ECON 615: Final Exam

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Wednesday, December 8, 2010

Instructions:

- You have three hours to complete the exam.
- The exam is worth a total of 135 points. The weight on each question is given in parenthesis.
- Read all questions carefully before attempting to answer. If you make any additional assumptions you think are necessary, state clearly what assumptions you are making.
- If you get stuck in the algebra derivations, remember you can get partial marks for explaining the equilibrium concept, the strategy for finding a solution, the results you expect to find, and the intuition for expecting these results.
- Please write **legibly**.

1. This question examines a simple Robinson Crusoe economy. Specifically, Crusoe produces the consumption good using only capital (i.e. labor is inelastically supplied) according to the production function:

$$y_t = f(k_t)$$

Capital depreciates at rate δ. Crusoe has preferences over consumption given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

Each period, from output and un-depreciated capital, Crusoe decides how much to consume, and how much capital to invest for next period (This is exactly the basic Crusoe model we studied in class). Answer the following questions regarding this model.

(a) (10 points) Write down the Bellman equation determining the solution to this problem. What are the state and control variables?

(b) (10 points) Find the Euler equation(s) from this recursive formulation.

(c) (10 points) Find an equation determining the steady state level of capital, $\bar{k}$.

2. Consider the CIA model from class. Recall, we use the Hansen model with indivisible labor, so that preferences are given by:

$$u(c_t^i, h_t^i) = \ln(c_t^i) + Bh_t^i$$

The production function is given by

$$y_t = \lambda_t K_t^\theta H_t^{1-\theta}$$

Here we assume that λ_t is stochastic, and follows the process $\ln(\lambda_{t+1}) = \gamma \ln(\lambda_t) + \varepsilon_{t+1}$. Furthermore, we also assume that the money growth rate remains stochastic, following the process $\ln(g_{t+1}) = (1-\pi)\bar{g} + \pi \ln(g_t) + \varepsilon_t^g$.

Finally, recall that in this model we assume that the agent receives a transfer from the government each period of $(g_t - 1)M_{t-1}$. The agent sells labor to the firm at rate w_t and rents capital at the rate r_t. Answer the following questions regarding this model.

(a) (5 points) Write down the CIA constraint and the household's budget constraint.

(b) (15 points) Show that the household's Euler equations can be written as:

$$\frac{1}{\beta} = E_t\left(\frac{w_t}{w_{t+1}}\left[(1-\delta) + r_{t+1}\right]\right) \tag{1}$$

$$\frac{B}{w_t \hat{p}_t} = -\beta E_t\left(\frac{1}{\hat{p}_{t+1} c_{t+1}^i g_{t+1}}\right) \tag{2}$$

where $\hat{p}_t = \frac{p_t}{M_t}$.

(c) (10 points) Show that equation (1) can be log-linearized to:

$$\tilde{w}_t \approx \beta E_t\left[\bar{r}(\tilde{r}_{t+1} - \tilde{w}_{t+1}) - (1-\delta)\tilde{w}_{t+1}\right]$$

(d) (10 points) Write the problem down recursively (i.e. write down the Bellman equation). What are the state and control variables?

(e) (10 points) Now suppose that instead of a transfer of $(g_t - 1)M_{t-1}$, the government transfers $(g_t - 1)m^i_{t-1}$; that is, the transfer now depends on the household's previous money holdings, not the entire previous money supply. Explain intuitively how this affects the problem. Does it change the problem?

3. Consider the following overlapping generations economy. Each period, there are N_t young agents born, who live for two period ("young" and "old"). The population evolves according to:

$$N_t = nN_{t-1}$$

Agents have preferences over consumption given by:

$$u(c^y_t, c^o_{t+1}) = \ln(c^y_t) + \ln(c^0_{t+1})$$

In this economy, agents own capital and supply labor to firms, who produce according to the production function

$$Y_t = K_t^{\theta} N^{1-\theta}$$

where K_t is the aggregate capital stock. There is no depreciation (i.e. $\delta = 1$). At time $t = 0$, there are initial old agents, collectively endowed with K_0 units of capital (this is the total capital stock in period 0). Finally, each young agent is endowed with 1 unit of time, which is *inelastically* supplied as labor. When old the agent has no endowment of time. Answer the following questions regarding this model.

(a) (5 points) Define $k_t = \frac{K_t}{N_t}$; that is k_t is the per-capita capital stock. Further, notice that the economy faces the following resource constraint:

$$Y_t + K_t = K_{t+1} + N_t c^y_t + N_{t-1} c^o_t$$

Re-write this constraint in terms of per-capita capital.

(b) (15 points) Solve the social planner's problem for this economy, and determine the optimal steady state value of per-capita capital, $k_t = k_{t+1} = \bar{k}$.

(c) (15 points) Now consider the decentralized economy, where households sell their labor to firms at rate w_t and rent capital at rate r_t. Recall that each young agent has 1 unit of time which is inelastically supplied as labor. Let s_t denote the savings (in capital) of a young agent born in period t. This agent faces the following two budget constraints:

$$c_t^y + s_t = w_t$$
$$c_{t+1}^o = (1 + r_{t+1})s_t$$

Determine the optimal choice of s_t for the agent.

(d) (10 points) Determine the equilibrium steady state value of per-capita capital, $k_t = k_{t+1} = \hat{k}$.

(e) (10 points) If $n > 1$, does this match the Pareto optimal allocation? i.e. is $\bar{k} = \hat{k}$? If no, provide some intuition for why the competitive equilibrium is not Pareto optimal.