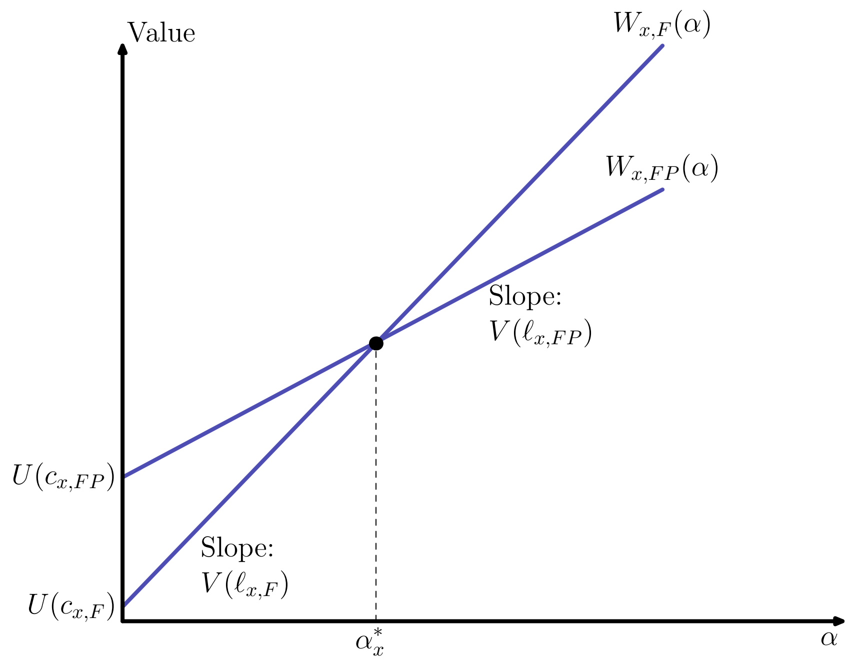


Figure 8: The determination of labor supply

3.2 Analysis

In this Section we discuss the determination of labor supply and, specifically, the type of employment a worker chooses. We discuss first the effect of productivity, then the effect of the number of children. Finally, we analyze the conditions under which the model can simultaneously imply a negative correlation between multiple jobholdings and productivity conditional on education, and a positive correlation between multiple jobholdings and education. Throughout the section we illustrate our discussion with a simplified version of a worker's decision problem: the choice between a full-time job ($e = F$) versus a full- and a part-time job ($e = FP$). This approach simplifies the discussion while still demonstrating the key mechanisms at work in the model.

The value functions for workers of skill $x \in \{S, U\}$ with type-F employment and type-FP employment are

$$W_{x,F}(\alpha) = U(c_{x,F}) + \alpha V(\ell_{x,F})$$

and

$$W_{x,FP}(\alpha) = U(c_{x,FP}) + \alpha V(\ell_{x,FP}),$$

respectively. Figure 8 represents these value functions and two points deserve to be mentioned. First, by construction the value functions are affine in α with an intercept given by U and a slope given by V. Second, even though Figure 8 represents the value functions to be increasing in α, this is only for representation's sake. The sign of the slope is the sign of V and has no particular meaning.

Figure 8 shows that workers with $\alpha > \alpha_x^*$ choose to work one full-time job while workers with $\alpha < \alpha_x^*$ prefer to work both a full-time and a part-time job. This results from (i) the fact that the intercept of $W_{x,F}(\alpha)$ is lower than the intercept of $W_{x,FP}(\alpha)$; and (ii) the fact that the slope of $W_{x,F}(\alpha)$ is larger than that of $W_{x,FP}(\alpha)$. To see that the first of these two conditions is generally satisfied, note that $c_{x,F} = w_{x,F}n_F - \theta k_x$ while $c_{x,FP} = w_{x,F}n_F + w_{x,FP}n_P - \theta k_x$. To see that the second condition is generally satisfied, it suffices to note that $\ell_{x,F} = 1 - n_F$ while $\ell_{x,FP} = 1 - n_F - n_P$. Hence $V(\ell_{x,F}) > V(\ell_{x,FP})$.

The marginal worker, that is the worker who is indifferent between one full-time job and two jobs (one full-time and one part-time) is defined by $W_{x,F}(\alpha_x^*) = W_{x,FP}(\alpha_x^*)$. Using this condition, the critical value for α_x^* is given by:

$$\alpha_x^* = \frac{U(w_{x,F}n_F + w_{x,P}n_P - \theta k_x) - U(w_{x,F}n_F - \theta k_x)}{V(1 - n_F) - V(1 - n_F - n_P)}. \tag{4}$$

The effect of productivity

How does multiple jobholding change when $w_{x,F}$ and/or $w_{x,P}$ increase? Several effects must be discussed. First, suppose the ratio $w_{x,F}/w_{x,P}$ remains constant as both wages increase proportionally. There are then standard income and substitution effects at work. If the income effect dominates, workers will tend to choose employment types requiring fewer hours of work. In the simple model we consider here this means a single, full-time job. If the substitution effect dominates, the opposite occurs. That is, workers will tend to choose employment types requiring more hours of work and, therefore, they will be more likely to take two jobs.

In addition to the standard income and substitution effects there can also be "relative price effects" when the ratio $w_{x,F}/w_{x,P}$ changes. We use the term "relative price effect" to refer to the relative wages between full-time and part-time jobs and not to the relative price of leisure and consumption which, of course, changes even when $w_{x,F}$ and $w_{x,P}$ increase proportionally.

Suppose, for instance, that $w_{x,F}$ is multiplied by a factor 2 and $w_{x,P}$ is multiplied by a factor 3. This could be viewed first as multiplying both wages by a factor 2 and, second, as increasing $w_{x,P}$ alone. The first part implies income and substitution effects as described earlier. In the second part, that is when $w_{x,P}$ alone increases there is (i) a standard income effect, again, because the worker becomes richer; (ii) a standard substitution effect, again, because leisure becomes more expensive; and (iii) what we refer to as the "relative price effect" which indicates that time spent working the full time job is becoming relatively more expensive. The latter effect makes part-time labor more attractive and induces workers to choose to work part-time. In the simple model we consider here this means workers tend to choose to work two jobs. In the general model that may or may not be the case depending upon the strength of the income and substitution effects. If, for example, $w_{x,P}$ increases while $w_{x,F}$ remains constant fewer workers would want to work the full time job and more would want to work the part-time job ("relative price effects"). If the standard income effect is strong enough, workers will then tend to choose to hold only one, part-time job. If, instead, the standard substitution effect is strong enough, workers will tend to choose to hold two part-time jobs.

The interplay between income, substitution and relative price effects is, ultimately, a quantitative question that we address in Section 4. For now we show, formally, how these effects operate in our simplified model. We start with a proportional increase in both $w_{x,F}$ and $w_{x,P}$. Suppose that $w_{x,P} = \rho w_{x,F}$ and that ρ is a positive constant. Equation (4) implies

$$d\alpha_x^*\bigg|_{\frac{w_{x,P}}{w_{x,F}}=\rho} = \frac{U'(c_{x,FP})\left(1+\rho\frac{n_P}{n_F}\right) - U'(c_{x,F})}{V(\ell_{x,F}) - V(\ell_{x,FP})} n_F dw_{x,F}. \tag{5}$$

The sign of this expression is the sign of the numerator. When $w_{x,P} = \rho w_{x,F}$, consumption of the two types of workers is related by

$$c_{x,FP} = c_{x,F}\left(1+\rho\frac{n_P}{n_F}\right) + \theta k_x \rho \frac{n_P}{n_F}.$$

The numerator in Equation (5) becomes

$$U'\left(c_{x,F}\left(1+\rho\frac{n_P}{n_F}\right) + \theta k_x \rho\frac{n_P}{n_F}\right)\left(1+\rho\frac{n_P}{n_F}\right) - U'(c_{x,F}) \tag{6}$$

Suppose the utility index U is such that $U'(cz)z$ is a decreasing function of z. We claim this type of utility index implies that the income effect from a change in wages dominates the substitution effect. We prove this claim in Appendix B. Note that if $U'(cz)z$ is decreasing in z and if $\theta = 0$, the expression in (6) is negative. Note also that the first element in (6) is decreasing in θ, therefore the expression in Equation (6) is negative for all $\theta \geq 0$ whenever $U'(cz)z$ is decreasing in z.

We have shown that when the income effect dominates, α_x^* decreases whenever wages increase proportionally. The interpretation of this result is that worker seek to increase their leisure when wages increase and that they can achieved this by using the extensive margin of employment and, in particular, by choosing to work only one job.

Next we turn to the individual effects of $w_{x,F}$ and $w_{x,P}$ which are described in Equation (7).

$$d\alpha_x^* = \frac{1}{V(\ell_{x,F}) - V(\ell_{x,FP})}\Big[\big(U'(c_{x,FP}) - U'(c_{x,F})\big)n_F dw_{x,F} + U'(c_{x,FP})n_P dw_{x,P}\Big]. \quad (7)$$

It is immediate that the effect of $w_{x,F}$ alone is to reduce α_x^* and, therefore, to increase the number of workers with one job (in this simplified model). This follows from the fact $c_{x,F} < c_{x,FP}$. Similarly, it is immediate that the effect of $w_{x,P}$ alone is to increase α_x^* and, therefore, to increase the number of workers with two jobs (in this simplified model).

The effect of children

In our model children cost goods and constitute an endowment. Thus, a decrease in the number of children is akin to an increase in income holding all relative prices constant. Since leisure is a normal good workers with fewer children tend to work fewer hours which, again, is achieved by adjusting their labor supply at the extensive margin. Thus, the decrease of the number of children of unskilled workers (see Figure 4) is conducive to a decline in the fraction of multiple jobholders while the increase for skilled workers has the opposite effect.

In the context of the simple example of this Section, the effect of a change in the number of children is

$$\frac{d\alpha_x^*}{dk_x} = -\theta \frac{U'(c_{x,FP}) - U'(c_{x,F})}{V(\ell_{x,F}) - V(\ell_{x,FP})} > 0, \quad (8)$$

where the inequality follows from the facts that $c_{x,F} < c_{x,FP}$ and that $\ell_{x,F} > \ell_{x,FP}$. Thus, a decrease in the number of children reduces α_x^* and, therefore decreases the proportion of workers with two jobs. This effect is qualitatively consistent with the estimated coefficients of the Probit models $1M$, $1A$, $2M$ and $3M$ displayed in Table 1

Comparative advantage

In Section 2 we pointed out an apparent contradiction. On the one hand, conditional on education the most productive workers are the least likely to hold multiple jobs; on the other hand the most educated workers are the most likely to hold multiple jobs. Our model reconciles this apparent contradiction through the following mechanisms.

First, we have shown that as a long as preferences are such that the income effect from wages dominate the substitution effects, the most productive workers seek to work fewer hours. They achieve this by adjusting labor supply at the extensive margin, working only one job instead of two. This mechanism explains the time series correlation of multiple jobholdings with productivity, as well as the cross-sectional correlation conditional on education.

Second, to see how our model explains the higher prevalence of multiple jobholders among skilled workers, we show the conditions under which $\alpha^*_S/\alpha^*_U > 1$ in the simplified model. To do this, we consider the special case where $U(c) = (1-\sigma_C)^{-1}(c-\bar{c})^{1-\sigma_C}$ with $\sigma_C > 0$. In the quantitative analysis of Section 4 we use this functional form for U. It follows that

$$\frac{\alpha^*_S}{\alpha^*_U} = \underbrace{\left(\frac{w_{S,F}}{w_{U,F}}\right)^{1-\sigma_C}}_{A} \underbrace{\frac{\left(n_F + \frac{w_{S,P}}{w_{S,F}} n_P - \frac{\theta_K k_S - \bar{c}}{w_{S,F}}\right)^{1-\sigma_C} - \left(n_F - \frac{\theta_K k_S - \bar{c}}{w_{S,F}}\right)^{1-\sigma_C}}{\left(n_F + \frac{w_{U,P}}{w_{U,F}} n_P - \frac{\theta_K k_U - \bar{c}}{w_{U,F}}\right)^{1-\sigma_C} - \left(n_F - \frac{\theta_K k_U - \bar{c}}{w_{U,F}}\right)^{1-\sigma_C}}}_{B}. \tag{9}$$

Two points are worth noting. First, given a college premium, that is given $w_{S,F}/w_{U,F} > 1$, the features of A depend on the utility parameter σ_C. If $\sigma_C > 1$, then $A < 1$ and is decreasing in $w_{S,F}/w_{U,F}$. If $\sigma_C \in (0,1)$, then $A > 1$ and is increasing in $w_{S,F}/w_{U,F}$. This obtains because when $\sigma_C > 1$ the income effect of an increase in $w_{S,F}$ dominates the substitution effect, causing skilled workers to work less. In this simple model this is achieved by not taking a second job; as a result, α^*_S decreases. When $\sigma_C \in (0,1)$ the substitution effect dominates and α^*_S increases in response to an increase in $w_{S,F}$. Second, part B of Equation (9) is increasing in $w_{S,P}/w_{S,F}$ and decreasing in $w_{U,P}/w_{U,F}$.

Suppose that $\sigma_C > 1$ as in our quantitative analysis of Section 4. It follows from the discussion above that $\alpha_S^*/\alpha_U^* > 1$ whenever $w_{S,P}/w_{S,F}$ is sufficiently large relative to $w_{U,P}/w_{U,F}$. We refer to the inequality

$$\frac{w_{S,P}}{w_{S,F}} > \frac{w_{U,P}}{w_{U,F}} \tag{10}$$

as describing a comparative advantage of skilled workers over unskilled workers in part time jobs.

When $\sigma_C > 1$ the "college-premium" effect embodied in A tends to reduce the prevalence of multiple jobholders among college educated workers in the cross-section. The "comparative advantage" effect embodied in B acts in the opposite direction, however. With a large enough comparative advantage the model reconciles the higher prevalence of multiple jobholders among college-educated workers in the cross-section, with the negative correlation between multiple jobholding and productivity in both the time series and the cross-section (conditional on education).[8]

The economics behind this discussion is illustrated in Figure 9. Consider two workers, one skilled and the other unskilled. Suppose that they both work one full-time job and have identical preferences, i.e., the same α. Under what condition would the skilled worker take a second job while the unskilled worker would not? The cost of taking the second job, forgone leisure time, is the same for each worker. The benefit, however, is not the same. The skilled worker's marginal utility is lower because of the skill premium. Thus, in order for the utility gain from the second job to be larger for the skilled worker (green vertical arrow) than for the unskilled (red vertical arrow), the associated consumption gain must be larger for the skilled worker relative to the unskilled. Hence the need for a comparative advantage of skilled workers in part time jobs.

We also note that the presence of children matters in this discussion. To see this in Figure 4, suppose the number of children increases, from 0 to k_U for an unskilled worker, and from 0 to k_S for a skilled worker. Further assume that $k_S < k_U$. Both $c_{S,F}$ and $c_{S,FP}$ move to the left by $\theta k_S/w_{S,F}$ while $c_{U,F}$ and $c_{U,FP}$ move to the left by $\theta k_U/w_{U,F}$. Note that the total cost of children is higher for the unskilled worker: $\theta k_U/w_{U,F} > \theta k_S/w_{S,F}$ since $k_U > k_S$ and $w_{S,F} > w_{U,F}$. Thus, the benefit of a second job increases for the unskilled worker more than

[8]When $\sigma_C \in (0, 1)$ both the "college-premium" and the "comparative advantage" effects operate in the same direction. Therefore the comparative advantage needed to reconcile the cross-section and time series observations is lower.

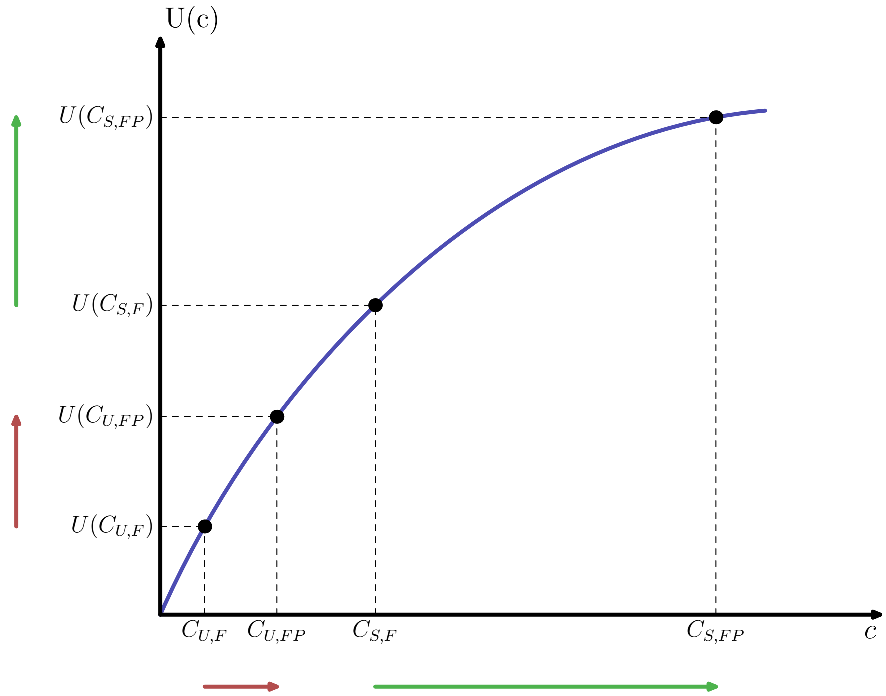


Figure 9: The comparative advantage of skilled workers in part-time jobs

for the skilled worker. This implies that a higher comparative advantage is necessary for the skilled worker to choose multiple jobs while the unskilled worker would not.[9]

4 Quantitative Analysis

Our quantitative analysis proceeds in several steps. First, we calibrate our model to the U.S. data in 1994–the first year for which the CPS reports statistics on multiple jobholders. We discipline the parameters of the model by targeting the proportion of multiple jobholders by education, as well as the distribution of multiple jobholders across different employment types (i.e. F, P, FP, PP, FF). We also target the college premium and a measure of the goods cost of children. Key among the parameters to be determined in this first exercise are the four skill- and job-specific technology parameters, $z_{x,j}$ for $(x, j) \in \{S, U\} \times \{F, P\}$

Second, we compute a new equilibrium corresponding to the U.S. data in 2017. There are three differences between the 1994 (initial) equilibrium and the 2017 (final) equilibrium: (i)

[9]Note that $k_S < k_U$ is not necessary for this result to hold.

we allow productivity parameters to change in such a way as to reproduce income growth in the U.S. economy between 1994 and 2017, and the increase in the college premium; (ii) we let the number of children for high school- and college-educated workers change as in the U.S. data; and (iii) we let the proportion of college-educated workers change as in the U.S. data.

Third, we compare the model's predicted evolution of multiple jobholding to the actual U.S. data. We also conduct a decomposition of the contribution of changes in the aforementioned variables: productivity, the number of children and educational attainment. Finally, we report empirical evidence of the comparative advantage of skilled workers in holding multiple jobs.

4.1 Calibration

We interpret a time period as lasting one week, and assume that there is a total of $7 \times (24 - 8) = 112$ hours available for either work or leisure in the week. A full-time job requires 40 hours, implying $n_F = 40/112 = 0.36$. We use Figure 3 to justify that a part-time job requires 10 hours of work; therefore, we set $n_P = 10/112 = 0.09$. According to the CPS data, in 1994 56 percent of workers had at least some college education. Thus, we set $\mu_S = 0.56$. The data also show that college-educated workers ages 20-55 had 1.15 children under 18 years of age. High school-educated workers ages 20-55 had 1.22 children under the age of 18. Thus, we set $k_S = 1.15$ and $k_U = 1.22$. We choose the following functional form for the production function

$$Y = \left(L_S^\eta + L_U^\eta\right)^{1/\eta}. \tag{11}$$

The elasticity of substitution between skilled and unskilled labor is $1/(1-\eta)$. We follow Goldin and Katz (2007) and use an elasticity of substitution of 1.6, implying $\eta = 1 - 1/1.6$. We use the same value for ϕ_S and ϕ_U, which determine the elasticity of substitution between full-time and part-time labor for skilled and unskilled labor, respectively (Equation 2). Note that we do not weight L_S^η and L_U^η in Equation (11). This is because such weights could not be distinguished from the the productivity parameters $z_{x,j}$ for $(x, j) \in \{S, U\} \times \{F, P\}$.

For the utility function we specify:

$$U(c) = \frac{(c - \bar{c})^{1-\sigma_C}}{1 - \sigma_C} \text{ and } V(\ell) = \frac{\ell^{1-\sigma_L}}{1 - \sigma_L},$$

where $\sigma_C, \sigma_L > 0$. The distribution of α is assumed to be log-normal:

$$\ln(\alpha) \sim N(\mu_\alpha, \sigma_\alpha).$$

This implies 10 parameters to calibrate:

$$\omega = \{\sigma_C, \sigma_L, \bar{c}, \mu_\alpha, \sigma_\alpha, z_{S,F}, z_{S,P}, z_{U,F}, z_{U,P}\, \theta_K\}.$$

These parameters are calibrated to the following moments from the data: the proportion of two jobholders by education, the proportion of workers with two part-time jobs, two full-time jobs and the proportion with one part-time and one full-time job. We also target hourly earnings ratio between workers with one or two jobs, by education levels and the college premium. Finally, we target the goods cost of a child as fraction of a household's income. Thus there exists 10 moments to calibrate the 10 parameters. Practically, we define

$$M(\omega) = \begin{bmatrix} p_{S,FP} + p_{S,PP} + p_{S,FF} - 7.22\% \\ p_{U,FP} + p_{U,PP} + p_{U,FF} - 4.12\% \\ \mu_S p_{S,PP} + (1-\mu_S) p_{U,PP} - 1.30\% \\ \mu_S p_{S,FF} + (1-\mu_S) p_{U,FF} - 0.20\% \\ \mu_S p_{S,FP} + (1-\mu_S) p_{U,FP} - 3.40\% \\ e_{S,2}/e_{S,1} - 0.93 \\ e_{U,1}/e_{S,1} - 0.67 \\ e_{U,2}/e_{S,1} - 0.67 \\ w_{S,F}/w_{U,F} - 1.55 \\ \theta_K(\mu_S k_S + (1-\mu_S)k_U)/y - 0.2 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \tag{12}$$

where $y = \mu_S \sum_e p_{S,e} y_{S,e} + (1-\mu_S) \sum_e p_{U,e} y_{U,e}$, is the average worker's income. We then solve

$$\min_\omega M(\omega)' M(\omega). \tag{13}$$

A few comments are in order at this stage. The first two rows of $M(\omega)$ indicate the difference between the model's implied proportion of skilled and unskilled workers with two jobs, and the corresponding empirical moment in 1994. The third row relates to the proportion of workers with two part-time job–namely the difference between the statistics implied by the model and its empirical counterpart. These statistics are calculated using the CPS data, where a part-time, single jobholder is defined as a worker with only one job with less than

35 hours per week. Rows 4 and 5 relate to the proportion of workers with two part-time jobs (row 4) and two full-time jobs (row 5).

Rows 6-9 relate to the wage ratios between various categories of workers. We define the following variables: the average hourly earnings of a worker with skill x and one job: $e_{x,1}$, and the hourly earnings of a worker with skill x and two jobs: $e_{x,2}$. These averages are

$$e_{x,1} = \frac{1}{p_{x,F} + p_{x,P}} \left[p_{x,F} w_{x,F} + p_{x,P} w_{x,P} \right]$$

and

$$e_{x,2} = \frac{1}{p_{x,FP} + p_{x,PP} + p_{x,FF}} \left[p_{x,FP} \frac{w_{x,F} n_F + w_{x,P} n_P}{n_F + n_P} + p_{x,PP} w_{x,P} + p_{x,FF} w_{x,F} \right].$$

Our data reveal that in 1994, the relative hourly earnings of workers were $e_{S,2}/e_{S,1} = 0.93$, $e_{U,1}/e_{S,1} = 0.67$ and $e_{U,2}/e_{S,1} = 0.67$.[10] We use these statistics in row 6-8 of $M(\omega)$. For row 9 we define a measure of the college premium as the ratio of hourly earnings between workers with at least some college and workers with at most a high-school degree (each holding a single, full time job). Our data indicate this ratio to be 1.55 in 1994. The model's counterpart of this ratio is $w_{S,F}/w_{U,F}$. Finally, row 10 indicates that the cost of a child represents 20 percent of the household's income–see Greenwood et al. (2017, table 1 and 2).

Although the parameters are determined simultaneously through the minimization program (13), some moments matter more than others for some parameters of the model. First, the college premium data imposes discipline on $z_{S,F}$ and $z_{U,F}$ which, in part, determine the college premium $w_{S,F}/w_{U,F}$. Second, given the college premium, the minimization program (13) can adjust the parameters $z_{S,P}$ and $z_{U,P}$ in order to match the proportion of multiple job holders across education groups. This is because $z_{S,P}$ and $z_{U,P}$ affect the part-time wages $w_{S,P}$ and $w_{U,P}$ which, together with the full-time wages, determine the comparative advantage of skilled workers in part-time jobs (see Equation 9). Utility and distribution parameters are informed by the proportion of multiple jobholders in different employment types (e.g. FP, FF, etc.). Finally, the cost of children informs the value of parameter θ_K.

Table 3 reports the model's calibrated parameters and Table 4 presents the model's ability to match the targeted statistics. The model is able to reproduce the ranking of the prevalence of multiple jobholders across education: college-educated workers are more likely to be multiple

[10] See Appendix A, Figure A.4.

Table 3: Calibrated parameters

Preferences	$\sigma_C = +3.20$, $\sigma_L = +1.33$, $\bar{c} = -0.12$
	$\mu_\alpha = +3.05$, $\sigma_\alpha = +0.45$
Technology	$\eta = +0.38$, $\phi_S = +0.38$, $\phi_U = +0.38$
	$z_{S,F} = +0.61$, $z_{S,P} = +0.05$,
	$z_{U,F} = +0.36$, $z_{U,P} = +0.01$,
Worktime	$n_F = +0.36$, $n_P = +0.09$
Children	$k_S = +1.15$, $k_U = +1.22$, $\theta_K = +0.03$

Table 4: Model fit

Moment	Model	Data
Proportion of		
Two jobholders, skilled	6.50%	7.22%
Two jobholders, unskilled	3.92%	4.12%
Two jobholders, FP	3.81%	3.4%
Two jobholders, PP	1.36%	1.3%
Two jobholders, FF	0.20%	0.2%
Earnings relative to one jobholders, skilled		
Two jobholders, skilled	0.92	0.93
Two jobholders, unskilled	0.62	0.67
One jobholders, unskilled	0.68	0.67
College premium	1.47	1.55
Cost of children	0.20	0.20

jobholders because they have a comparative advantage in part-time jobs relative to high school-educated workers–see Section 3.2. Specifically:

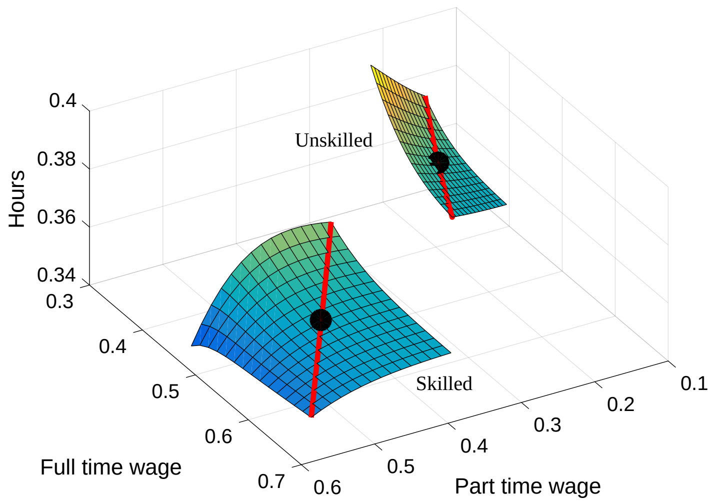
$$\frac{w_{S,P}}{w_{S,F}} = 0.84 > 0.48 = \frac{w_{U,P}}{w_{U,F}}.$$

Figure 10 displays aggregate labor supply, measured in hours, for skilled and unskilled workers. We make several observations. First, focus on the red lines on each surface. Along these lines, $w_{x,F}$ and $w_{x,P}$ change proportionately, maintaining $w_{x,P}/w_{x,F}$ at the appropriate equilibrium value. As workers face increasing wages and a constant relative price of full-time versus part-time jobs, total hours worked decreases. This is because the income effect from an increase in wages dominates and workers seek to reduce their hours by selecting into employment types requiring fewer hours.

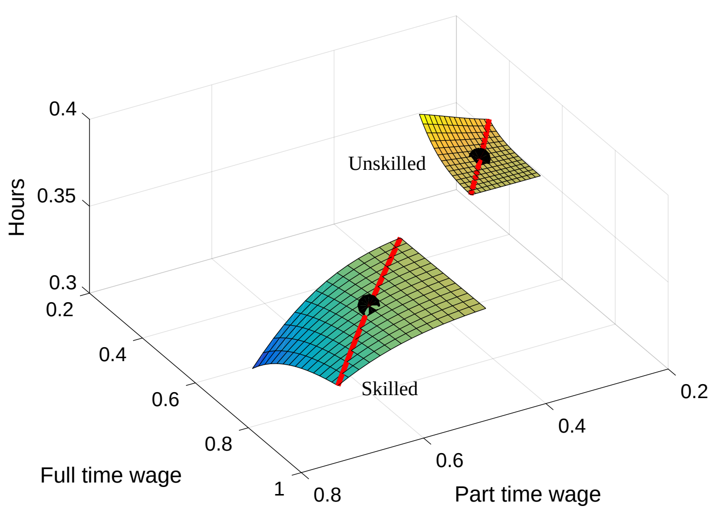
Second we note that labor supply is higher for unskilled workers than for skilled workers, while skilled workers are more likely to hold multiple jobs. This is, again, the result of the income effect: unskilled workers seek to work longer hours because they are paid less on average. They achieve this by selecting into employment types that require long hours, one full-time job, one full-time and one part-time job or two full-time jobs. Skilled workers, on the other hand, seek to work fewer hours, but are enticed to choose multiple jobs by their comparative advantage in part-time jobs. Thus, two part-time jobs are more prevalent among skilled workers. Since hours on two part-time jobs do not add up to the hours of one full-time job, this tends to lower the total hours supplied by skilled workers.

Before turning to several counterfactual experiments, we conduct a few additional calculations. The questions we seek to answer are: what are the directions and strength of the effects of the exogenous variables in our model? We ask this question because, in the experiments we conduct in Section 4.2, we change the exogenous variables in ways that are consistent with data. It is then difficult to assess whether a variable has a "small" effect because the elasticity of the model with respect to this variable is "small," or because the change in the variable is "small" in the data.

We proceed as follows. For each exogenous variable, we consider a 1 percent change relative to calibrated value. Namely, we multiply each exogenous variable by 1.01, leaving the other variables constant. We then compute the relative (in percent) change in the proportion of multiple jobholders for each education group. Table 5 reports the results.



A - Initial equilibrium



B - Final equilibrium

Figure 10: Labor supply in calibrated model

Note: The figure shows the labor supply functions of skilled and unskilled workers, measured in hours. The black dots indicate the equilibrium solution. The red line indicates a constant (at equilibrium value) $w_{x,P}/w_{x,F}$ ratio for $x \in \{S, U\}$.
Source: Authors' calculations.

Table 5: The elasticity of multiple jobholding, percent

Variable	Skilled	Unskilled
$z_{S,F}$	-5.03	-1.93
$z_{S,P}$	1.01	-0.02
$z_{U,F}$	-1.15	-4.07
$z_{U,P}$	-0.00	1.02
μ	1.05	-1.90
k_S	0.59	-0.03
k_U	-0.00	0.89

Note: The table report the percentage change in the fraction of multiple jobholders after a 1 percent increase in a particular exogenous variable.
Source: Authors' calculations.

We make a few observations. First, the model's response is the strongest for the productivity parameters, and weakest for the number of children. Second, an increase in $z_{S,F}$ ($z_{U,F}$) reduces the proportion of skilled (unskilled) multiple jobholders via the mechanism described in Section 3.2, i.e. more workers seek to work one, full-time job when full-time jobs become more productive (see Equation (7)). Second, an increase in the proportion of skilled workers, μ, increases the proportion of skilled multiple jobholders and reduces the proportion of unskilled multiple jobholders. This obtains because, all else equal, when there are more skilled workers and fewer unskilled workers, wages decrease for the skilled and increase for the unskilled. The income effect then induces the skilled to work longer hours (and therefore to take more jobs) and the unskilled to work fewer hours (and therefore to work fewer jobs). Finally, the effect of the number of children is positive (see Section 3.2) but small relative to the effect of productivity.

4.2 Experiments

The calibrated model is now used to conduct the following experiment. Changing a few key parameters, calculate an equilibrium for 2017, and compare the model's predictions to the U.S. data in 2017. How well does the model predict the observed changes in multiple jobholding? For this experiment, three sets of parameters change between 1994 and 2017. First, the number of children increases for the skilled from $k_S = 1.15$ to $k_S = 1.16$ and decreases from $k_U = 1.22$ to $k_U = 1.18$ for the unskilled. Second, the proportion of college-educated workers increases from 56 to 66 percent, thus $\mu_S = 0.66$ in the new equilibrium. Finally, productivity changed. In order to discipline the change in productivity, we impose that productivity growth is the same for workers of a given skill, regardless of whether their employment is full-time of part-time. Denoting the growth rate of productivity for skilled and unskilled as g_S and g_U, respectively, then

$$\begin{aligned} z_{S,j}(\text{new}) &= (1+g_S)z_{S,j}, \quad \text{for } j \in \{F, P\} \\ z_{U,j}(\text{new}) &= (1+g_U)z_{U,j}, \quad \text{for } j \in \{F, P\} \end{aligned}$$

We set g_x to satisfy two conditions. First output is 40 percent higher than in the first equilibrium, which corresponds to a growth rate of 1.4 percent per year between 1994 and 2017: $1.014^{24} = 1.4$. Second, the college premium is 1.66 as revealed by CPS data in 2017.

The first column of Table 6 reports the results of simultaneously changing productivity, fertility and educational attainment. The model predicts a sizable decline in the proportion of two-job holders. For skilled workers the model-predicted decline accounts for 68.7 percent of the observed decline, while for unskilled workers the model accounts for 133.7 percent of the observed decline. That is, the model under-predicts the decline of multiple jobholding among skilled workers by 31.3 percent; and over-predicts the decline by 33.7 percent for unskilled workers.

The model predicts that the prevalence of multiple jobholders remains higher for college-educated workers, (5.26 percent), than for high school-educated workers (2.50 percent). These numbers compare with 5.42 percent and 3.06 percent in the U.S. data, respectively. That is, college-educated workers retained their comparative advantage in part-time jobs over high school-educated workers.

We conduct a few additional experiments. In the first three additional experiments we change only one variable at a time: either we change only the productivity parameters ("Prod. only"), or the number of children ("Fert. only") or educational attainment ("Educ. only"). Columns 2-4 of Table 6 report the change in the proportion of two-job holders in each case.

These experiments offer several interesting observations. First, consistent with the elasticities presented in Table 5, the effect of productivity alone is significant. In the absence of changes

Table 6: Experiments

	All	Prod. only	Fert. only	Educ. only	Prod. + Educ.	Prod. + Fert.	Educ. + Fert.
Skilled	68.7	52.8	-2.0	-75.2	67.9	53.7	-77.9
Unskilled	133.7	23.9	10.6	105.7	128.4	33.2	111.6

Note: The table reports the decline in the proportion of multiple jobholders implied by the model, as a percentage of the actual decline between 1994 and 2017.
Source: Authors' calculations.

in fertility and educational attainment, the model still predicts 52.8 percent of the decline in the prevalence of multiple jobholders for skilled workers and 23.9 percent of the decline for unskilled workers. Second, the decline in the number of children for unskilled workers (from 1.22 to 1.18) accounts, on its own, for 10.6 percent of the decline in the prevalence of multiple jobholders among the unskilled. This leads us to conclude that children are, indeed, a significant determinant in the decision to hold a second job. For skilled workers the number of children actually increased (see Bar et al., 2018) from 1.15 to 1.16, which by itself increases the proportion of multiple jobholders.

Third, education has opposite effects on skilled and unskilled workers. This obtains because as the number of college-educated workers increases, *ceteris paribus*, their wages decrease. In the main experiment, however, the wage of skilled workers increases because of productivity growth. This pushes the prevalence of multiple jobholders downward. Here however, the former effect prevails, and multiple jobholding among college-educated workers increases. For high school-educated workers, the rise in wages is not as pronounced as in the main experiment; as a result, the decline in multiple jobholders is weaker, although still slightly stronger than in the U.S. data.

The magnitudes in the "Productivity-only" and the "Education-only" experiments are large (with opposite signs for skilled vs. unskilled workers in "Education-only"). Given this, we consider a fourth experiment combining both changes in productivity and changes in education, while keeping the number of children constant. This is labeled "Prod.+Educ." in Table 6. This experiment implies results that are close to the main "All" experiment, underlining the important interactions between education and productivity in the model. Similarly, the column labeled "Prod. + Fert." shows the effects of changing both Productivity and Fertility (holding Education fixed), while the column labeled "Educ. + Fert." shows the effects of changing both Education and Fertility (holding Productivity fixed). In general, the strong effects of productivity and education are consistent with the elasticities presented in Table 5.

4.3 Evidence of comparative advantage

In Section 3.2 we concluded that the most educated workers must have a comparative advantage in taking a second job in order to explain the higher prevalence of multiple jobholders among them. In this section, we offer evidence of the presence of such comparative advantage in the data. To do so, we estimate two separate earnings equations for single and multiple jobholders. For each equation we use Heckman's (1976) sample correction to account for the fact that selection into the group of multiple or single jobholders is not random. We then use the earnings equations to compute the average hourly earnings of workers with one or multiple jobs, by education.

The models estimated are

$$\begin{aligned} \ln(e_j) &= \theta X + \epsilon \\ \Pr(\text{number of jobs} = j) &= \Pr(\gamma \tilde{X} + \nu > 0) \end{aligned}$$

where ϵ and ν are jointly normally distributed random variables with mean zero. The variable e_j indicates hourly earnings (all jobs) for workers with one job ($j = 1$) or workers with multiple jobs ($j = m$). The variable X contains the workers age (and the square of age), indicator functions for education, sex and race as well as state, year and occupation fixed effects. The selection equation follows the models we estimated in Table 1. Thus $\tilde{X}$ contains X as well as the number of children and the marital status of the workers.

We estimate the two models ($j = 1, m$) separately using Heckman's two-step estimator. When estimating the model for single (multiple) jobholders, that is $j = 1$ ($j = m$), we consider observations for multiple (single) jobholders to be missing. Figure 11 reports the ratio of mean earnings between workers with multiple jobs and workers with one job, by education. The first lesson from Figure 11 is that the earnings ratio is increasing with education. This indicates that workers with the highest education have the most to gain from working multiple jobs. Importantly, this provides evidence of the comparative advantage mechanism that our model implies is necessary to explain the higher prevalence of multiple

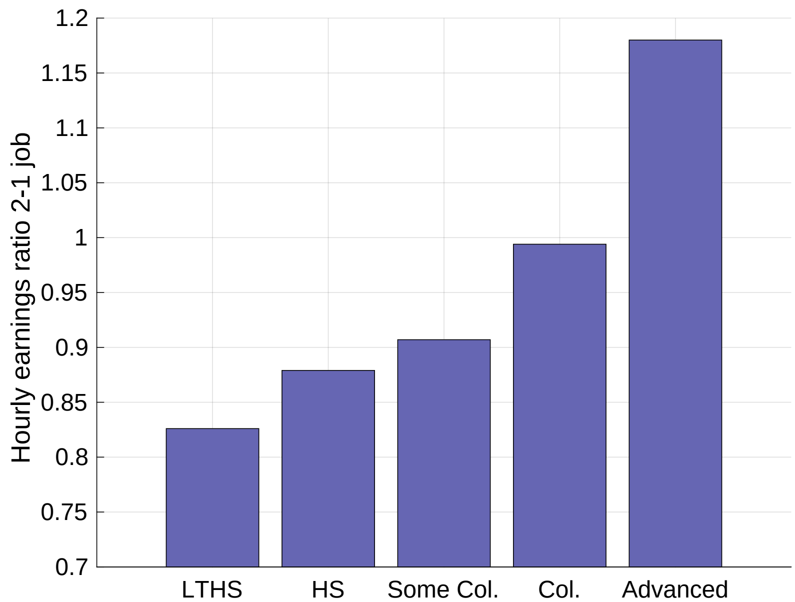


Figure 11: Hourly earnings difference between 1- and 2-jobholders

Source: Current Population Survey and authors' calculations jobholding among college-educated workers.

The second message from Figure 11 is that the hourly earnings of multiple jobholders tend to be lower than that of single jobholders. Since a large fraction of multiple jobholders work one full- and one part-time job, this suggests that part-time wages are in general lower than full-time wages. Note the exception for college-educated workers, though. For these workers the second job does not reduce (college) and may even increase the hourly earnings (advanced).

5 Conclusion

Since the mid 1990s the proportion of multiple jobholders, conditional on education, decreases when productivity increases, both in the time series and in the cross-section. It is, however, increasing with education in the cross-section. These features remain after controlling for demographic and other economic variables. To explain these two seemingly contradictory facts, we develop an equilibrium model of the labor market where workers are heterogeneous in their preference for leisure, as well as in education. Workers adjust labor supply at the extensive margin only, deciding between various combinations of full-time and part-time jobs.

A version of the model with only two types of employment is analyzed to illustrate the key mechanisms. First, an income effect explains the negative correlation of multiple jobholding with productivity both in the time series and in the cross-section. That is, as workers become more productive they seek to increase their leisure time. This is achieved by foregoing the second job opportunity. Second, the higher prevalence of multiple jobholding among college-educated requires a comparative advantage effect. That is, skilled workers are relatively more productive in part-time jobs compared to unskilled workers.

The model is calibrated to U.S. data in 1994 and provides insights into what factors explain the aforementioned facts. Specifically, the model accounts for 68.7% of the decrease in multiple jobholdings for college-educated workers, and over-predicts it by 33.7 percent for high school-educated workers. We found that the role of productivity and education are, quantitatively, the most important. Even though the changing number of children has a non-negligible effect, it remains a second-order contributor.

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A Data appendix

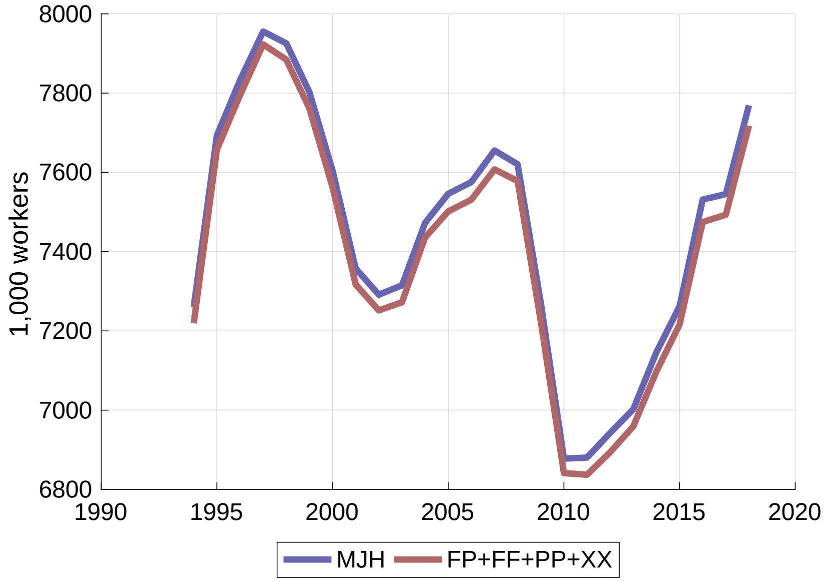

Figure A.1: Number of workers with two jobs

Note: MJH reports the number of multiple jobholders. The term FP+FF+PP+XX refer to the sum of people holding one full-time and one part-time job (FP), two full-time jobs (FF), two part-time jobs (PP), or two jobs with variable hours on either the primary or the secondary job (XX). The difference between the two lines indicates the number of workers with more than two jobs.
Source: Bureau of labor statistics.

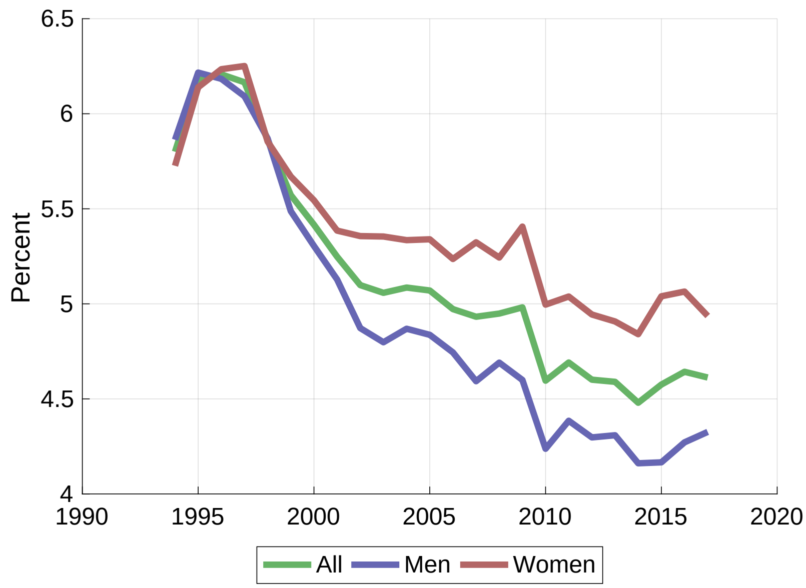

Figure A.2: Proportion of employees with multiple jobs

Source: Current Population Survey.

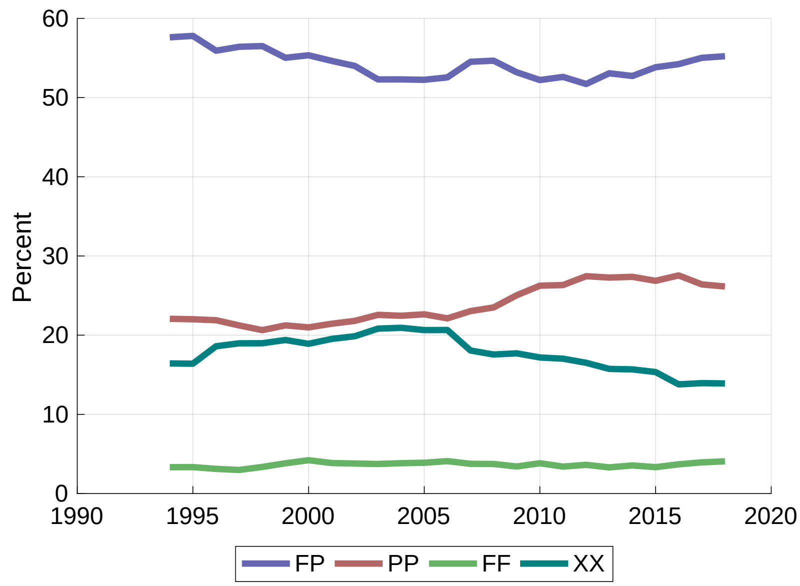

Figure A.3: Proportion of multiple jobholders by type of jobs

Note: "FP" means one full-time and one part-time job, "FF" means two full-time jobs (FF), "PP" means two part-time jobs, and "XX" mean that hours vary either on the primary or the secondary job.
Source: Bureau of labor statistics.

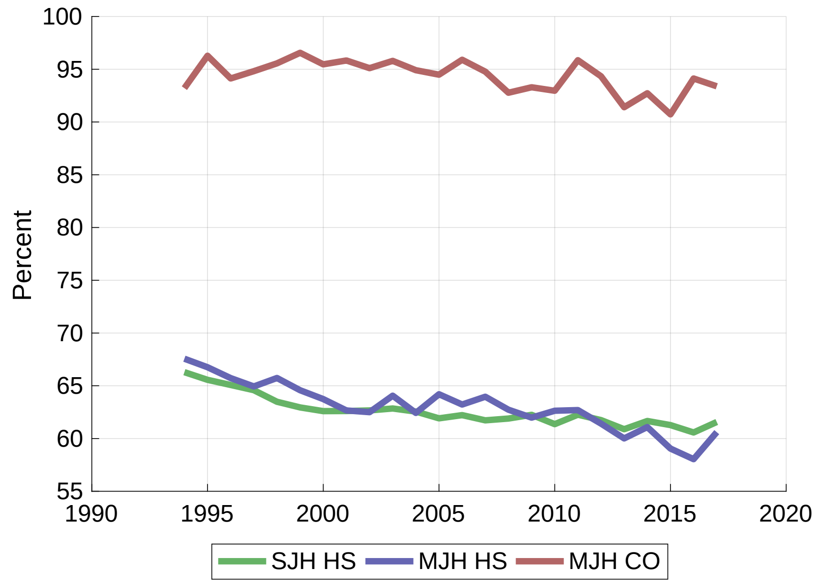

Figure A.4: Hourly earnings relative to 1-job holder, college educated

Note: "SJH" means single-job holder and "MJH" means multiple jobholders. "HS" means at most high school graduate and "CO" means some college education.
Source: Current Population Survey.

Table 7: Probit Model Multiple Jobholding Dependent Variable

Variables	(1) Model 1F	(2) Model 1M	(3) Model 1A	(4) Model 2F	(5) Model 2M	(6) Model 3F	(7) Model 3M
Education (Less than HS reference group)							
HS	1.179***	1.259***	1.217***	1.182***	1.258***	1.185***	1.260***
	(0.0178)	(0.0175)	(0.0131)	(0.0177)	(0.0174)	(0.0187)	(0.0174)
Some college	1.407***	1.499***	1.453***	1.411***	1.498***	1.407***	1.464***
	(0.0217)	(0.0236)	(0.0171)	(0.0217)	(0.0236)	(0.0235)	(0.0203)
College	1.456***	1.544***	1.501***	1.465***	1.543***	1.432***	1.469***
	(0.0272)	(0.0277)	(0.0210)	(0.0273)	(0.0277)	(0.0303)	(0.0253)
Advanced	1.646***	1.755***	1.703***	1.668***	1.755***	1.598***	1.609***
	(0.0304)	(0.0358)	(0.0280)	(0.0308)	(0.0358)	(0.0311)	(0.0272)
Number of children	0.976***	1.026***	1.004**				
	(0.00227)	(0.00294)	(0.00220)				
Part-time in usual job (full-time reference group)							
Part-time, <20 hrs	1.579***	1.537***	1.551***	1.592***	1.537***	1.575***	1.506***
	(0.0168)	(0.0233)	(0.0168)	(0.0168)	(0.0232)	(0.0165)	(0.0258)
Part-time, 21-34 hrs	1.340***	1.373***	1.346***	1.346***	1.373***	1.334***	1.364***
	(0.0130)	(0.0160)	(0.0127)	(0.0130)	(0.0159)	(0.0124)	(0.0184)
Hours vary on usual job	0.834***	0.842***	0.837***	0.833***	0.842***	0.823***	0.851***
	(0.00948)	(0.0111)	(0.00908)	(0.00950)	(0.0111)	(0.0104)	(0.0115)
Age of Children (No children under 18 reference group)							
Child 0-2				0.820***	1.028***	0.808***	1.023***
				(0.00594)	(0.00751)	(0.00653)	(0.00846)
Child 3-5				0.927***	1.038***	0.923***	1.036***
				(0.00642)	(0.00861)	(0.00690)	(0.00850)
Child 6-13				0.948***	1.020***	0.943***	1.015**
				(0.00625)	(0.00522)	(0.00604)	(0.00594)
Child 14-17				1.068***	1.062***	1.060***	1.066***
				(0.00619)	(0.00831)	(0.00574)	(0.00882)
Female (Y/N)			0.885***				
			(0.00625)				
Real wage, 2017 $s	0.999***	0.997***	0.998***	0.999***	0.997***	0.999***	0.998***
	(0.000339)	(0.000293)	(0.000286)	(0.000337)	(0.000293)	(0.000337)	(0.000265)
Married	0.829***	1.038***	0.894***	0.835***	1.040***	0.834***	1.052***
	(0.00544)	(0.0129)	(0.00581)	(0.00558)	(0.0130)	(0.00640)	(0.0147)
Age (Older than 60 reference group)							
Age 20-29	1.092***	1.072***	1.066***	1.181***	1.074***	1.177***	1.067***
	(0.0162)	(0.0138)	(0.0120)	(0.0174)	(0.0134)	(0.0192)	(0.0149)
Age 30-39	1.177***	1.111***	1.133***	1.223***	1.116***	1.217***	1.111***
	(0.0133)	(0.0135)	(0.0102)	(0.0139)	(0.0135)	(0.0145)	(0.0132)
Age 40-49	1.278***	1.155***	1.210***	1.253***	1.154***	1.245***	1.152***
	(0.0122)	(0.0110)	(0.00838)	(0.0113)	(0.0112)	(0.0119)	(0.0118)
Age 50-59	1.226***	1.157***	1.187***	1.214***	1.154***	1.211***	1.158***
	(0.0130)	(0.0122)	(0.00834)	(0.0126)	(0.0123)	(0.0128)	(0.0128)
Race (White reference group)							
Black	0.937***	1.020	0.976	0.938***	1.021	0.935***	1.003
	(0.0155)	(0.0189)	(0.0148)	(0.0157)	(0.0190)	(0.0156)	(0.0180)
Hispanic	0.847***	0.894***	0.873***	0.847***	0.894***	0.849***	0.881***
	(0.0162)	(0.0123)	(0.0118)	(0.0163)	(0.0124)	(0.0168)	(0.0125)
Other	0.883***	0.814***	0.848***	0.886***	0.813***	0.888***	0.795***
	(0.0178)	(0.0235)	(0.0183)	(0.0181)	(0.0235)	(0.0179)	(0.0243)
Constant	0.165***	0.151***	0.176***	0.162***	0.151***	0.148***	0.140***
	(0.00374)	(0.00358)	(0.00325)	(0.00369)	(0.00357)	(0.00399)	(0.00314)
Observations	1,018,409	978,118	1,996,527	1,018,409	978,118	846,628	811,143
State FE	YES	YES	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES	YES	YES
Occupation FE						YES	YES

*** p<0.01, ** p<0.05, * p<0.1
Odds ratios presented. Robust z statistics in parentheses. Standard errors clustered at the state level.

B Income and substitution effects

In this section we consider a simple consumption-leisure problem along the intensive margin of labor supply:

$$\max_{c,\ell} \{U(c) + V(\ell) : c = w(1-\ell)\}.$$

We show that the income effect from a change in w dominates whenever $U'(c)c$ is a decreasing function, and the substitution effect dominates whenever $U'(c)c$ is increasing. When $U(c) = \ln(c)$ the function $U'(c)c$ is a constant (equal to 1) and the two effects offset each others. In the model of Section 3 labor supply adjustments do not operate along an intensive margin but along an extensive margin. The logic of income and substitution effects is the same, however.

The first-order condition for ℓ is $U'(c)w = V'(\ell)$. Combining this expression with the budget constraint implies that the optimal choice of leisure is implicitly defined by

$$U'(c)c = V'(\ell)(1-\ell).$$

The right-hand side of this equation is a decreasing function of ℓ. When w rises consumption increases. If $U'(c)c$ is decreasing, leisure must increase, i.e. the substitution effects dominates. If $U'(c)c$ is increasing, leisure must decrease, i.e. the substitution effects dominates. Finally, if $U'(c)c$ is a constant (as it is with log utility), leisure is invariant to changes in w.

In Section 3.2 we refer to $U'(cz)z$ being a decreasing (increasing) function of z as implying that the income (substitution) effect dominates. Note that

$$\frac{\partial}{\partial z}U'(cz)z = U''(cz)cz + U'(cz) \text{ and } \frac{\partial}{\partial c}U'(c)c = U''(c)c + U'(c),$$

therefore

$$\text{sign}\left[\frac{\partial}{\partial z}U'(cz)z\right] = \text{sign}\left[\frac{\partial}{\partial c}U'(c)c\right].$$

Hence, the statement that $U'(cz)z$ is decreasing in z is equivalent to the statement $U'(c)c$ is decreasing in c.