

# Productivity Insurance: The Role of Unemployment Benefits in a Multi-Sector Model\*

David L. Fuller<sup>†</sup>

University of Wisconsin-Oshkosh and CIREQ

Marianna Kudlyak<sup>‡</sup>

The Federal Reserve Bank of Richmond

Damba Lkhagvasuren<sup>§</sup>

Concordia University, CIREQ and Mongolian National University

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## Abstract

We construct a multi-sector search and matching model where the unemployed receive idiosyncratic productivity shocks that make working in certain sectors more productive than in the others. Agents must decide which sector to search in and face moving costs when leaving their current sector for another. In this environment, unemployment is associated with an additional risk: low future wages if mobility costs preclude search in the appropriate sector. This introduces a new role for unemployment benefits – productivity insurance while unemployed. For plausible parameterizations unemployment benefits increase per-worker productivity. In addition, the welfare-maximizing benefit level decreases as moving costs increase.

**Keywords:** unemployment insurance, search, mobility, productivity, multi-sector model, sectoral mismatch

**JEL classification:** J62, J63, J64, J65

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<sup>†</sup>Department of Economics, University of Wisconsin-Oshkosh, 800 Algoma Blvd., Oshkosh, WI, 54901; E-mail: [fullerd@uwosh.edu](mailto:fullerd@uwosh.edu).

<sup>‡</sup>Research Department, The Federal Reserve Bank of Richmond, 701 E Byrd St., Richmond, VA, 23219. E-mail: [marianna.kudlyak@rich.frb.org](mailto:marianna.kudlyak@rich.frb.org).

<sup>§</sup>Department of Economics, Concordia University, 1455 de Maisonneuve Blvd. West, Montreal, Quebec, H3G 1M8 Canada; E-mail: [damba.lkhagvasuren@concordia.ca](mailto:damba.lkhagvasuren@concordia.ca).

# 1 Introduction

The existing literature on the provision of unemployment insurance (hereafter UI) has focused on the role of UI in smoothing consumption between employment states. This role implies a trade-off between insurance and incentives (e.g., [Hopenhayn and Nicolini, 1997](#)). More insurance implies reduced output as the duration and incidence of unemployment increases. The relationship between UI and output, however, may change if one recognizes that UI also encourages unemployed workers to seek higher productivity jobs. Allowing the composition of jobs to respond endogenously, [Acemoglu and Shimer \(1999, 2000\)](#) show that UI benefits encourage firms to create higher productivity jobs, which in turn might lead to an increase in aggregate output.

In this paper, we introduce an additional role for unemployment benefits: insurance against idiosyncratic sector-specific productivity shocks while unemployed. Similarly to [Acemoglu and Shimer \(1999, 2000\)](#) and [Marimon and Zilibotti \(1999\)](#), we allow the composition of jobs to be endogenously determined. In particular, we consider an environment in which, upon becoming unemployed, individuals are subject to idiosyncratic shocks that render their current skills less suitable for their most recent sector of employment. If a move from one sector to another is costless, such a shock poses no additional risk to the unemployed; they simply move to the most productive sector for their particular skills. If, however, the move requires paying moving costs, then unemployment poses a risk to future wage prospects if unemployed workers search in the relatively less productive sector. In such an environment, unemployment benefits may help insure individuals against this risk by effectively reducing the costs of moving.

Specifically, we analyze a directed search model with matching frictions, multiple sectors, and risk-averse agents. An unemployed agent receives an idiosyncratic productivity shock that makes the agent more productive in one sector relative to the other sectors. The unemployed agent must decide which sector to search in, and she faces moving costs when leaving her current sector for another. Moving costs take two forms: a direct utility cost and

a time cost, as changing sectors involves an additional period of unemployment. Mobility between sectors is directed. Workers know their productivity in another sector before leaving their current sector as in [Roy \(1951\)](#). In each sector, firms post wages and agents direct their search to a specific job as in [Moen \(1997\)](#). Therefore, the model represents a blend of a dynamic Roy model and a competitive search model.

In contrast to standard sectoral reallocation theory (e.g., [Lucas and Prescott, 1974](#)), the model allows for explicit distinction between inter-sectoral mobility and within-sector trading frictions. Such distinction is essential for examining the link between UI and sectoral mobility.<sup>1</sup>

We first analyze how unemployment benefits affect equilibrium outcomes, focusing on the productivity effects. We show that the mobility decision is characterized by a reservation rule for productivity shocks. For idiosyncratic shocks above the reservation value the agent moves sectors, and remains in the current sector for shocks below the reservation value. This feature implies that the effect of benefits on productivity depends on how changes in benefits affect the reservation value. We illustrate two main effects on the reservation value, one that has a negative effect on productivity and one that has a positive effect.

The negative effect on productivity acts as a “moral hazard” effect. It occurs because the benefit acts as a subsidy to search. Increasing benefits increases the value of unemployment, reducing the gain from moving to higher productivity sectors. Agents require larger idiosyncratic shocks to be willing to move sectors. This effect increases the reservation productivity, decreasing per-worker productivity. This effect resembles the moral hazard effects in a McCall search model ([McCall, 1970](#)): a higher value of unemployment implies workers become more selective, resulting in longer unemployment durations. The difference between the effect in [McCall \(1970\)](#) and in our model is that in our multi-sector model, the increased selectiveness of when to switch sectors implies a decrease in the productivity of matches that do occur. In contrast, more selective workers in the McCall search model implies higher

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<sup>1</sup>[Lkhagvasuren \(2012\)](#) shows that the interaction of between-sector mobility and within-sector trading frictions might be key to accounting for the negative correlation between unemployment and sectoral mobility.

wages/productivity once employed.

An increase in unemployment benefits may also decrease the reservation productivity. Higher UI benefits help the worker incur the cost associated with mobility. Specifically, an increase in UI benefits reduces the gap between the lifetime utility of moving and the lifetime utility of remaining in the current sector, for all values of the idiosyncratic productivity shock. This decreases the reservation productivity, increasing mobility and thus per-worker productivity.

The combined effect of benefits on productivity remains ambiguous. To quantitatively illustrate which effect dominates, we calibrate the model to the U.S. economy. We find that the positive effect dominates, so that increasing benefits increases per-worker productivity as workers move more frequently in response to idiosyncratic productivity shocks. Unemployment increases, while vacancies decrease. Quantitatively, we find that a 25% increase in the benefit level increases per-worker productivity by 0.1%.

We also analyze the welfare-maximizing unemployment benefit level and find that it depends on the size of the moving cost. As moving costs increase, the optimal benefit decreases. Determining the optimal benefit requires managing the aforementioned trade-off between the positive and negative productivity effects, in addition to the traditional within sector trade-offs. As the moving cost increases, the negative productivity effect becomes stronger, putting downward pressure on the optimal benefit level. We find that in response to a 1 percent increase in the moving cost, the optimal benefit decreases by 6 percent. Thus, while the magnitude of the productivity effects is relatively small, the overall role of UI as productivity insurance may be quite important.

The productivity results in our model relate to the efficient UI literature, most notably, [Acemoglu and Shimer \(1999, 2000\)](#) and [Marimon and Zilibotti \(1999\)](#). Our model is about the risks posed by sectoral/skill mismatch. Inefficient matches are formed if workers continue to search in the “wrong” sector. In this sense, [Marimon and Zilibotti \(1999\)](#) represents the closest work to ours, as they also examine the issue of mismatch to analyze differences in

U.S. and European unemployment and productivity outcomes. The productivity effects in our model work through encouraging costly mobility. In contrast, the productivity effects in [Marimon and Zilibotti \(1999\)](#) work similarly to [McCall \(1970\)](#); workers have a higher reservation “wage” and are willing to wait longer to find a better match. In [Marimon and Zilibotti \(1999\)](#) an increase in UI benefits necessarily implies an increase in the productivity of the matches that do form. In our model, however, this is not necessarily true, as discussed above with regards to [McCall \(1970\)](#).

In our model, the impact of UI benefits on productivity is theoretically ambiguous; however, for plausible parameterizations, unemployment benefits increase per-worker productivity. Another important difference is that in our model workers direct their search to more suitable jobs by incurring an explicit search or moving cost, whereas, in [Marimon and Zilibotti \(1999\)](#), search is undirected in the absence of such a cost. In their model selection occurs as workers can reject randomly matched poor quality jobs. As indicated earlier, the combination of directed search and the explicit search or moving cost allows us to analyze the interdependence of the search cost, the optimal level of benefits, and endogenous productivity.

[Acemoglu and Shimer \(1999, 2000\)](#) also show that higher UI benefits can increase per-worker productivity. The important distinction is that they do not analyze mismatch. Rather, in their model, higher productivity jobs are more costly for firms to create, and since they arrive less frequently, more costly for workers to direct their search towards. Workers, however, are ex-ante homogeneous, and thus there is no mismatch. UI benefits increase productivity by encouraging workers to search for the “riskier” high wage jobs.

No such economy wide trade-off for creating higher productivity jobs exists in our model. Provided the worker searches in the appropriate sector, the high wage job arrives faster than the lower wage job in the previous sector. That is, a worker searching for a job more suitable for their particular skills is more likely to find a job than if they search for less suitable jobs. The worker may still incur a longer duration, however, as changing sectors involves an

additional period of unemployment.

Finally, the productivity effects in both [Acemoglu and Shimer \(1999, 2000\)](#) and [Marimon and Zilibotti \(1999\)](#) work by convincing workers to wait longer for better opportunities. In our model, the productivity effects work by increasing mobility.

The remainder of the paper is as follows. [Section 2](#) describes the environment and agent decisions. [Section 3](#) characterizes the equilibrium of the model economy. [Section 4](#) discusses the productivity effects of benefits, and [Section 5](#) presents our quantitative analysis. We conclude in [Section 6](#). Appendices contain further proofs and model details.

## 2 Model

This section describes the model environment and equilibrium definition.

### 2.1 Environment

The economy is composed of two sectors, denoted by 0 and 1, populated by a measure one of risk-averse workers and a continuum of risk-neutral firms. Individuals are either employed or unemployed. Employed workers are matched with a firm. In each period an unemployed worker chooses to search in the current sector or move and search in the other sector. While employed, workers do not engage in on-the-job search. Therefore, every mover is unemployed, while not all unemployed workers are movers.

Each period, firms search for workers by creating vacancies. The flow cost of a vacancy is  $k$ . Free entry drives the expected present value of an open vacancy to zero. Vacant jobs and unemployed workers are matched according to a matching technology. Without loss of generality, we assume that each firm employs at most one worker. All matches are dissolved exogenously with probability  $\lambda$ .

Let  $b$  denote per-period income of an unemployed worker. Flow utility of an unemployed worker searching for a job within his own sector is  $\log(b)$ , while that of an unemployed worker

moving across sectors is  $\log(b) - c$ , where  $c > 0$  is the utility cost of moving.<sup>2</sup> Moving between two sectors involves an additional cost: it takes one period. The flow utility of a worker is  $\log(w)$ , where  $w$  denotes the wage. Workers and firms discount the future at the same rate,  $\beta$ .

## 2.2 Production Technology

Let  $y_i(x)$  be the production function describing per-period output produced by a firm that employs a worker with productivity  $x$  and operates in sector  $i \in \{0, 1\}$ . We assume that

$$y_0(x) = 1 - x \tag{1}$$

and

$$y_1(x) = 1 + x. \tag{2}$$

Equations (1) and (2) imply that individual productivity is perfectly negatively correlated across sectors: the best workers of sector 0 are the worst workers of sector 1.<sup>3</sup>

We present the model in terms of two sectors,  $i \in \{0, 1\}$ . The model, however, can be generalized to an economy with  $N$  sectors by interpreting  $y_i(x)$  as the agent's productivity shock in the current sector, and  $y_{1-i}(x)$  as the highest of the  $N - 1$  productivity shocks from the remaining  $N - 1$  sectors.

By construction, idiosyncratic productivity does not change within a given match. If an employed worker becomes unemployed, she draws her new productivity from the uniform

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<sup>2</sup>Consistent with the literature on sectoral reallocation (e.g. [Lucas and Prescott \(1974\)](#)), we have focused on the time cost and modelled the utility cost of moving as separable, which may be interpreted as a leisure cost. An alternative would be to assume moving involves a direct consumption cost, i.e.  $\log(b - c)$ . We discuss this alternative further in Section 6.

<sup>3</sup>See [Moscarini \(2001\)](#) and [Lkhagvasuren \(2014\)](#) for related dynamic extensions of [Roy's \(1951\)](#) framework. One can consider labor income shocks that are not perfectly negatively correlated across sectors. For example, suppose that  $e_0$  and  $e_1$  are productivity of a worker in sectors 0 and 1, respectively. Further suppose that these two shocks are not perfectly negatively correlated; i.e.,  $\text{Corr}(e_0, e_1) > -1$ . Then, consider the following decomposition:  $e_0 = z + x$  and  $e_1 = z - x$ , where  $z$  and  $x$  are uncorrelated shocks and  $z \geq 1$ . Under such specifications, although the mobility decision is affected by both  $x$  and  $z$ , mobility affects only the sector-specific component  $x$  (or  $-x$ ). Thus, the main equilibrium effect emphasized in the paper remains the same.

distribution on the interval  $[-\omega, \omega]$ .<sup>4</sup> For relatively high values of the productivity shock  $x$ , the unemployed worker prefers to search in the current sector; for relatively low values of  $x$ , the worker prefers to move and search in another sector. We also assume that  $0 < b < 1 - \omega$ . If upon unemployment the worker decides to search in another sector, she incurs the moving cost  $c$ .

## 2.3 Wages

We assume that wages are determined through competitive search, as in Moen (1997) and Rogerson, Shimer and Wright (2005). A firm decides whether or not to post a vacancy. A vacancy is fully characterized by the productivity level,  $x$ , the wage,  $w$ , and the sector,  $i$ . If a firm decides to post a vacancy, it chooses these three variables in order to maximize its expected profits. An unemployed worker directs her search towards the most attractive job. Let  $\mathbb{W}_i(x)$  denote the set of wages posted at the productivity level  $x$  in sector  $i$ .

## 2.4 Matching Technology

Let  $n_\tau$  denote the number of unemployed workers searching for a job of type  $\tau = (w, x, i)$ , and  $v_\tau$  denote the number of vacant type  $\tau$  jobs. The total number of type  $\tau$  matches is given by

$$M_\tau = \mu n_\tau^\eta v_\tau^{1-\eta}$$

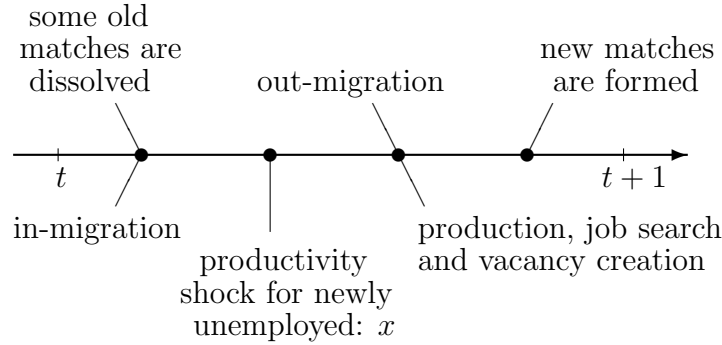
where  $0 < \eta < 1$  and  $\mu > 0$ . Let  $q_\tau = n_\tau/v_\tau$ . We refer to  $q_\tau$  as the queue length for a vacancy of type  $\tau$ . A type  $\tau$  vacant job is filled with probability  $\alpha(q_\tau) = \mu q_\tau^\eta$ , and any of the  $n_\tau$  workers finds a job with the probability  $f(q_\tau) = \mu/q_\tau^{1-\eta}$ .

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<sup>4</sup> Note that when a worker draws an idiosyncratic shock upon separation, this can also be reinterpreted as an all-in-one shock capturing wage risk over the expected employment duration.



**Figure 1:** Timing of the Events



## 2.5 Timing of the Events

Figure 1 displays the timing of events. Each period consists of four stages. At the beginning of each period, a fraction  $\lambda$  of the existing matches is dissolved. At the same time, the pool of unemployed workers in a given sector is augmented by new workers arriving from the other sector. In the second stage, the workers separated from their matches observe their new idiosyncratic shock,  $x$ . In the third stage, some of the newly unemployed individuals could decide to leave their current sector to search for a better opportunity in the other sector. These workers arrive at the other sector at the beginning of the next period (recall moving takes one period). Also in the third stage, production and vacancy creation occur, while the unemployed workers who do not move across sectors search for a job. In the last stage, new matches are realized.

## 2.6 Two key features

We have added two elements to the basic competitive search model described in Rogerson et al. (2005): sector-specific productivity and moving costs. Specifically, if there is no sector-specific productivity dispersion (i.e.,  $\omega = 0$ ), the economy converges to the standard one-sector search and matching model. Moreover, if moving across sectors is prohibitive ( $c = \infty$ ), the economy is equivalent to a one-sector model with exogenous idiosyncratic productivity.

Therefore, endogenous idiosyncratic productivity in the presence of costly mobility is the key equilibrium channel considered in this paper.

In this economy, unemployment imposes two risks. First, a worker loses her employment earnings; i.e. income drops from  $w$  to  $b$ . Second, an unemployed worker also risks an idiosyncratic productivity shock that renders her skills unsuitable for the sector the worker is currently in. Since moving is costly, the unemployed worker may prefer to continue searching in the relatively less productive sector. Below we show that the productivity risk affects the future lifetime earnings of an unemployed worker in two ways: a lower future wage *and* a lower job-finding probability.

## 2.7 Value Functions

This section presents the value functions for workers and firms.

### 2.7.1 Workers

Consider a worker who is unemployed at the beginning of the current period, in sector  $i$  with productivity  $x$ . Let  $S_i(x)$  denote the lifetime utility value to the worker of searching for a job in the current sector,  $i$ . Let  $M_i(x)$  denote the lifetime value to the worker of moving from sector  $i$  to sector  $1 - i$ . The value function of the unemployed worker is

$$U_i(x) = \max \{S_i(x), M_i(x)\}. \quad (3)$$

Given the moving cost,  $c$ , and the timing of mobility, the value of moving from sector  $i$  to sector  $1 - i$  is given by

$$M_i(x) = \log(b) - c + \beta S_{1-i}(x). \quad (4)$$

Let  $W_i(w, x)$  denote the value of being employed, in sector  $i$  with productivity  $x$ , by a

firm who pays wage  $w$ :

$$W_i(w, x) = \log(w) + \beta(1 - \lambda)W_i(w, x) + \beta\lambda \int_{-\omega}^{\omega} U_i(x') dG(x'), \quad (5)$$

where  $G$  denotes the uniform distribution function on the interval  $[-\omega, \omega]$ . A worker takes  $q_\tau$  as given. Therefore, the expected lifetime utility value of searching for a job in sector  $i$  is given by

$$S_i(x) = \max_{w \in \mathbb{W}(x, i)} \{ \log(w) + \beta f(q_{w, x, i}) W_i(w, x) + \beta(1 - f(q_{w, x, i})) S_i(x) \}. \quad (6)$$

### 2.7.2 Firms

Now consider a matched firm operating at productivity level  $x$  in sector  $i$ . Given the wage  $w$ , the value of the match to the firm is given by

$$J_i(w, x) = y_i(x) - w + \beta(1 - \lambda)J_i(w, x). \quad (7)$$

Let  $V_i(x)$  denote the value of posting a vacancy at productivity level  $x$  in sector  $i$ .  $V_i(x)$  is defined by

$$V_i(x) = \max_w \{ -k + \beta\alpha(q_{w, x, i}) J_i(w, x) \} \quad (8)$$

Due to free entry and profit maximization, all rents from vacancy creation are exhausted in the economy. Thus, for any pair  $(x, i)$ :

$$V_i(x) = 0. \quad (9)$$

## 2.8 Unemployment and Mobility

Let  $\psi_i^u(x)$  denote the number of unemployed workers searching for jobs in sector  $i$  at productivity level  $x$ . Similarly, let  $\psi_i^e(x)$  denote the number of workers employed at productivity

level  $x$  in sector  $i$ . Since the total population is normalized to one,

$$\sum_i \int_{-\omega}^{\omega} (\psi_i^u(x) + \psi_i^e(x)) dx = 1. \quad (10)$$

The economy-wide unemployment rate is given by

$$u = \sum_i \int_{-\omega}^{\omega} \psi_i^u(x) dx. \quad (11)$$

Let  $\Omega_i$  denote the decision rule governing whether an unemployed worker in sector  $i$  stays in her current sector:

$$\Omega_i(x) = \begin{cases} 1 & \text{if } S_i(x) \geq M_i(x), \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

Then, the measure of workers moving from sector  $i$  to  $1-i$  is given by  $\psi_i^m = (1 - \Omega_i(x))\psi_i^u(x)$ .

Therefore, the economy-wide mobility rate is given by

$$m = \sum_i \int_{-\omega}^{\omega} \psi_i^m(x) dG(x). \quad (13)$$

## 2.9 Definition of the Equilibrium

The equilibrium consists of a set of value functions  $\{U_i, S_i, W_i, J_i, V_i\}$ , a decision rule  $\Omega_i$ , sets of posted wages  $\mathbb{W}_i$  for any  $i \in \{0, 1\}$ , and measures  $\{n, v\}$  such that

1. Given  $(S_0, S_1)$ , the decision rule  $\Omega_i(x)$  and the value function  $U_i(x)$  solve (3);
2. Given  $U_i$ , the value function  $W_i(w, x)$  solves (5);
3. Given  $q_i, U_i$  and  $W_i$ , the value function  $S_i(x)$  solves (6) for each  $w \in \mathbb{W}_i(x)$ ;
4. The value function  $J_i(w, x)$  solves (7);
5. Given  $J_i, n$  and  $v$ , the value function  $V_i(x)$  solves (8) for each  $w \in \mathbb{W}(x)$ ; and

6. Free entry:

$$\begin{cases} v(w, x, i) > 0 \text{ and } V_i(x) = 0 \text{ if } w \in \mathbb{W}_i(x), \\ v(w, x, i) = 0 \text{ and } V_i(x) = 0 \text{ if } w \notin \mathbb{W}_i(x) \text{ or } \mathbb{W}_i(x) \text{ is an empty set.} \end{cases}$$

### 3 Equilibrium Characterization

We solve the model in two steps. First, we find the local labor market equilibrium, treating  $\bar{U}_i = \int U_i(x)dG(x)$  as a parameter. After obtaining workers' and firms' decisions within a local market, we determine  $\bar{U}_i$  using equation (3).

#### 3.1 Queue Length and Wages

Taking  $\bar{U}_i$  as given, equation (5) can be re-written as

$$W_i(w, x) = \frac{\log(w) + \beta\lambda\bar{U}_i}{A}, \quad (14)$$

where  $A = 1 - \beta(1 - \lambda)$ . Inserting the latter into (6), we have

$$\log(w) = \frac{A(1 - \beta)S_i(x) - A\log(b)}{\beta f(q_{w,x,i})} + AS_i(x) - \beta\lambda\bar{U}_i \quad (15)$$

Using equations (7) and (8), a firm's problem can be written as:

$$\max_{q_{w,x,i}} \{ \alpha(q_{w,x,i}) (y_i(x) - w) \}. \quad (16)$$

A firm posting a vacancy at the productivity level  $x_i$  takes  $S_i(x)$  and  $\bar{U}_i$  as given. Therefore, combining equations (15) and (16), a firm's problem becomes

$$\max_{q_{w,x,i}} \left\{ \alpha(q_{w,x,i}) \left( y_i(x) - \exp \left( \frac{A(1 - \beta)S_i(x) - A\log(b)}{\beta f(q_{w,x,i})} + AS_i(x) - \beta\lambda\bar{U}_i \right) \right) \right\}. \quad (17)$$

Taking the FOC in (17) and combining the result with the free entry condition, it can be shown that

$$q_{w,x,i} = \frac{\eta k}{1 - \eta} \frac{\exp\left(-\frac{A(1-\beta)S_i(x) - A\log(b)}{\beta f(q_{w,x,i})} - AS_i(x) + \beta\lambda\bar{U}_i\right)}{(1 - \beta)S_i(x) - \log(b)}. \quad (18)$$

**Proposition 1 (Queue length).** *All firms creating a vacancy at the productivity level  $x$  in sector  $i$  choose the same queue length  $q_i(x)$ .*

**Corollary 1 (Wage).** *The free entry condition,  $V_i(x) = 0$ , and Proposition 1 imply that the wage is also unique for each pair  $(x, i)$  and is given by*

$$w_i(x) = y_i(x) - \frac{kA}{\beta\alpha(q_i(x))}. \quad (19)$$

Therefore, each job is fully characterized by the productivity level,  $x$ , and the sector,  $i$ . To summarize, given  $\bar{U}_0$  and  $\bar{U}_1$ , the local labor market equilibrium is given by (15), (18) and (19). In Appendix A, we show that the wage,  $w_i(x)$ , and the value of searching for a job in the current sector,  $S_i(x)$ , increase with productivity,  $y_i(x)$ , while the queue length,  $q_i(x)$ , and the value of moving,  $M_i(x)$ , decrease with productivity for each  $i$ .

These results also imply the following two corollaries:

**Corollary 2 (Queue length).** *The queue length  $q_{x,i}$  decreases with productivity  $y_i(x)$  for each  $i$ .*

**Corollary 3 (Wage).** *The wage  $w_{x,i}$  increases with productivity  $y_i(x)$  for each  $i$ .*

These two corollaries highlight the productivity insurance aspect of UI. Specifically, a shock  $x$  that implies higher productivity in sector  $i$ ,  $y_i(x)$ , also implies a higher wage and a higher job finding probability. Notice, this represents a different trade-off from the models of Acemoglu and Shimer (1999, 2000) and Marimon and Zilibotti (1999). There, the trade-off is between higher wages and lower job-finding probabilities. In contrast, in our model, higher wages are associated with higher job-finding rates, provided the worker searches in the appropriate sector. Note, however, that it may still be the case that a mover experiences

a longer expected duration, since moving requires an additional period of unemployment (see Section 5.2.1 for more discussion of this).

Given the results above, we also characterize the effects of productivity on the value of staying in the current sector or moving, respectively. These results are important for understanding the impact of unemployment benefits on mobility (and thus on productivity).

**Corollary 4 (Value of staying).** *The value of searching in the current sector,  $S_i(x)$ , increases with productivity  $y_i(x)$  for each  $i$ .*

**Corollary 5 (Value of moving).** *The value of moving from sector  $i$  to sector  $1-i$ ,  $M_i(x)$ , decreases with productivity  $y_i(x)$  for each  $i$ .*

## 3.2 Mobility Decision

Next we characterize a worker's mobility decision and determine the equilibrium values of  $\bar{U}_0$  and  $\bar{U}_1$ . If the moving cost is too high, mobility is zero. Let  $\bar{c}$  denote the lowest moving cost that prohibits mobility. For moving costs below  $\bar{c}$ , each period a certain fraction (but not all) of unemployed workers move between the two sectors. Thus, for each  $i$ , there exists a minimum productivity level  $\hat{x}_i$  such that

$$S_i(\hat{x}_i) = M_i(\hat{x}_i). \quad (20)$$

Productivity level  $\hat{x}_i$  represents a *reservation value* for the mobility decision. Depending on the current sector, for values of  $x$  above (sector  $i = 0$ ) or below (sector  $i = 1$ )  $\hat{x}_i$ , the agent prefers to search in the other sector. The presence of moving costs ( $c > 0$  and the time cost) distorts the reservation value  $\hat{x}_i$ .

Figure 2 shows the determination of  $\hat{x}_i$ , for  $i = 0, 1$ . Note that  $0 \leq \hat{x}_0 < \omega$  and  $-\omega < \hat{x}_1 \leq 0$ . Given the symmetry of the productivity shock in equations (1) and (2), the curve  $S_1(x)$  is a reflection of the curve  $S_0(x)$  with respect to a vertical line  $x = 0$ :

$$S_1(x) = S_0(-x). \quad (21)$$

Moreover,  $M_1(x)$  is also a reflection of  $M_0(x)$  with respect to the same line. Consequently, as shown in Figure 2, the decision rule for moving across sectors is symmetric with respect to 0:  $\hat{x}_0 = -\hat{x}_1$ . The minimum per-period match output is also the same between the sectors, i.e.,  $y_{\min} = 1 - |\hat{x}_1| = 1 - \hat{x}_0$ . In the event of a transition from employment to unemployment, the probability of moving to another sector upon job separation is  $p = (\omega - |\hat{x}_1|)/(2\omega) = (\omega - \hat{x}_0)/(2\omega)$ , recalling that  $G(x)$  is a uniform distribution.

Given  $\hat{x}_0$  and  $\hat{x}_1$ , the continuation values  $\bar{U}_0$  and  $\bar{U}_1$  are given by

$$\bar{U}_0 = \int_{-\omega}^{\hat{x}_0} S_0(x)dG(x) + \int_{\hat{x}_0}^{\omega} M_0(x)dG(x) \quad (22)$$

and

$$\bar{U}_1 = \int_{-\omega}^{\hat{x}_1} M_1(x)dG(x) + \int_{\hat{x}_1}^{\omega} S_1(x)dG(x). \quad (23)$$

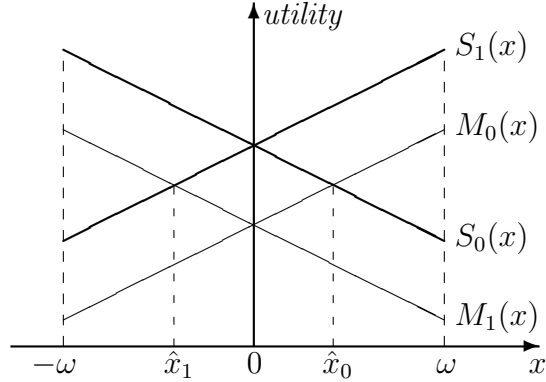
Equilibrium is fully characterized by equations (15), (18) to (20), (22) and (23). Unemployment benefits affect equilibrium outcomes primarily through two channels. First, as in a standard one-sector search and matching model, the benefit level  $b$  affects the queue length  $q_i(x)$  for each  $(x, i)$ , and thus also affects the job-finding rate. Second, the level of  $b$  affects  $\hat{x}_i$  for each  $i$ , which determines the mobility decision. Below we characterize the role of each factor to determine the impact of benefits on productivity.

## 4 Impact of Benefits on Productivity

Mobility in response to idiosyncratic productivity shocks represents the key addition of our model, relative to the standard one-sector model. Thus, to determine whether or not unemployment benefits can insure unemployed workers against these shocks, we need to understand how the mobility decision (i.e.  $\hat{x}_i$ ) responds to benefits.



**Figure 2:** Mobility Decision



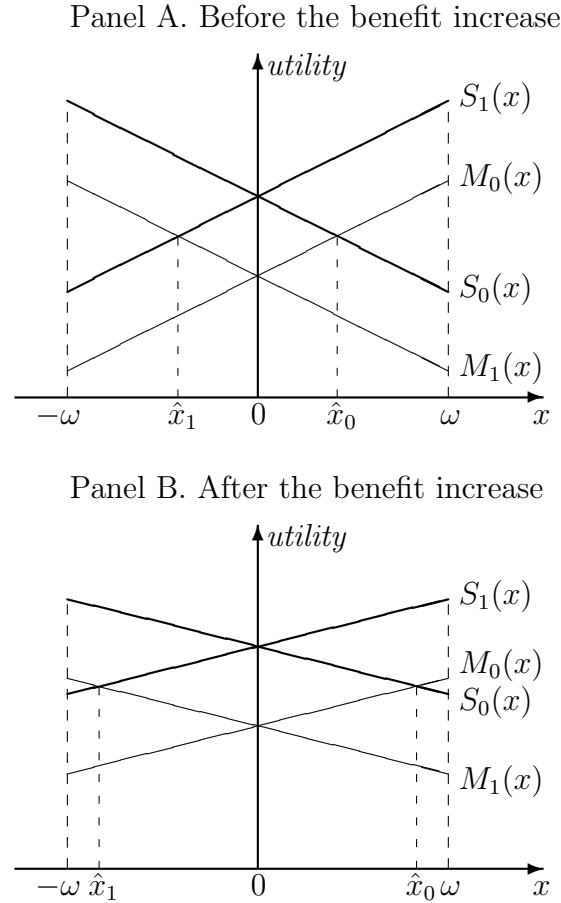
**Notes:**  $S_i(x)$  denotes the lifetime utility value to the worker of searching for a job on her current island  $i$  when her productivity level is  $x \in [-\omega, \omega]$ .  $M_i(x)$  denotes the lifetime value to the worker of moving from sector  $i$  to sector  $1-i$  when her productivity level is  $x \in [-\omega, \omega]$ . The value of searching for a job in the current sector  $S_i(x)$  increases with productivity  $y_i(x)$ , while  $M_i(x)$  decreases with productivity for each  $i$  (see Corollaries 4 and 5). Therefore, an unemployed worker of sector 1 moves to sector 0 if her productivity shock is below  $\hat{x}_1$ . Analogously, an unemployed worker of sector 0 moves to sector 1 if her productivity shock is above  $\hat{x}_0$ .

Benefits affect mobility through two channels. Whether an increase in benefits works to increase mobility, and thus help insure the unemployed against the risk of productivity shocks, depends on the relative size of the two effects. Without loss of generality, we consider the mobility decision of workers in sector 1, as the case of a sector 0 worker is symmetric.

The first channel concerns within-sector trading frictions. Specifically, unemployment benefits affect the slope of the value function  $S_1(x; b)$  with respect to  $x$ : higher benefits lower the utility differences between high and low productivity jobs, making  $S_1(x; b)$  flatter with respect to  $x$ . In other words,  $|\partial S_i(x; b)/\partial x|$  and  $|\partial M_i(x; b)/\partial x|$  decrease for each  $(x, i)$ . Figure 3 displays this effect.

This effect occurs because the attractiveness of high wage jobs diminishes relative to low wage jobs. Consequently, moving across sectors become less rewarding, and individuals become more selective. Thus, as benefits increase, the slope decreases, putting upward

**Figure 3:** Negative Productivity Effect

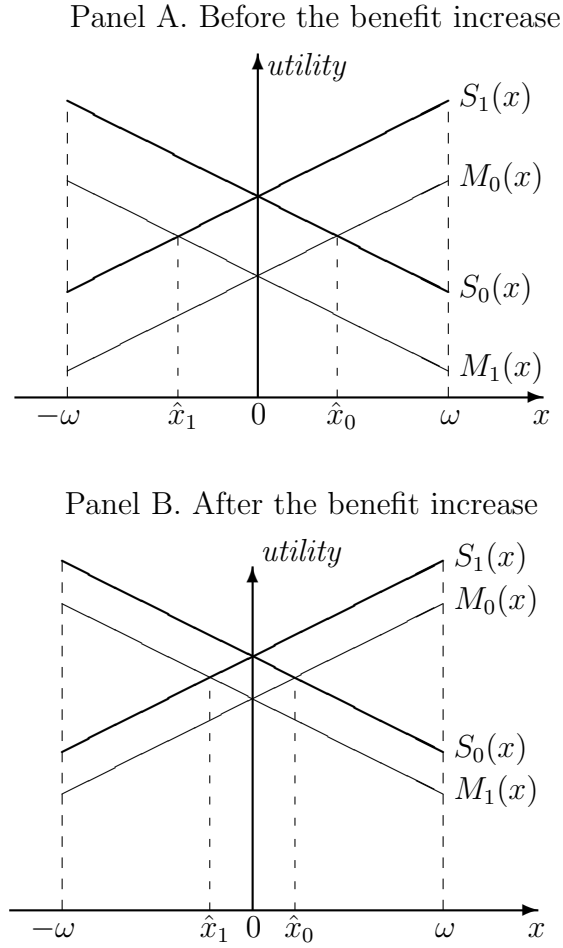


**Notes:** A higher benefit level lowers the value of searching for high productivity jobs relative to that for low productivity jobs (i.e., it makes the curves  $S_i(x)$  and  $M_i(x)$  flatter for each  $x$  and  $i$ ), putting downward pressure on productivity.

pressure on  $|\hat{x}_1|$ . Indeed, Figure 3 shows that a decrease in the slope lowers  $\hat{x}_1$  (raises  $\hat{x}_0$ ).

When  $\hat{x}_1$  decreases, mobility also decreases. This implies that unemployed workers become less responsive to the idiosyncratic productivity shocks. That is, more unemployed remain in a relatively unproductive sector for their particular skills; as a result, average productivity decreases. This represents a “moral hazard” effect. That is, unemployment benefits make workers more selective. Below in Section 5.2.1 we discuss the novelty of this effect relative to the results in Acemoglu and Shimer (1999, 2000) and Marimon and Zilibotti (1999).

**Figure 4:** Positive Productivity Effect



**Notes:** Higher benefits reduce the gap between  $S_i(x)$  and  $M_i(x)$  for each  $x$  and  $i$ , putting upward pressure on productivity.

An increase in UI benefits also has a positive effect on per-worker productivity, which helps insure workers against the idiosyncratic shocks. This second effect works through the marginal utility of consumption. This effect changes the “level” of  $S_1$  relative to  $M_1$ .

Specifically, benefits affect the value of staying in the current sector relative to the value of moving across sectors. As benefits increase, the flow utility of moving across sectors,  $\log(b) - c$ , grows faster than the constant flow utility associated with staying in the current sector,  $(1 - \beta)S_1(0, b)$ . This reduces  $|\hat{x}_1|$  (see Figure 4), increasing mobility, and thus average productivity. The total effect on productivity remains ambiguous, depending on which effect

dominates. For the remainder of the paper, we evaluate these effects quantitatively. In this sense, the main goal of our quantitative analysis is illustrative: to show that under plausible parametrizations, the positive impact of benefits on productivity can dominate the negative moral hazard effect of UI.

## 5 Quantitative Evaluation

Below we first describe our baseline parametrization, and then we present the quantitative results.

### 5.1 Calibration

The time period is one month. We set the discount factor  $\beta = 1/1.05^{1/12}$ , which reflects an annual interest rate of 5%. The separation rate is set to  $\lambda = 0.033$ , consistent with [Hagedorn and Manovskii \(2008\)](#). The elasticity of the matching function,  $\eta$ , is set to 0.5. The flow utility of unemployed workers staying in their current sector is  $b = 0.4$ . The volatility of the idiosyncratic shocks is set to  $\omega = 0.2$ . This value gives us approximately 10 percent wage variation.<sup>5</sup>

Empirically, sectors can be thought of in terms of geographical locations, industries, occupations, or a combination of these. The cost of moving across sectors is chosen to target an annual mobility rate of 10 percent.<sup>6</sup> The moving cost is  $c = 4.66$ , while the average wage defined as  $\bar{w} = \frac{1}{1-u} \sum_i \int_{-\omega}^{\omega} w_i(x) \psi_i^e(x) dx$  is slightly greater than one. This implies a moving

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<sup>5</sup>According to [Guvenen \(2009\)](#), the wage variation caused by the overall labor income shock is 20-30 percent. Noting that the sectoral mismatch shock is a part of the overall income shock, we target a value much lower than those estimated by [Guvenen \(2009\)](#). Moreover, a recent work of [Auray, Fuller, Lkhagvasuren and Terracol \(2014\)](#) estimates that the dispersion of the income shock specific to a worker-industry pair is approximately 15 percent of the average wage, a value comparable to that assumed in the benchmark model. Below, in Appendix B, we show that when the magnitude of the idiosyncratic shock becomes negligible, the benefits have no impact on mobility.

<sup>6</sup>In terms of the empirical mobility rates, [Murphy and Topel \(1987\)](#) report industry annual mobility rates at 6-10 percent. [Kambourov and Manovskii \(2009\)](#) report annual occupational mobility rate at 16 percent in the early 1970s and at 21 percent in the mid-1990s. For geographical labor markets in the U.S., depending on the distance, mobility can be 3-10 percent per year.

**Table 1:** Parameters of the Benchmark Model

<i>Parameters</i>	<i>Values</i>	<i>Description</i>
$\beta$	0.9959	the time-discount factor
$\lambda$	0.033	the job separation rate
$\eta$	0.50	the elasticity of the matching technology
$b$	0.4	flow utility of unemployment
$\mu$	0.4875	the efficiency of the matching technology
$k$	1.2661	the vacancy cost
$\omega$	0.2	volatility of the sector-specific shock
$c$	4.6574	moving cost

**Notes:** This table summarizes the key parameters of the model.

**Table 2:** Prediction of the Model

<i>Variables</i>	<i>Benchmark</i>	<i>Higher benefit levels</i>		<i>Description</i>
	$b = 0.40$	$b = 0.45$	$b = 0.50$	
$u$	0.0586	0.0642	0.0703	unemployment
$m$	0.0990	0.0996	0.1000	annual mobility
$v/u$	0.6198	0.5398	0.4679	the vacancy-unemployment ratio
$y_{\min}$	0.9052	0.9065	0.9076	minimum observed productivity
$\bar{y}$	1.0770	1.0776	1.0781	per-worker output
$\bar{w}$	1.0004	1.0061	1.0115	the average wage

cost equal to approximately four months of labor income.

As [Shimer \(2005\)](#) shows, a Cobb-Douglas matching technology implies that the average vacancy-unemployment ratio is intrinsically meaningless for the job-finding rate and the unemployment rate. Given this, we normalize the average vacancy-unemployment ratio to 0.6 ([Hagedorn and Manovskii, 2008](#)), and target an economy-wide unemployment rate of 6 percent. These imply  $k = 1.2661$  and  $\mu = 0.4875$ . [Table 1](#) summarizes the key parameters. The column labeled *benchmark* in [Table 2](#) displays the predictions of the baseline model.

## 5.2 Policy Experiments

We first explore if unemployment benefits can insure unemployed workers against the idiosyncratic productivity shocks. The answer to this question depends on which effect dominates. To analyze these effects, we first simulate the benchmark model for different levels of  $b$ . Table 2 summarizes the results.

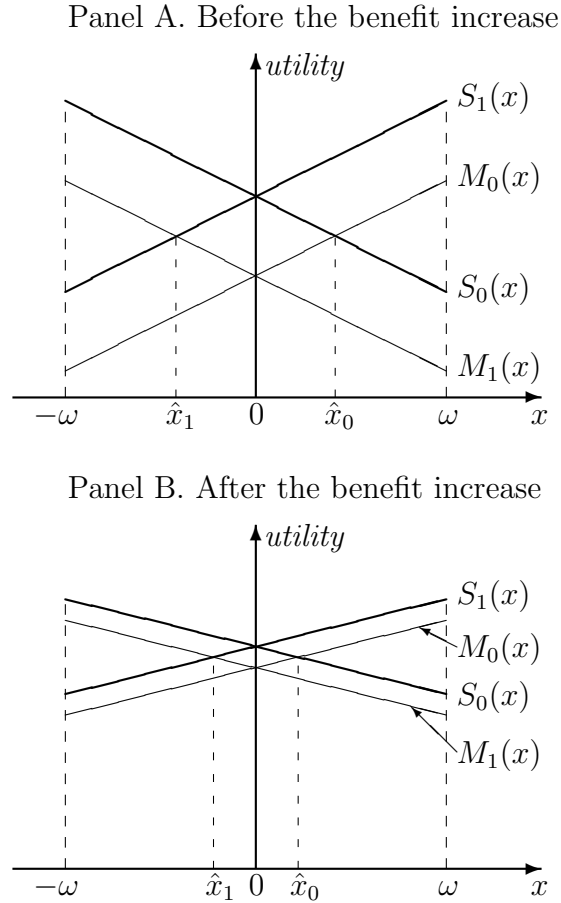
### 5.2.1 Productivity and Unemployment

Table 2 indicates that benefits lower  $|\hat{x}_j|$ . As a result, higher benefits raise both the minimum productivity  $y_{\min} = 1 - |\hat{x}_1|$  and average productivity  $\bar{y} = \frac{1}{1-u} \sum_i \int_{-\omega}^{\omega} y_i(x) \psi_i^e(x) dx$ . Therefore, the results imply that the positive effect of UI on productivity dominates the negative effect (see Figure 5).

As can be seen in Table 2, as benefits increase, unemployment increases, the ratio of vacancies to unemployment decreases, and average productivity increases. While qualitatively similar to the results in Acemoglu and Shimer (1999, 2000) and Marimon and Zilibotti (1999), the mechanism driving the results in this paper is quite different. As  $|\hat{x}_j|$  decreases with benefits, unemployed workers become *less* selective and, on average, search for a job at a higher productivity level. Workers being less selective helps reduce the level of mismatch in the economy, increasing per-worker productivity. In contrast, in Marimon and Zilibotti (1999), UI benefits make workers more selective which increases per-worker productivity.

In addition, in the model in this paper, more workers moving to the higher productivity sectors puts upward pressure on the probability of finding a job. As  $|\hat{x}_j|$  decreases, the probability of moving, given a transition from employment to unemployment, increases. Since unemployed workers move to a sector where their productivity is higher, job offers are more likely to occur. Once at the destination, movers have an average job finding probability of 0.40, compared to an average of 0.38 for those remaining. This represents another distinction between the mechanism in our paper relative to Acemoglu and Shimer (1999, 2000) and Marimon and Zilibotti (1999). In these environments, more productive

**Figure 5:** Productivity Effects



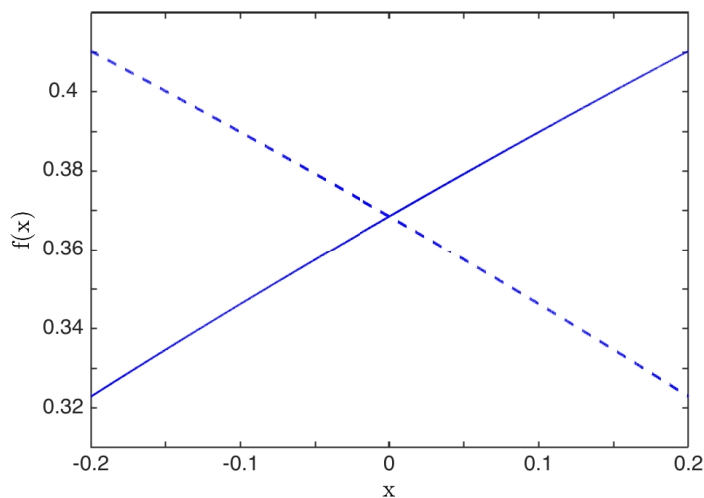
**Notes:** Case where an increase in UI benefits increases per-worker productivity.

jobs take longer to find. In contrast, in our model, the unemployed workers who move to their higher productivity sector have a higher probability of finding job; there does not exist an economy-wide trade-off in terms of job-creation as in [Acemoglu and Shimer \(1999, 2000\)](#).

It is true, however, that in our model, the time-cost of mobility can erase some of the job-finding probability gains from moving. For the average mover, the average unemployment duration is around 2.5 months once they arrive in the new sector. Incorporating the one period spent moving, the average unemployment duration for these workers is about 3.5 months. For a worker who decides to stay in the current sector, the average unemployment duration is around 2.7 months. Thus, the time-cost of mobility plays an important role in

the analysis.<sup>7</sup>

**Figure 6:** Job-Finding Probability by Productivity



**Notes:** This figure shows that the job-finding probability increases with productivity. The solid line plots the job-finding probability of workers in sector 1,  $f_1(x)$ , as a function of  $x$ , while the dashed line plots that of those in sector 0,  $f_0(x)$ .

As in standard one-sector search and matching models, as benefits increase the vacancy-unemployment ratio decreases at each productivity level, exerting downward pressure on the job finding rate. Table 2 shows that as the benefits rise, unemployment increases, indicating that the effect through the vacancy-unemployment ratio is much stronger than the other two effects. This is not surprising since the fraction of unemployed workers searching for a job in their own sector is much larger than those moving across sectors and, thus, unemployment is mainly determined by within-market search frictions. In Appendix A.2 we characterize these effects *within* a particular sector.

### 5.2.2 Mobility

What is the impact of an increase in benefits on mobility? Using the accounting equations of labor market flows and stocks in Section 2.8, the mobility rate is given by

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<sup>7</sup>We would like to thank an anonymous referee for highlighting the importance of this channel.



**Table 3:** Robustness: Zero Moving Cost

<i>Variables</i>	<i>Benchmark</i>	<i>Higher benefit levels</i>		<i>Description</i>
	$b = 0.40$	$b = 0.45$	$b = 0.50$	
$u$	0.0613	0.0662	0.0716	unemployment
$m$	0.1741	0.1749	0.1753	annual mobility
$v/u$	0.6764	0.5944	0.5209	the vacancy-unemployment ratio
$y_{\min}$	0.9855	0.9873	0.9889	minimum observed productivity
$\bar{y}$	1.0988	1.0989	1.0990	per-worker output
$\bar{w}$	1.0189	1.0240	1.0289	the average wage

This table shows that benefits can raise mobility and productivity even when there is no moving cost. For the results reported in the table, we simulated the benchmark model by considering the zero moving cost (i.e.,  $c = 0$ ) and different values of the benefit level,  $b$ .

$$m = \lambda(1 - u)p, \quad (24)$$

where  $1 - u$  is employment and  $p = (\omega - |\hat{x}_1|)/(2\omega)$  is the probability of moving across sectors (given a transition from employment to unemployment).

The results in Table 2 show that higher benefits lower employment,  $1 - u$ . At the same time, workers become more selective (i.e.,  $|\hat{x}_j|$  decreases), which increases  $p$ , the probability of moving upon separation. Therefore, the impact of benefits on overall mobility is analytically uncertain. Table 2 shows that benefits raise overall mobility, indicating that the probability of moving across sectors, given a transition from employment to unemployment, responds to benefits more than employment, in percentage terms.

As discussed in Section 2, moving involves two costs: a time cost (as workers spend an additional period unemployed), and a direct utility cost  $c$ . To understand the role played by each cost, consider Table 3. This table provides the results when  $c = 0$  under the baseline parameterization; that is, the time cost represents the only cost of moving. There are several key points to note.

First, relative to the baseline case in Table 2, mobility,  $y_{\min}$ , and  $\bar{y}$  are higher. This is the result of  $c > 0$  reducing mobility, and thus per-worker output. The jump in  $\bar{y}$  from 1.0770

in the baseline parameterization of  $c$ , to 1.0988 when  $c = 0$ , underscores the importance of mobility for productivity. The case of  $c = 0$  implies a much higher  $y_{min}$  and thus higher mobility.

Second, increases in UI benefits still have an impact on per-worker output. In the case of  $c = 0$ , a 25% increase in benefits increases per-worker output by 0.0182%, compared to 0.11% in the baseline case (where  $c > 0$ ). UI benefits still have a positive effect on productivity here since they help alleviate the time cost of mobility. Indeed, the increase in mobility from  $b = 0.4$  to  $b = 0.5$  is similar in Table 3 to that reported in Table 2 (where  $c > 0$ ). Thus, even when  $c = 0$ , increasing UI benefits increases mobility and per-worker productivity. The overall productivity gains are small relative to the baseline case, but part of this is due to the fact that when  $c = 0$  annual mobility remains relatively high. That is,  $y_{min}$  is already close to 1 so that there is less potential for UI benefits to impact productivity.

### 5.3 Moving Costs and Optimal Benefit Level

In this section, we analyze the relationship between moving costs and the welfare-maximizing benefit level. Determining the optimal benefit level is an exercise in balancing the traditional insurance vs. incentives (i.e. negative productivity effect and increase in unemployment), and the additional insurance motive provided by the positive productivity effect. We determine the optimal benefit level and how it changes with moving costs. In the welfare comparisons, we consider benefits above the benefit level in the benchmark economy,  $= BM$ , that are financed by a lump sum tax  $\tau$ . Then, aggregate welfare is given by:

$$\mathcal{U} = u \log(b - \tau) - mc + \sum_i \int_{-\omega}^{\omega} \log(w_i(x) - \tau) \psi_i^e(x) dx. \quad (25)$$

The optimal benefit level is determined by finding the combination of  $\tau$  and  $b$  that maximizes  $\mathcal{U}$ , subject to the budget constraint

$$\tau = u(b - b^{BM}). \quad (26)$$

**Table 4:** Optimal Benefit Level by Moving Cost

Moving cost $c$	Optimal benefit level	Tax $\tau$	Consump. gain (%)	Minimum prod. $y_{\min}$	Average prod. $\bar{y}$	Unemp. $u$	Mobility $m$
0	0.6909	0.0284	0.0058	0.9937	1.0992	0.0977	0.1748
1	0.6788	0.0268	0.0057	0.9762	1.0979	0.0963	0.1592
Benchmark (4.66)	0.6638	0.0253	0.0053	0.9124	1.0802	0.0958	0.1016
7	0.6509	0.0237	0.0049	0.8715	1.0583	0.0944	0.0647
12	0.6148	0.0189	0.0040	0.8000	1.0000	0.0882	0

**Notes:** Consump. gain refers to the percentage increase in the average flow consumption measured by  $(\exp((1 - \beta)(\mathcal{U}(b) - \mathcal{U}(b^{BM}))) - 1) \times 100\%$  where  $\mathcal{U}(b)$  is given by equation (25).

The latter equation stresses that we set  $\tau$  to zero in the benchmark model. This is merely a normalization as we assume a lump sum tax. Thus, it should be kept in mind that the optimal tax rates calculated below are relative to the benchmark tax rate.

Table 4 describes how the optimal benefit level varies with moving costs. Our main finding from this experiment is that the optimal benefit level is decreasing in moving costs. Moreover, the total welfare gains from adopting the optimal benefit level are also decreasing in moving costs. There are many factors behind this result.

Consider first the case where  $c = 12$ . This economy essentially functions as two separate standard Pissarides (2000) economies with a productivity shock ( $x$ ). In this world, higher benefits increase unemployment (via reduced vacancy creation by firms). The advantage of these higher benefits is strictly improved consumption smoothing, or the standard insurance aspect of UI benefits. Relative to the economies with mobility, the  $c = 12$  economy has a lower optimal benefit level and lower unemployment rate.

As  $c$  decreases, annual mobility increases. Now, in addition to the standard Pissarides model trade-offs, an increase in UI benefits also has positive productivity effects. As a result, the optimal benefit level and unemployment rate are higher as  $c$  decreases (and annual mobility increases).

Finally, recall that the productivity effects of UI in this model involve a further trade-off (see Section 4). On the negative side is the “moral hazard” effect that makes workers more selective, decreasing mobility and the productivity effects. When  $c$  remains relatively low this effect is less troublesome as annual mobility is already relatively high; as a result, the optimal level of benefits is relatively high.

## 6 Conclusion

We construct a search-matching model with sectoral mobility and analyze the provision of unemployment benefits. Unemployment in this environment poses an additional risk because the unemployed workers are subject to idiosyncratic productivity shocks that affect future wage prospects. Unemployment benefits increase mobility, which reduces the risks of the idiosyncratic productivity shocks the unemployed face. Our results show that unemployment benefits can have a substantial impact on productivity through higher mobility. The optimal replacement ratio decreases with the costs of moving, and the welfare gains decrease from 0.006% when moving is costless to 0.004% when moving is prohibitively costly.

Moving costs are modelled as a separable utility cost. Modelling moving costs as a consumption cost directly is an interesting alternative. In this case, UI benefits potentially have a larger impact on mobility and thus productivity since with risk-averse workers, the marginal utility of consumption is higher for movers relative to stayers. This enhances the positive impact of benefits on productivity relative to the negative effect. One potential issue, however, is quantitative: restricting to positive consumption limits the range of possible moving costs, making calibration with respect to mobility more difficult. Despite this numerical issue, modelling the costs to moving in this manner represents an interesting direction for future research.

# A Analytical Details

Here we prove a set of claims made in the text.

## A.1 Wages, Queue Length and Productivity

**Proof of Proposition 1.** Recall that  $f$  is a strictly decreasing function of  $q_{w,x,i}$ . Moreover, for the value of search to exceed the value of unemployment, it must be that  $S_i(x) > \log(b)/(1 - \beta)$ . Thus, the right-hand side of equation (18) strictly decreases with  $q_{w,x,i}$ , while the left-hand side increases. Thus, for each  $(x, i)$  there is a unique queue length,  $q_{w,x,i}$ , which we denote by  $q_i(x)$ . □

**Proof of Corollary 1.** See Proof of Proposition 1. □

**Proof of Corollary 2.** Rewrite equation (15) using the uniqueness result,

$$\log(w_{x,i}) = C(b) + \left(1 + \frac{1 - \beta}{\beta f(q_{x,i})}\right) K_i(x), \quad (\text{A.1})$$

where

$$C(b) = \frac{A \log(b)}{1 - \beta} - \beta \lambda \bar{U}_i \quad (\text{A.2})$$

and

$$K_i(x) = A \left( S_i(x) - \frac{\log(b)}{1 - \beta} \right). \quad (\text{A.3})$$

Note that the term  $C(b)$  is common across different productivity levels and different sectors. It is important to keep this in mind in the analysis below. On the other hand, using equation (18),

$$K_i(x) = \frac{r}{w_{x,i} q_{x,i}}, \quad (\text{A.4})$$

where  $r = \frac{\eta k A}{(1 - \eta)(1 - \beta)}$ . Inserting equation (A.4) for  $K_i(x)$  and then equation (19) for  $w$  into

equation (A.1), respectively, one can get

$$\log\left(y_i(x) - \frac{kA}{\beta\alpha(q_{x,i})}\right) = C(b) + \left(\frac{r}{q_{x,i}} + \frac{r(1-\beta)}{\beta\alpha(q_{x,i}(x))}\right) \left(y_i(x) - \frac{kA}{\beta\alpha(q_{x,i})}\right)^{-1}. \quad (\text{A.5})$$

Without loss of generality let  $i = 1$ . Also, notice that the left-hand side of equation A.5 increases with  $q_{x,i}$ , while the right-hand side decreases with  $q_{x,i}$ . Since  $y_1(x) = 1 + x$ , an increase in  $x$  raises the left-hand side of equation A.5 while lowering its right-hand side. Therefore, the equilibrium queue length  $q_{x,1}$  decreases with  $x$ . Using the symmetric production function, it can be seen that the equilibrium queue length  $q_{x,0}$  increases with  $x$ . Therefore, an increase in productivity  $y_i(x)$  lowers the queue length.  $\square$

**Proof of Corollary 3.** Combining equations (A.1) and (A.4), one can write

$$\log(w_{x,i}) - \left(\frac{1}{q_{x,i}} + \frac{1-\beta}{\beta\alpha(q_{x,i})}\right) \frac{r}{w_{x,i}} = C(b). \quad (\text{A.6})$$

Recall that  $\alpha$  is a strictly increasing function. Therefore, the left hand side of equation (A.6) is an increasing function of  $q_{x,i}$ . Thus,  $w_{x,i}$  and  $q_{x,i}$  are negatively related between different values of  $x$ . Therefore, since the queue length  $q_{x,i}$  decreases with productivity, the wage  $w_{x,i}$  increases with productivity.  $\square$

**Proof of Corollaries 4 and 5.** Combine equation (4) with Corollaries 2 and 3.  $\square$

## A.2 Impact of Benefits

In Section 5.2.1 we discuss the effects of within sector frictions on the average duration of unemployment. The following two results summarize the effects of unemployment benefits *in* a particular sector.

**Proposition A.1 (Queue length and benefits).** *Benefits raise the queue length at each*

*productivity level.*

*Proof.* Recall that the left-hand side of equation A.5 increases with  $q_{x,i}$ , while the right-hand side decreases with  $q_{x,i}$ . Moreover, the benefit level  $b$  affects the right-hand side through the term  $C(b)$ . Specifically, an increase in  $b$  raises  $C(b)$ , since the first term on the right-hand side of equation (A.2),  $A \log(b)/(1 - \beta)$ , dominates the second term  $\beta \lambda \bar{U}_i$ . Therefore, the benefit level will also raise the equilibrium queue length  $q_{x,i}$  for each pair  $(x, i)$ .  $\square$

**Proposition A.2 (Wage and benefits).** *Benefits raise the wage at each productivity level.*

*Proof.* The productivity specific wage increases with the queue length (see equation (19)). Then using Corollary 4, it can be seen that the wage  $w_{i,x}$  increases with the benefit level  $b$  for each pair  $(i, x)$ .  $\square$

## B Robustness

Table B.1 show the results of Table 2 for the case of a low  $\omega$ . In this case, mobility is zero; as a result, UI benefits have no impact on per-worker output. Thus, as  $\omega$  increases, so do the potential productivity gains from UI benefits (assuming the other parameters remain constant).

**Table B.1:** Low Productivity Dispersion,  $\omega = 0.002$ 

<i>Variables</i>	<i>Benchmark</i>	<i>Higher benefit levels</i>		<i>Description</i>
	$b = 0.40$	$b = 0.45$	$b = 0.50$	
$u$	0.0468	0.0524	0.0586	unemployment
$m$	0	0	0	annual mobility
$v/u$	0.6873	0.5950	0.5122	the vacancy-unemployment ratio
$y_{\min}$	0.9980	0.9980	0.9980	minimum observed productivity
$\bar{y}$	1.0000	1.0000	1.0000	per-worker output
$\bar{w}$	0.9330	0.9377	0.9422	the average wage

This table shows that when the productivity dispersion,  $\omega$ , is sufficiently low, mobility becomes zero and may not respond to a further increase in benefits. For the results reported in the table,  $\omega = 0.002$ ,  $k = 1.0525$  and  $c = 0$ , while the rest of the parameters are kept at their benchmark values.



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