ECON 615: Final Exam

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Instructions:

- You have three hours to complete the exam.
- Read all questions carefully before attempting to answer. If you make any additional assumptions you think are necessary, state clearly what assumptions you are making.
- If you get stuck in the algebra derivations, remember you can get partial marks for explaining the equilibrium concept, the strategy for finding a solution, the results you expect to find, and the intuition for expecting these results.
- Please write **legibly**.

1. This question examines the Robinson Crusoe economy with variable labour. Specifically, Crusoe produces the consumption good using capital and labour according to the production function:

$$y_t = f(k_t, h_t)$$

Capital depreciates at rate δ. Crusoe has preferences over consumption given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

where $u_c(c_t, h_t) > 0$ and $u_h(c_t, h_t) < 0$. Each period, from output and un-depreciated capital, Crusoe decides how much to consume, and how much capital to invest for next period (This is exactly the version we studied in class). Answer the following questions regarding this model.

(a) Write down the Bellman equation determining the solution to this problem. What are the state and control variables?

(b) Find the Euler equations from this recursive formulation and interpret them.

Now suppose we modify Crusoe's problem as follows. Specifically, Crusoe now accumulates *human capital*, denoted by n_t. Production is now given by $y_t = f(k_t, n_t h_t)$. Human capital grows according to:

$$n_{t+1} = (1 - \gamma)n_t + \kappa s_t$$

where $\gamma > 0$ is the depreciation of human capital and $\kappa > 0$. s_t is time spent studying. Crusoe has an endowment of one unit of time and spends it according to the constraint:

$$1 = l_t + h_t + s_t$$

where l_t is leisure and Crusoe's utility function is given by $u(c_t, l_t) = \ln(c_t) + A\ln(l_t)$. Capital works as above.

(c) Write down the recursive formulation for this problem. What are the state and control variables?

(d) Find the Euler equations in this case.

(e) Intuitively, how have Crusoe's trade-offs changed?

2. Consider the CIA model from class. Recall, we use the Hansen model with indivisible labor, so that preferences are given by:

$$u(c_t^i, h_t^i) = \ln(c_t^i) + Bh_t^i$$

The production function is given by

$$y_t = \lambda_t K_t^\theta H_t^{1-\theta}$$

Here we assume that λ_t is constant, following $\lambda_t = 1, \forall t$. Furthermore, we also assume that the money growth rate remains fixed; i.e. $g_t = \bar{g}, \forall t$.

Finally, recall that in this model we assume that the agent receives a transfer from the government each period of $(\bar{g}-1)M_{t-1}$. The agent sells labor to the firm at rate w_t and rents capital at the rate r_t. Given this, the household faces two constraints:

$$\hat{p}_t c_t^i \leq \frac{\hat{m}_{t-1}^i + \bar{g} - 1}{\bar{g}} \tag{1}$$

$$c_t^i + \frac{\hat{m}_t^i}{\hat{p}_t} + k_{t+1}^i = w_t h_t^i + r_t k_t^i + (1-\delta)k_t^i + \frac{\hat{m}_{t-1}^i + \bar{g} - 1}{\hat{p}_t \bar{g}} \tag{2}$$

Answer the following questions regarding this model.

(a) Show that the household's Euler equations can be written as:

$$\frac{1}{\beta} = \left(\frac{w_t}{w_{t+1}}\left[(1-\delta) + r_{t+1}\right]\right) \tag{3}$$

$$\frac{B}{w_t \hat{p}_t} = -\beta\left(\frac{1}{\hat{p}_{t+1} c_{t+1}^i \bar{g}}\right) \tag{4}$$

where $\hat{p}_t = \frac{p_t}{M_t}$.

(b) Write the problem down recursively (i.e. write down the Bellman equation). What are the state and control variables?

(c) Using this recursive formulation, show that the household's Euler equations are given by equations (3) and (4)

3. This question explores a simple OLG model, similar to the one studied in class. The population size is fixed; that is, each period one new young agent is born, and an old agent dies. When young, each agent has an endowment of y_1 goods. When old, the agent receives an endowment of y_2 goods, with $y_1 > y_2$. Each initial old agent is endowed with m units of money, so the total money supply $M = N_0 m$ is fixed. Let p_t denoted the price of money in period t. To be explicit, each agent has preferences given by:

$$U(c_t^y, c_{t+1}^o) = \ln(c_t^y) + \ln(c_{t+1}^o)$$

and faces the consraints:

$$c_t^y + p_t m_t = y_1$$

$$c_{t+1}^o = y_2 + p_{t+1} m_t$$

Answer the following questions regarding this model.

(a) Define a competitive equilibrium in this economy.

(b) Intuitively, how should the equilibrium price change over time? Is it growing, shrinking, or constant? Explain.

(c) Find the competitive equilibrium allocations for c_t^y and c_{t+1}^o.

(d) Now consider the social planner's problem in this economy. Find the planner's allocation of c_t^y and c_t^o. Is the competitive equilibrium Pareto optimal?

(e) Now suppose that $y_2 > y_1$. What are the competitive equilibrium allocations in this case? Is it Pareto optimal?

(f) Does money serve any role in this economy? Explain the friction that $y_2 > y_1$ creates? What exchange does a young agent alive in period t want to make? What about an old agent? Can they make these trades?