Dynamic Comparative Advantage, Directed Mobility Across Sectors, and Wages*

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Abstract

This paper argues that evolving comparative advantage is important not only for worker flows across sectors, but also for wage growth and lifetime earnings. First, the main individual-level relationship between sectoral mobility and wages is established using Panel Study of Income Dynamics. Second, a dynamic, stochastic multi-sector model with worker-sector match productivity is introduced to account for the relationship. In the model, a sector may experience simultaneous inflows and outflows of workers that are much larger than the corresponding net flows. Movers tend to have a lower wage than non-movers both prior to, and after the move. Wages grow with sectoral tenure. Those who move more frequently tend to have lower lifetime earnings. Recent movers are more likely to move again. Labor mobility decreases with labor market experience. All these predictions of the model are consistent with data, but generated by a remarkably simple, evolving match productivity shock.

Keywords: Stochastic Multi-Sector Model, Excess and Net Mobility, Dynamic Sectoral Mismatch, Labor Income Shocks, Lifetime Earnings, Return to Tenure, Directed Mobility
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1 Introduction

There exists a large literature examining the role of mobility in employment and wage dynamics. Most of these studies focus on aggregate and sectoral level implications of labor mobility;[1] however, little is known about the mechanism underlying the individual-level relationship between mobility and wages. In the presence of sectoral mobility driven by labor income, neither movers nor stayers are a random sample with respect to income. In fact, as shown below, wages differ substantially between movers and stayers (see, for example, Figure 1). Analyzing such individual-level wage differences between movers and stayers might be the key to understanding labor income dynamics and the role of sectoral mobility for wages.

In this paper, we argue that evolving comparative advantage is important not only for worker flows across sectors, but also for many salient features of the individual-level relationship between mobility and wages. In this regard, the paper makes two contributions to the literature on the role of mobility for wages. First, it characterizes the main relationships between sectoral mobility, the wage level, sectoral tenure, wage growth and lifetime earnings of workers using Panel Study of Income Dynamics (PSID). The paper finds several novel results that existing theories of sectoral mobility and wages are unable to capture. Second, the paper develops a dynamic multi-sector model with persistent worker-sector match productivity that accounts for these facts.

The PSID data allow us to examine the wage profile before and after a worker changes sectors of employment. We find that workers changing sectors have wages below the mean in both their original and destination sectors. This pre- and post-move relationship has also been shown in Jovanovic and Moffitt (1990) and McLaughlin and Bils (2001). We extend these two-period, pre- and post-move wage comparisons by tracing the movers' wage over their lifetime. To our knowledge, this is the first attempt in the literature to characterize the wage as a function of sectoral tenure and to consider lifetime earnings. The empirical analysis yields two novel results.

First, the average wage of new arrivals steadily increases with sectoral tenure, but remains below the mean wage of the sector for nearly 10 years (see Figure 2). Second, individuals changing sectors more frequently tend to have significantly lower lifetime earnings. These patterns remain robust after controlling for individual characteristics

[1] A representative sample of earlier studies that examine the role of mobility for aggregate and sectoral dynamics include Lucas and Prescott (1974), Lilien (1982), Abraham and Katz (1986), Rogerson (1987), Alvarez and Veracierto (1999) and Alvarez and Shimer (2011).

including age and education.

We argue that these facts are consistent with dynamic idiosyncratic shocks to the worker-sector match. In particular, we account for these patterns by constructing a stochastic dynamic model of wages and mobility. In the model, worker productivity is subject to a dynamic, worker-sector match shock. This idiosyncratic, persistent shock affects the relative productivity of the *individual* in one sector relative to others. Workers may decide to move to the sector of highest comparative advantage, not necessarily to the sector with the highest average wage. This dynamic worker-sector match component represents the key innovation of our model, relative to existing models. For the remainder of the paper, the shock is referred to as a "worker-sector match" or "match" shock.

Moreover, the model includes a sector-specific skill premium, as in Rogerson (2005) and Kambourov and Manovskii (2008). The longer a worker stays in a sector, the more likely the person receives the skill "premium." There also exists a persistent sector-wide shock affecting all workers within a particular sector. This sectoral shock causes net mobility, while the dynamic match shock creates excess mobility.[2]

The notion of match specific idiosyncratic "comparative advantage" or idiosyncratic match quality can be traced back to Roy's (1951) seminal theory of sectoral selection and the wage distribution (see Heckman and Taber, 2008, for various extensions of Roy's model.) Our model differs from the existing versions of Roy's model with heterogenous match productivity in an important way: evolving match quality.

Dynamic match quality is not entirely new to the literature. There is a growing literature that introduces match quality to the Lucas and Prescott (1974) island model (*e.g.*, Coen-Pirani (2010), Lkhagvasuren (2012) and Carrillo-Tudela and Visschers (2014)). The model developed in this paper differs from the recent studies along two main dimensions. First, the aforementioned models focus mainly on sectoral dynamics and do not consider the individual-level relationship between mobility and wages, which we focus on exclusively. Second, mobility is *undirected* in the existing models with dynamic match quality. That is, if a worker is hit by an adverse match shock and decides to change sectors, she moves across sectors without knowing their wage at the destination. In such a setting, a worker draws a new match shock at

[2]As in Davis and Haltiwanger (1992), net mobility refers to the gap between the simultaneous in- and outflows at the sectoral level, while excess mobility is overall (or gross) mobility minus the net mobility. In other words, excess mobility refers to in- and outflows that cancel at the sectoral level.

the destination from an exogenous distribution, uncorrelated with the worker's past wages. In contrast, mobility is *directed* in our model, as in Roy's (1951) model. In our model, a worker knows her match quality across sectors, and thus knows the wage at the destination, before switching sectors. Moreover, both movers and stayers draw productivity shocks from an endogenous distribution that is conditional on their past wage.

We consider directed mobility and thus an endogenous distribution for two important reasons. First, with directed mobility, there is no need to specify additional and different restrictions on the income processes of stayers versus movers. This is important, especially when one analyzes the wage differences between movers and stayers. Second, with directed mobility, the parameterization of the model becomes highly parsimonious. Specifically, we introduce dynamic match productivity through only two parameters: the persistence and dispersion of an AR(1) process. We show both analytically and quantitatively that such a simple productivity process can account for many key features of the mobility patterns and wage distribution among movers and non-movers, including the new facts established in this paper. Moreover, empirically, undirected mobility appears to be an untenable assumption when considering wage data of annual frequency, since workers can revise their mobility decisions in less than a year. Thus, the key novelty of our model is the combination of directed mobility inherent in sectoral selection literature (with permanent match quality) built on the Roy (1951) model, and a stochastic match shock in the recent sectoral dynamics literature built on the island model of Lucas and Prescott (1974).

Our results show that directed mobility driven by dynamic, evolving comparative advantage is consistent with many features of mobility and wages, including the novel facts we establish. Specifically, our model accounts for the observed mover-stayer wage gap, wage growth among movers, and the negative correlation between mobility and lifetime earnings. The model is also able to capture the negative relationship between mobility and labor market experience seen in the PSID data. In addition, our data analysis includes the two-sample nonparametric Kolmogorov-Smirnov test to show that the wage distributions of movers and stayers are significantly different. The combination of the persistent match shock and directed mobility introduced in this paper offers a natural explanation for why these different wage distributions may arise when mobility and wages are jointly determined.

To further illustrate the role played by evolving worker-sector comparative advantage, we consider a version of the model where the sector-specific skill premium is absent. Our analytical and quantitative results show that when the match shock is highly persistent, the model is still capable of capturing the key features of the data, in the absence of a sector-specific skill premium. Intuitively, high persistence in the dynamic match shock implies that those who remain in the current sector are those who experience further improvements in relative productivity. This dynamic selection effect is able to generate the negative wage gap between movers and stayers, the positive wage-tenure relationship, and the negative correlation between mobility and lifetime earnings. However, when the persistence of comparative advantage is too high, the model is unable to capture certain quantitative features of the distribution of workers by sectoral tenure and repeat mobility.[3] Nevertheless, the model with no skill premium shows that directed mobility driven by dynamic match productivity goes a long way to account for the wage-mobility relationship, including the novel facts established in the paper.

We also consider a model where mobility is primarily driven by the sectoral level shock rather than the match shock. While such a model performs well along many dimensions, it generates implausibly high volatility in sectoral employment. This further underscores the important role played by the dynamic worker-sector match shock not only for the individual level wage-mobility relationship, but also for sectoral level dynamics.

The remainder of the paper is organized as follows. Section 2 details the facts characterizing the wage-mobility relationship. Section 3 describes the model. Section 4 considers a highly simplified version of the model, and presents analytical results on the impact of the dynamic match shock on the wage gap between movers and stayers and the wage-tenure profile. Section 5 describes the calibration and the main results. Section 6 performs a set of numerical experiments to disentangle the effects driving the key features of wage-mobility relationship. Section 7 considers a re-calibrated version of the model where the skill premium is omitted. Section 8 concludes.

2 Data analysis

In this section, we characterize the relationships between wages, mobility, and sectoral tenure in the PSID data. There exist several key patterns linking sectoral-mobility

[3] Repeat mobility is defined as the share of workers moving twice within two consecutive years relative to overall mobility.

and wages discussed in this Section. The goal is to provide the foundation of empirical facts the model of Section 3 is compared against. In this regard, the model will uncover what role dynamic evolving comparative advantage plays in the relationship between wages, mobility, and sectoral tenure, taking the empirical facts as given.

Specifically, we characterize the dynamics of wages, mobility, and sectoral tenure in way not yet captured in the existing literature. This includes examining wages in the periods before a change in sectoral employment, and then following wage evolution after the move occurs. In the model of Section 3 (and in general all "Roy" type models), the wage of movers relative to that of the other workers in the sector represents the key variable. Workers move to the sector of highest comparative advantage, not necessarily to the sector with the highest average wage. Given this, our analysis focuses on the relative mover-stayer wage gap, at both the origin and destination, the evolution of this wage gap following mobility, and lifetime earnings.

We begin by first describing the PSID data and some key sectoral mobility patterns. Then we present a characterization of relative wages before a change in sectoral employment, and follow the evolution for the 12 years following mobility. We then discuss what the evidence implies regarding the forces driving the wage and sectoral-tenure relationship. Finally, we present a novel finding linking lifetime earnings to mobility. This finding provides a crucial piece of evidence to discipline the model.

2.1 Sample description

The analysis uses data from PSID of 1968-1997; however, for certain mobility measures, the analysis further restricts attention to the Retrospective Occupation-Industry Supplemental Data Files, released in 1999. Kambourov and Manovskii (2009a) find that the Retrospective Files for the period of 1968-1980 provide a more accurate measure of labor mobility across industries and occupations than the main PSID data. This enhanced precision is essential as we focus on the individual-level relationship between mobility and wages. For example, when using the less accurate measure of mobility in the main PSID data of 1968-1997, the relationship between mobility and earnings becomes weaker (see Table A.4) while the main data patterns discussed in this section remain robust.[4]

[4] While we focus on the individual-level relationships, others have analyzed more aggregate features of sectoral dynamics. For examples, see Lee and Wolpin (2006), Kambourov and Manovskii (2009a), Jaimovich and Siu (2012) and Autor and Dorn (2013).

Our sample of Retrospective Files consists of 3057 male household heads aged 20-65, totaling 28,443 observations over the period 1968-1980. In the analysis, "sectors" are defined as industries, with four broad industries considered: Agriculture, Manufacturing and mining (hereafter Manufacturing), Services, and Public Sector.

The main reason for considering these broad industries is the sample size. McLaughlin and Bils (2001) argue that to measure the wage gap between inter-sectoral movers and stayers, one needs large sectors, as movers are a small fraction of the labor force. On the other hand, in PSID, the wages and industries are available for a few thousand workers. Then, if one considers much smaller (or finer) industries, the number of movers within each industry will be too low to measure the wage gap between movers and stayers while making it hard, if not impossible, to construct a reliable wage-tenure profile in data. For this measurement consideration, we focus on the above broad sectors.[5]

Sectoral mobility occurs if an individual switches industries between two consecutive years in which he is employed. Wages are measured as real hourly wages, computed as annual labor income divided by annual hours and deflated by the *Consumer Price Index for All Urban Consumers* provided by the Bureau of Labor Statistics (BLS).

2.2 The basic fact: large gross and small net mobility

Why workers change sectors represents a central question to our analysis. While our analysis below explores this question in detail, some basic facts in the PSID sample also shed light on the question.

In particular, net mobility constitutes less than 1% of total sectoral employment. Gross mobility, however, is relatively large, constituting 7% of the full sample. For example, in the 1968-1980 Retrospective Files, 541 Manufacturing workers moved to the Service sector, and 534 Service sector workers moved to Manufacturing. This amounts to net mobility of only 7 workers.

With respect to the question of why workers change sectors, one could argue that workers move from Manufacturing to Services if the Service sector becomes more productive than Manufacturing, on average. That is, a shock hits the entire sector, changing relative productivities for all workers. Kambourov and Manovskii (2009a),

[5] Lee and Wolpin (2006) also consider broad industries such as Manufacturing and Service sectors.

for example, adopt this approach with respect to occupational mobility. In terms of mobility across industries, however, large gross mobility relative to net mobility suggests that sectoral level effects alone cannot explain observed mobility.

Jovanovic and Moffitt (1990) are among the first to explore the fact that the percentage of people moving across sectors is much higher than the percentage change in sectoral employment. Indeed, they argue that sector-wide effects cannot explain observed labor mobility across sectors and that idiosyncratic comparative advantage represents an important factor of the mobility decision. Under this view, a worker changes sectors when they find a better match quality. For example, a worker in the Service sector may find a relative comparative advantage in Manufacturing and change sectors, while another worker in Manufacturing finds the opposite.

The goal of our paper is to understand the nature of this idiosyncratic comparative advantage, or the match component. The existing literature examining wages and mobility has focused on a permanent match component (see, for example, Moscarini (2001)). Below we further characterize the key relationships that shed light on this match component, and argue this evidence suggests an idiosyncratic match component that is dynamic and evolving over time, rather than permanent.

2.3 Wage dynamics, mobility and sectoral tenure

We begin with two features previously characterized in the literature: (i) workers who are about to change sectors have lower wages relative to those workers who stay in the sector and (ii) once they do change sectors, they also have lower wages relative to the incumbents (see for example McLaughlin and Bils (2001)). Here, we also show that the wage distributions of movers and stayers are significantly different. Building on this pre- and post-move wage comparisons, we then present new evidence on the evolution of wages over time following the move.

2.3.1 The mover-stayer wage gap

Consider workers moving from Service to Manufacturing between time $t-1$ and t. Their wage before changing industries is 74 percent of the average wage among those remaining in the Service sector. Once these movers arrive at Manufacturing, on average their wage is 70 percent of the Manufacturing incumbents. Once the movers from Manufacturing arrive at the Service sector, on average their wage is 74 percent of the Service sector incumbents. Below we measure the mover-stayer wage gap, controlling for worker characteristics such as age and education along with sector effects. The pattern also holds for the reverse flow.

2.3.2 The wage distribution among movers versus stayers

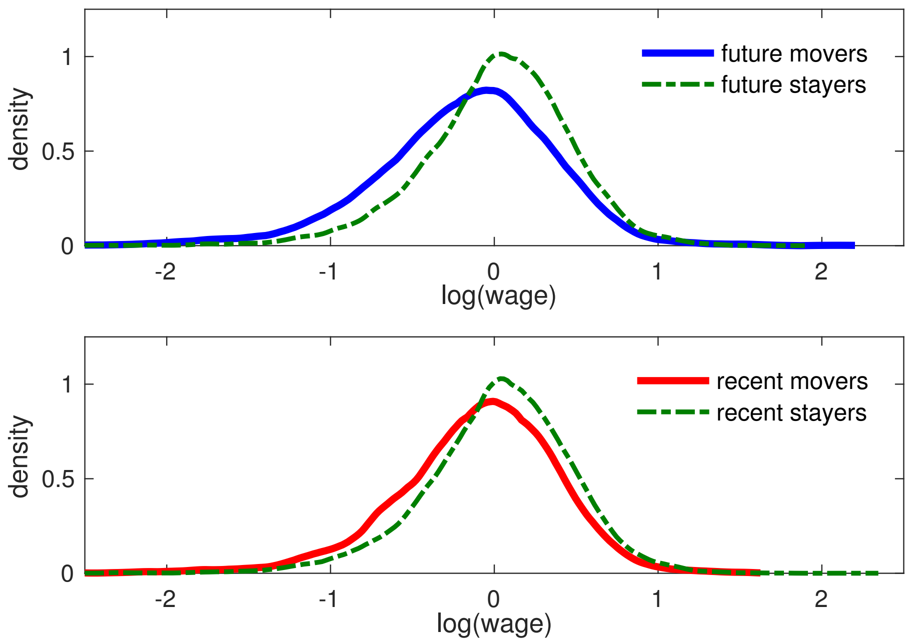
The aforementioned facts, movers have lower wages before and after a sectoral change, suggests that movers and stayers may be distinctly different. For example, the same factors influencing low wages may also be related to a worker's propensity to move. To examine this, in Figure 1, we plot the wage distributions of movers and stayers, before and after the move, respectively. The distributions in Figure 1 suggest that movers and stayers might have different wage distributions. To validate this, we perform a formal statistical test of the equality of the two distributions. According to the two-sample nonparametric Kolmogorov-Smirnov test, the wage distribution of movers is different than the wage distribution of stayers, both before and after mobility, with highly significant p-values of 0.001. Thus, movers are indeed drawn significantly more from the lower half of the wage distribution.

2.3.3 The wage-tenure profile

Exploring the evolution of wages after arriving in the new sector represents one contribution of this paper. To understand the full evolution of wages before and after a move, we plot the log hourly wage difference between movers and stayers as a function of sectoral tenure, starting one year before the move occurs and following a worker for up to 12 years of tenure in the new sector. Figure 2 plots this wage-tenure profile, controlling for different worker characteristics, including industry, year, age, and education. In Figure 2, it is evident that a worker's wage is below the group median prior to, and after the move. Importantly, wages increase with tenure in the new sector, but do so relatively slowly. Wages remain below the median of identical workers (in observable characteristics) for about ten years following mobility. This wage-tenure profile is central to our dynamic analysis of wages and mobility below.

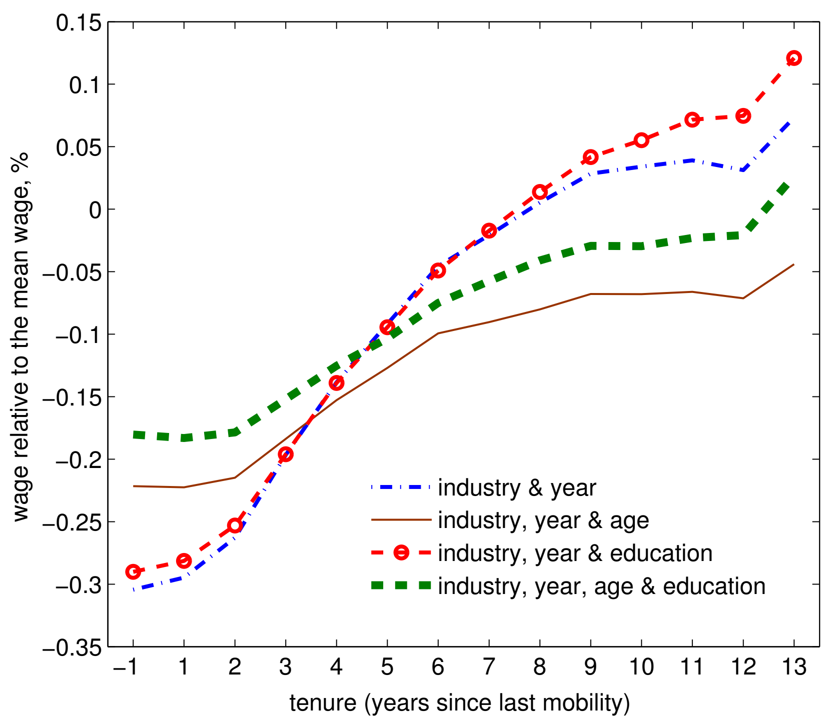
Previous work on the relationship between wages and tenure have focused on determining whether or not a statistically significant positive relationship exists. For example, Topel, 1991 finds that wages increase with tenure in a particular job, and Kambourov and Manovskii (2009b) find a positive relationship between wages and occupational tenure. While these studies uncover the positive relationship between wages and tenure, they do not shed light on how dynamic match productivity affects the wage-mobility relationship and the wage-tenure profile.

Figure 1: The Wage Distribution of Movers and Stayers



Notes: The figure compares the log-wage distributions of industry movers and stayers in PSID. The log-wages are measured as the residuals of a regression of the real log wages on year, industry, age group, education group dummies and their interactions. The upper panel displays the kernel density estimates of the residual log-wage distributions of those who will be working in a different industry in the next year (*future movers*) and those who will be working in the same industry in the next year (*future stayers*). The lower panel displays the kernel density estimates of the residual log-wage distributions among those who moved within the last year (*recent movers*) and those who did not change their industry within the last year (*recent stayers*). According to the finite sample, nonparametric Kolmogorov-Smirnov test, the distributions within each panel are different at a significance level of 0.001. So, movers are drawn significantly more from the lower half of the wage distribution (see Section 2.3).

Figure 2: The Wage-Tenure Profiles in PSID



Notes: This figure plots the mean log hourly wage difference between movers and stayers as a function of sectoral tenure. Tenure of 1 means the first year after mobility. Tenure of -1 refers to the wage difference in the year before mobility. The difference is measured within the specific group the individual belongs to. Specifically, the profile labeled "industry & year" plots the average wage difference within industry-year cells. The other profiles control for different effects, including age and education. Each profile is smoothed using local polynomials. We consider four broad industries: Agriculture, Manufacturing and mining (hereafter Manufacturing), Services, and Public Sector.

2.4 New evidence on mobility and lifetime earnings

The link between sectoral mobility (computed from the 1968-1980 Retrospective Files) and lifetime earnings (computed from the full sample, 1968-2007) is now examined. Identifying this relationship plays an important role in the quantitative analysis in Section 5.

The logical outcome of our results above is the following. Mobility is associated with lower wages both before and after a move. Given this, one may ask whether lower wages are caused by a transitory effect, or if they stem from persistent productivity differences. In the latter case, one may expect to see a substantial negative relationship between lifetime earnings and mobility. Indeed, we find that individuals with lower lifetime earnings tend to be more mobile in the PSID.

2.4.1 Individual-level mobility

To measure an individual's propensity to move, we construct several mobility indexes. First, consider the most parsimonious index: the individual-specific mean of the mobility dummy (over the period covered by the Retrospective Files). Denoting this index by $\mathcal{M}_i^{\mathrm{a}}$, we define it as: $\mathcal{M}_i^{\mathrm{a}} = \frac{1}{T_i-1}\sum_{t=1}^{T_i-1} m_{it}$, where T_i is the number of years of observations for individual i and m_{it} is a dummy variable for changing industries between the periods $t-1$ and t.

Second, to control for the fact that mobility varies with age and education (see Table A.2), consider the following normalized index: $\mathcal{M}_i^{\mathrm{b}} = \frac{1}{T_i-1}\sum_{t=1}^{T_i-1} m_{it}/\tilde{m}_{it}$, where $\tilde{m}_{it}$ is the average mobility rate among individuals in the same age and education group as person i at time t. This measures average mobility for the *individual*, relative to otherwise similar workers. To ensure the robustness of these measures, we also compute quantile versions of these two indexes. Let $\mathcal{M}_i^{\mathrm{c}}$ and $\mathcal{M}_i^{\mathrm{d}}$ denote the quantile versions of $\mathcal{M}_i^{\mathrm{a}}$ and $\mathcal{M}_i^{\mathrm{b}}$, respectively.[6]

2.4.2 Life-time earnings

Similarly, lifetime earnings is measured with several indexes. $\mathcal{E}_i^{\mathrm{a}}$ is the individual fixed effect estimated from a fixed-effect regression of log hourly wage on total sector

[6]Because of the finite number of age and education cells and the low mobility rate, more than one person can share a particular value of the level index $\mathcal{M}_i^{\mathrm{a}}$ or $\mathcal{M}_i^{\mathrm{b}}$. For example, there are 37 unique values of $\mathcal{M}^{\mathrm{a}}$. To avoid any possible bias caused by the discrete nature of the indexes, we construct the quantile indexes by imposing the same quantile on those who are tied to the associated level index.

Table 1: Individual-Level Mobility and Life-Time Earnings

Individual-level mobility	*Lifetime earnings*			
	$\mathcal{E}^{\mathrm{a}}$	$\mathcal{E}^{\mathrm{b}}$	$\mathcal{E}^{\mathrm{c}}$	$\mathcal{E}^{\mathrm{d}}$
$\mathcal{M}^{\mathrm{a}}$	-0.200	-0.141	-0.205	-0.124
$\mathcal{M}^{\mathrm{b}}$	-0.157	-0.152	-0.161	-0.145
$\mathcal{M}^{\mathrm{c}}$	-0.220	-0.168	-0.226	-0.155
$\mathcal{M}^{\mathrm{d}}$	-0.189	-0.172	-0.197	-0.169

Notes: *Individual-level mobility* ($\mathcal{M}$) refers to the number of moves a worker made during the sample period. *Lifetime earnings* ($\mathcal{E}$) measures the average of the residual log hourly wages of a particular worker over the sample period. The table displays pairwise correlations of various measures of the two variables described in Section 2.4. The p-values associated with these correlations are all less than 0.01.

experience, and dummy variables for age, year, state, education, and sector. $\mathcal{E}_i^{\mathrm{b}}$ is the individual-specific mean residual from an OLS regression of log hourly wage on total sector experience, and full sets of dummies (age, year, state, education, and sector). This life-time earnings index is proposed by Moffitt and Gottschalk (2002) using PSID. $\mathcal{E}_i^{\mathrm{c}}$ and $\mathcal{E}_i^{\mathrm{d}}$ are the quantile versions of $\mathcal{E}_i^{\mathrm{a}}$ and $\mathcal{E}_i^{\mathrm{b}}$, respectively.

2.4.3 Correlation of life-time earnings and mobility

Table 1 displays the correlations for each pair lifetime earnings and mobility indices. All of the correlations are negative and significant, indicating that individuals with lower lifetime earnings are more mobile.

On the surface, the aforementioned relationships between wages and mobility appear in contradiction to the theory that workers move to pursue better employment opportunities and wages. What a worker's wage *would have been* had they decided to stay in the original sector is unobservable, however. This represents the key difficulty in drawing conclusions from the data. To disentangle the potential forces driving the patterns established above, the next section presents a dynamic model with joint determination of wages and mobility.

3 Model

To uncover the forces underlying the relationship between wages and sectoral mobility, the model builds on McLaughlin and Bils (2001) who use Roy's (1951) framework to study mover-stayer wage differences. As mentioned earlier, McLaughlin and Bils (2001) consider a two-period model with permanent match quality. In contrast, we consider a dynamic Roy model with a *persistent* idiosyncratic productivity shock specific to the worker-sector match. Also, as in Rogerson (2005) and Kambourov and Manovskii (2009a), in our model, workers may acquire a sector-specific skill premium and workers within a particular sector are subject to a common productivity shock referred to as the sectoral shock. The sectoral shock causes net mobility, while the worker-sector match shock creates excess mobility.

3.1 Environment

We present the model in terms of two sectors. The model, however, can be recast as an economy with $N > 2$ sectors. (Section 6.1 provides further details.) The two sectors are denoted by 0 and 1. Each sector is inhabited by a large number of workers. A worker's wage in a particular sector is determined by three components: a sector-specific skill premium, a sectoral shock, and the worker-sector match shock.

3.1.1 The sector-specific skill premium

For sector-specific skill, we adopt the specification of Kambourov and Manovskii (2009a). Individuals are either skilled or unskilled in their current sector, and a worker can only be skilled in one sector at a time.[7] A skilled worker is more productive than an otherwise identical unskilled worker (in the same sector). In each period, an unskilled worker becomes skilled in the current sector with probability p. Let π denote the skill premium that this worker receives. Notice that the longer an agent remains in the current sector, the more likely they are to be skilled; therefore, tenure is required to become skilled. In addition, this skill premium is "general" in the sense that all skilled workers within a sector receive the same premium. This is in contrast to the idiosyncratic dynamic comparative advantage we introduce below. Each period

[7]See Lazear (2009) and Gathmann and Schonberg (2009) for alternative views of human capital where there is transferability of skills across sectors. Also, similarly to Kambourov and Manovskii (2009a), we do not allow workers to exert effort to increase their specific skills.

a worker exits labor market with probability δ, while newly born workers enter the economy.

3.1.2 The sectoral shock

There exists a sectoral shock. It affects the productivity of all workers in one sector relative to the other sector. Specifically, all workers in sector 1 are subject to the shock z_t. This shock has a stationary transition function $\Pr(z_{t+1} < z' \mid z_t = z) = G(z' \mid z)$ given by the following autoregressive process:

$$z_{t+1} = rz_t + u_t, \tag{1}$$

where $r \geq 0$ and u_t is a zero-mean random variable. Let s denote the unconditional standard deviation of z_t: $s = \mathrm{Std}(z_t)$.

3.1.3 The match shock

Each worker's productivity is subject to an idiosyncratic shock. The magnitude of the shock depends on the worker's current sector of employment. Let (e_0, e_1) denote these labor income shocks. The pair of shocks is drawn for each worker in each period. The labor income shocks e_0 and e_1 are correlated across time and sectors, and given by the following autoregressive process:

$$\begin{cases} e_{0,t} = \rho e_{0,t-1} + u_{0,t} \\ e_{1,t} = \rho e_{1,t-1} + u_{1,t}, \end{cases} \tag{2}$$

where $\rho \geq 0$ and the innovations $(u_{0,t}, u_{1,t})$ are drawn from a zero-mean, bivariate distribution.

3.2 Wages

The current wage for workers in sector 0 is given by,

$$w_{0,t} = \omega + \pi h_{0,t} + e_{0,t} \tag{3}$$

and for workers in sector 1,

$$w_{1,t} = \omega + \pi h_{1,t} + e_{1,t} + z_t, \tag{4}$$

where $h_{j,t}$ is an indicator variable equal to 1 if the worker is skilled in sector $j \in \{0, 1\}$ at time t, or equal to zero otherwise. The constant term ω is introduced merely to normalize the average wage to 1. Below we decompose the shocks $(e_{0,t}, e_{1,t})$ to further simplify the wages in equations (3) and (4).

3.3 Comparisons with earlier models

As mentioned earlier, Jovanovic and Moffitt (1990) and McLaughlin and Bils (2001) are among the first to analyze the wage difference between movers and stayers.

3.3.1 Undirected mobility and exogenous distribution

In Jovanovic and Moffitt (1990), idiosyncratic productivity remains constant if a worker stays in the current sector, but can be drawn from an *exogenous* distribution if the worker moves to a new sector. Moreover, in Jovanovic and Moffitt (1990), a worker leaves the current sector without knowing match productivity at the destination. For example, consider a worker in sector 0. If e_0 is too low, the worker decides to move to sector 1. In Jovanovic and Moffitt (1990), for such a worker, e_1 is realized only after the person arrives at sector 1, and e_0 and e_1 are uncorrelated. In other words, a worker moves across sectors without knowing his or her wage at the destination. This type of mobility is referred to as undirected mobility. In the literature, the main reason for using undirected mobility is computational. Under the assumption of undirected mobility, the state space of the dynamic problem and the simulation reduces substantially (see, for example, Coen-Pirani, 2010 and Lkhagvasuren, 2012).[8]

3.3.2 Directed mobility with permanent productivity

In McLaughlin and Bils (2001), however, a worker's productivity across sectors are correlated and known to the worker. In other words, a worker knows his or her wage at the destination before making the mobility decision. This feature of their model is inherent in Roy's (1951) model and present in much of the literature on sectoral selection (see, for example, Heckman and Taber, 2008). This key feature of Roy's model is referred to as directed mobility, and appears essential to analyzing wages, especially when considering the wage gap between movers and stayers.

[8]As explained below in Section 5.1, under directed mobility, keeping the record of sectoral history and wages of each worker and using this information for every iteration of the calibration procedure represents a major computational demand for our analysis.

However, in McLaughlin and Bils (2001), match productivity is permanent (*i.e.*, e_0 and e_1 are constant over time). Therefore, their model considers only net mobility driven by the z shock, but ignores large excess flows discussed in Section 2.2.

3.3.3 Directed mobility with evolving match quality and skills

The main deviation of our model from the models in earlier work on the wage gap between movers and stayers is the combination of two elements: (i) directed mobility, and (ii) evolving match quality. With this combination, the wage distribution gap among movers versus stayers arises endogenously, and thus there is no need to specify additional and different restrictions on the income processes of stayers versus movers. This is important, especially when one analyzes the wage-mobility relationship.

Furthermore, in the models of both Jovanovic and Moffitt (1990) and McLaughlin and Bils (2001), a worker lives for only two periods, and there is no skill premium. We extend these two-period, pre- and post-move wage comparison analyses by tracing the movers wage over their lifetime while considering a skill premium and an evolving match shock.

3.4 Timing of the events

In our model, each period consists of four stages. In the first stage, individuals observe the sectoral shock, z, and the match shock e_i. In the second stage, after observing these shocks, individuals decide which sector to work in. A worker moving between sectors start as unskilled in the new sector. In the third stage, each worker supplies one unit of labor and receives the wage. That is, production or work occurs during the third stage. In the fourth stage, some of the unskilled workers become skilled. Simultaneously, some workers leave the labor market and new, unskilled workers enter the market.

From a data-to-modeling perspective, our analysis requires matching moments for employment to employment changes in *wages*. Thus, productivity represents the key variable in the model. While we do not explicitly include unemployment in the model, one could view unemployment as a particularly bad productivity shock either via $e_{j,t}$ or via z_t. Although this does not capture the full essence and potential impact of unemployment, given the data limitations (for example, both the wage and sectoral mobility in the PSID can be measured only at an annual frequency), this is the most appropriate modeling strategy. Our parameterization should capture the effects of unemployment on wages via the two shock processes, $e_{j,t}$ ($j = 0, 1$) and z_t.[9]

3.5 Decomposing the match shock

While the wage is subject to the idiosyncratic shocks e_0 and e_1, a certain component of the two shocks are not important for our main purposes as these components do not affect mobility and the mover-stayer wage gap. To see this, let us consider the following decomposition of the innovations of e_0 and e_1:

$$\begin{cases} u_{0,t} = \zeta_t + \epsilon_t \\ u_{1,t} = \zeta_t - \epsilon_t, \end{cases} \tag{5}$$

where ζ_t and ϵ_t are *independent*, zero-mean, transitory shocks. Then, by repeatedly substituting for the lagged values of e_0 and e_1 in equation (2), one can write

$$\begin{cases} e_{0,t} = y_t + x_t = \rho(y_{t-1} + x_{t-1}) + \zeta_t + \epsilon_t \\ e_{1,t} = y_t - x_t = \rho(y_{t-1} - x_{t-1}) + \zeta_t - \epsilon_t, \end{cases} \tag{6}$$

where x_t and y_t are independent shocks. According to this decomposition, x_t and y_t are independent AR(1) processes with the common persistence ρ. More important, x_t is the part of the match shock that drives a worker's mobility decision, thus it is important for mobility and the wage gap between movers and stayers (see Appendix B.1 for a more detailed discussion). For this consideration, below we characterize workers' mobility decision using $(x_t, -x_t)$ shocks, rather than $(e_{0,t}, e_{1,t})$ shocks.

3.6 Re-defining the wages

Given the above decomposition, redefine the wages of those working in sector 0 as

$$w_{0,t} = \omega + \pi h_{0,t} + x_t, \tag{7}$$

and the wages of those working in sector 1 as

$$w_{1,t} = \omega + \pi h_{1,t} - x_t + z_t. \tag{8}$$

[9]See Moscarini (2001), Rogerson (2005), Lkhagvasuren (2012) and Carrillo-Tudela and Visschers (2014) for related multi-sector models of unemployment.

We solve the model using the wages in equations (7) and (8), meaning that the y shock in equation (6) is omitted in the model. This is because the y shock does not affect a worker's mobility decision, the mean wage gap between movers and stayers, and the wage tenure profile. However, when we calculate certain simulated moments that require the wage dispersion, we take into account the effect of the y shock (see Section 5.3.5 and Appendix B.10).

Now, in the model, individual productivity is *perfectly-negatively* correlated across sectors; the best-matched workers in sector j are the worst-matched workers of sector $1-j$.[10] For example, suppose an unskilled worker currently employed in sector 1 receives a shock $x_t > 0$ at the beginning of period t. This shock makes the worker more productive in sector 0 relative to sector 1 when $z_t = 0$. Therefore, the worker may prefer to move to sector 0. Rogerson (2005) and Moscarini and Vella (2008) consider similar, *perfectly-negatively* correlated individual productivity across sectors. However, unlike in their models, sector-specific productivity is stochastic in the current model.

Let $F(x' \mid x)$ denote the transition function given by the following AR(1) process

$$x_{t+1} = \rho x_t + \epsilon_{t+1}, \tag{9}$$

and let σ denote the standard deviation of x_t: $\sigma = \mathrm{Std}(x_t)$.

3.7 Value functions

Let $U_j(h, x, z)$ denote the lifetime utility of a worker in sector $j \in \{0, 1\}$ with skill level $h \in \{0, 1\}$, where x and z represent the match and sector shocks, respectively. This represents the utility associated with the moment following the realization of the shocks, but preceding the mobility decision.

3.7.1 Skilled stayers

For a skilled worker in sector j, the lifetime utility of staying in j is given by

$$S_j(1, x, z) = w_j(1, x, z) + \beta(1-\delta) \iint U_j(1, x', z') dF(x' \mid x) dG(z' \mid z), \tag{10}$$

[10] As stated earlier, the model can be recast as an economy with $N > 2$ sectors, by interpreting x as the worker's match shock in the current sector (*i.e.*, sector 0), and $-x$ as the highest of the $N-1$ match shocks from the remaining $N-1$ sectors. Section 6.1 provides further details.

where β is the time-discount factor.

3.7.2 Unskilled stayers

For an unskilled worker in sector j, the lifetime utility of staying in j is given by

$$\begin{aligned} S_j(0,x,z) = \quad & w_j(0,x,z) + \beta(1-\delta)\left\{ p \iint U_j(1,x',z')dF(x' \mid x)dG(z' \mid z) \right. \\ & \left. +(1-p)\iint U_j(0,x',z')dF(x' \mid x)dG(z' \mid z) \right\}. \end{aligned} \tag{11}$$

3.7.3 Movers

The lifetime utility for a worker moving from sector j to $1-j$ is given by

$$M_j(x,z) = S_{1-j}(0,x,z). \tag{12}$$

3.7.4 The mobility decision

Given the value functions S_j and M_j, the lifetime utility of a worker with skill level h is given by

$$U_j(h,x,z) = \max\{S_j(h,x,z), M_j(x,z)\}. \tag{13}$$

Let Ω_j denote the decision rule governing whether a person in sector j stays in her current sector:

$$\Omega_j(h,x,z) = \begin{cases} 1 & \text{if } S_j(h,x,z) \geq M_j(x,z), \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

Further details of the mobility decision are provided in Appendix B.2.

3.8 Measures

Let τ denote the number of periods a person has worked in their current sector for (since entering the labor market or since the last move). This sector tenure τ is measured at the end of each period, *i.e.*, $\tau \in \{1,2,3,\cdots\}$. At any t, a worker in sector j is fully characterized by her skill level h, match shock x, and sector tenure τ. Let $\mu_{j,t}(h,x,\tau)$ denote the number of workers in state (h,x,τ) in sector j at the end of period t. Next period's measure $\mu_{j,t+1}(h,x,\tau)$, $j \in \{0,1\}$, is determined by the current measures $(\mu_{0,t}(h,x,\tau), \mu_{1,t}(h,x,\tau))$ and next period's sectoral shock z_{t+1}.

The total number of workers in the economy is given by

$$L_t = \sum_h \sum_\tau \int (\mu_{0,t}(h, x, \tau) + \mu_{1,t}(h, x, \tau))dx \tag{15}$$

for all t. The number of new workers born in sector j is proportional to the measure of the unskilled workers of the sector: for all x,

$$\mu_{j,t}(0, x, 0) = \frac{\delta L_t \sum_{\tau \geq 1} \int \mu_{j,t}(0, x, \tau)dx}{\sum_{\tau \geq 1} \int (\mu_{0,t}(0, x, \tau) + \mu_{1,t}(0, x, \tau))dx}. \tag{16}$$

The law of motion governing sectoral dynamics and the definition of the equilibrium are contained in Appendices B.3 and B.4.

4 What drives the wage-mobility relationship?

Before continuing to the quantitative analysis of the model, it is useful to discuss how mobility and wages are interrelated in this dynamic extension of Roy's (1951) model, and what drives the wage gap between movers and stayers.

4.1 The skill premium

It is straightforward to see how the skill premium affects wages and mobility. First, with a skill premium, a skilled worker has a higher wage than an unskilled worker and thus moves less frequently than an unskilled worker. Second, because of directed mobility, when moving across sectors, skilled workers "require" higher wages at the destination. These effects generate a positive wage gap between stayers and movers. Furthermore, since those with higher tenure are more likely to be skilled, the skill premium generates a positive wage-tenure profile. These predictions are consistent with many data features discussed in Section 2. The skill premium, however, does not represent the only explanation for the aforementioned wage-mobility patterns. We now discuss an alternative explanation.

4.2 The match shock in a simple economy

A persistent match shock can also generate the key features of the wage-mobility relationship. To demonstrate this effect both numerically and analytically, we consider a simplified version of the model with no skill premium and no sector-wide shock where workers are infinitely-lived; *i.e.*, $\pi = 0$, $z_t = 0$ for all t, and $\delta = 0$. This version of the model is referred to as the simple economy.

In this simple economy, if a worker decides to work in sector 0, the wage will be $\omega + x_t$. However, if the worker decides to work in sector 1, the wage will be $\omega - x_t$. This implies the following simple decision rule:

$$\begin{cases} \text{a worker in sector 0 moves at period } t \text{ if } x_t < 0, \text{ or stays if } x_t \geq 0; \\ \text{a worker in sector 1 moves at period } t \text{ if } x_t > 0, \text{ or stays if } x_t \leq 0. \end{cases}$$

4.2.1 Simple illustrative simulation

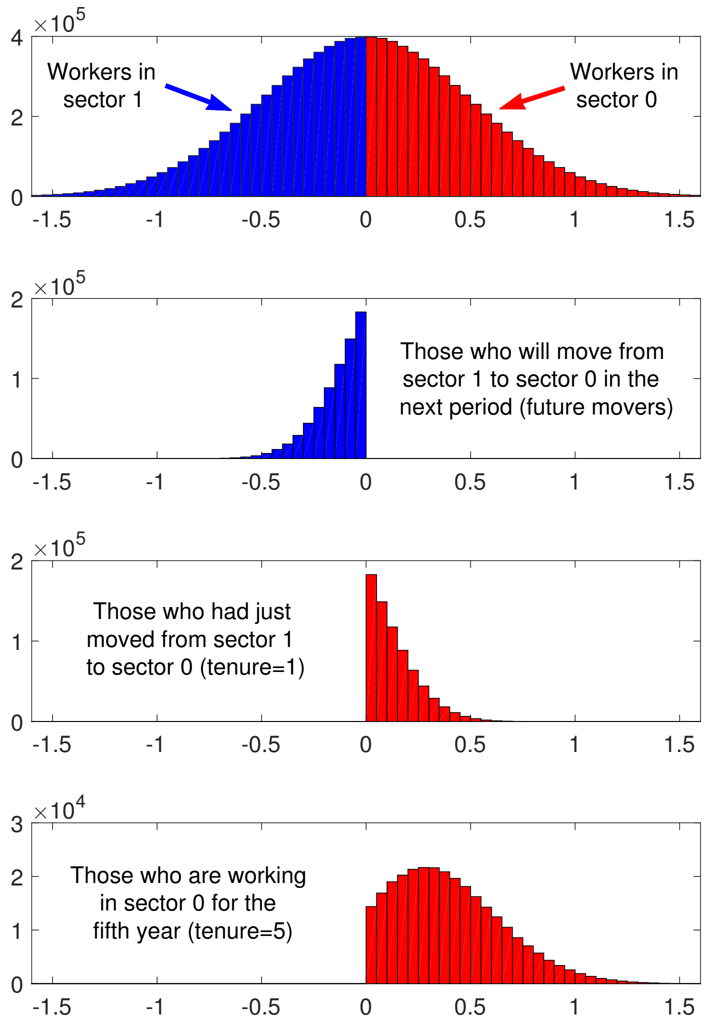
In this numerical example, we set the persistence of the match shock, ρ, to 0.9 and the unconditional standard deviation of the match shock, σ, to 0.5. We start with 10 million workers, and simulate the match shocks and mobility of these workers according to the simple algorithm outlined in Appendix B.5.

We summarize the main pattern of the distribution of the match shock in Figure 3. The distribution of all workers by the x shock is shown in the topmost (first) panel of Figure 3. The distribution is a mixture of two distributions. Specifically, because of the decision rule, the distribution of those in sector 0 is the upper half of the distribution for all workers, while the distribution for those in sector 1 is given by the lower half of the distribution for all workers.

In the second panel from top, we display the distribution of those in sector 1 who will move to sector 0 in the next period. Comparing this distribution of future movers with the lower half of the distribution in the topmost panel, it can be seen that the average shock for those who will move from sector 1 to sector 0 in the next period is higher than the average shock of the workers in sector 1. Since for sector 1 workers, the wage is negatively related to the match shock (*i.e.*, $w_{1,t} = \omega - x_t$), the average wage of future movers is lower than the average wage of stayers.

In the third panel, we display the distribution of shocks for those who moved from sector 1 to sector 0 (*i.e.*, those who are working for the first year in sector 0 since

Figure 3: The Simple Economy: Distribution of Workers by the Match Shock



Notes: This figure shows the distribution of the match shock in a simple economy with no skill accumulation, no sector-wide shock and no labor market exit ($\pi = 0$, $z_t = 0$ for all t and $\delta = 0$). In such a simple economy, the decision rule is given by the x shock only, as explained in Section 4.2. Recall that as shown in equations (7) and (8), the wages in sector 0 are positively related to the match shock x, while the wages in sector 1 are negatively related to the match shock x. The figure is based on 10 million workers. It shows that, on average, movers have a lower wage than stayers at both the origin and destination, and the average wage grows with sectoral tenure. See Propositions 1 to 3 and Figure 4 for further details.

their last move). Comparing this distribution with the upper half of the distribution in the topmost panel, the average value of the x shock among these newcomers is lower than the average shock of those in sector 0. Since for sector 0 workers, the wage is positively related to the match shock (*i.e.*, $w_{0,t} = \omega + x_t$), the average wage of newcomers is lower than the average wage of incumbents.

Finally, in the bottom panel, we show the distribution of match shocks for those working in sector 0 for the fifth consecutive year since their last move. Comparing this distribution with the distribution of the newcomers in the third panel, the average wage increases with sectoral tenure.

To further illustrate the role of a persistent match shock, we explore how the average value of the match shock evolves with sectoral tenure. In addition to the aforementioned simulated economy, we also consider two different values of ρ while keeping the standard deviation of the shock at 0.5 (*i.e.*, $\sigma = 0.5$). Those two values are $\rho = 0.5$ (less persistent) and $\rho = 0$ (transitory). The results are summarized in Figure 4. It shows that when the match shock is transitory, there is no wage gap between movers and stayers, and the wage-tenure profile is constant. More important, the figure shows that persistence of the match shock raises the wage gap between movers and stayers as well as the overall slope of the wage-tenure profile.

We now provide the analytical proof of these effects.

4.2.2 Analytical results

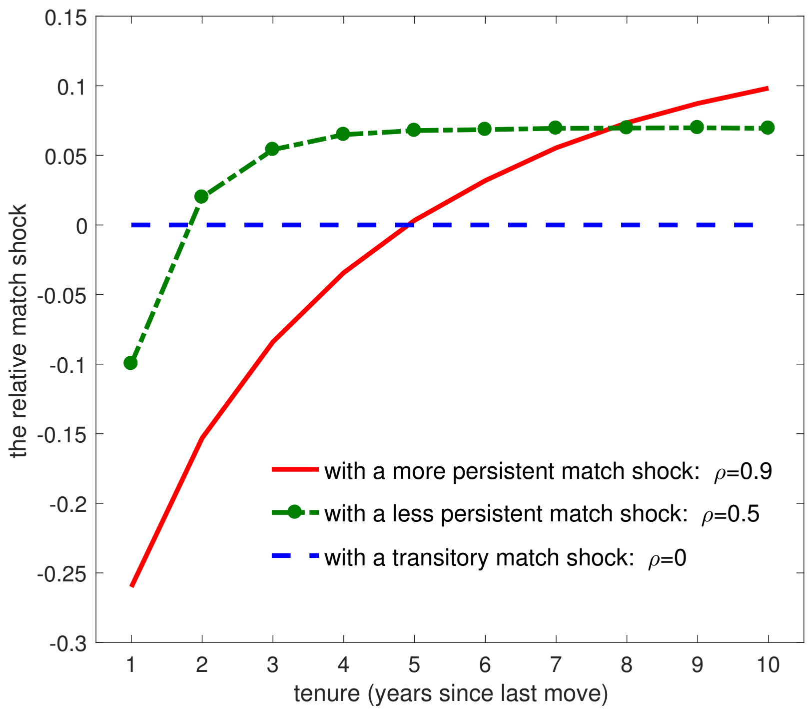
When the match shock is transitory, the average match shock remains constant with sectoral tenure. Specifically, the mean value of the match shocks at each level of sectoral tenure is given by $\mathbf{E}\big(\epsilon_t \mid \epsilon_t \geq 0\big)$. Then, with the transitory shock, the wage-mobility relationship in the simple economy can be summarized by the following claim.

Claim 1 (A transitory shock). *When the match shock is transitory (*i.e.*, $\rho = 0$),*

1. *the mean wage of newcomers is equal to the mean wage of incumbent workers,*
2. *the mean wage of future movers is equal to the mean wage of future stayers,*
3. *the mean wage of recent movers remains constant with sectoral tenure, and*
4. *overall earnings and individual-level mobility are uncorrelated over a finite number of periods.*

Proof. See Appendix B.6. □

Figure 4: The Simple Economy: The Average Match Shock by Sectoral Tenure



Notes: This figure shows wage-tenure profiles in a simple economy with no skill accumulation, no sector-wide shock and no labor market exits ($\pi = 0$, $z_t = 0$ for all t and $\delta = 0$). The three profiles are associated with three different values of ρ, persistence of the match shock x. The unconditional standard deviation of the match shock, σ, is set to 0.5. Each wage-tenure profile is based on the wages and mobility of 10 million workers. The vertical axis measures the magnitude of the match shock minus the mean value of the match shock in sector 0. The mean value of the match shock of sector 0 is given by $\mathbf{E}\big(x \mid x \geq 0\big)$ and equal to 0.399. Recall that the wages in sector 0 is positively related to the match shock. The figure shows that persistence of the match shock raises both the mover-stayer wage gap, in absolute terms, and the overall slope of the wage-tenure profile. See Propositions 1 to 3 and Figure 3 for further details.

However, when the match shock is persistent, the wage-mobility relationship becomes akin to that in data. We first look at the wage gap between movers and stayers.

Proposition 1 (**A persistent match shock and newcomers**). *When the match shock is nondegenerate and persistent (*i.e.*, $\sigma > 0$ and $\rho > 0$), the mean wage of the newcomers is lower than the mean wage of the incumbent workers.*

Proof. See Appendix B.7. □

Proposition 2 (**A persistent match shock and future movers**). *When the match shock is nondegenerate and persistent (*i.e.*, $\sigma > 0$ and $\rho > 0$), the mean wage of future movers is lower than the mean wage of future stayers.*

Proof. See Appendix B.8.

□

Next we analyze how a persistent match shock affects the wage-tenure profile.

Proposition 3 (**A persistent match shock and the wage-tenure profile**). *When the match shock is nondegenerate and persistent (*i.e.*, $\sigma > 0$ and $\rho > 0$), the average wage increases with sectoral tenure, over any finite range of sectoral tenure.*

Proof. See Appendix B.9. □

The results in Propositions 1 to 3 imply that, when the match shock is persistent, on average, both future movers and recent movers, and that the negative wage gap between movers and stayers persist over a finite number of periods. Thus, when the match shock is nondegenerate and persistent (*i.e.*, $\sigma > 0$ and $\rho > 0$), total earnings and individual-level mobility are negatively related, over a finite number of periods.

4.3 Discussion

We have shown that under directed mobility, not only a sector-specific skill premium, but also a persistent worker-sector match shock can generate the main patterns of the wage-mobility relationship in PSID. For example, in order to account for the substantial wage gap between movers and stayers in data, one can use either a highly persistent match shock in the absence of the skill premium, or the combination of a substantial skill premium and a transitory match shock. However, as shown below, these two approaches have very different quantitative implications on the labor mobility pattern. Therefore, in order to understand the relative role of the skill premium and the match shock, one needs to consider the actual mobility pattern measured in data. It is important to keep this in mind when going to the quantitative analysis in Section 5.

5 Quantitative analysis

The model in Section 3 provides a flexible framework to analyze the relationship between wages and mobility. This section examines, quantitatively, how well the model captures the main facts in Section 2. Importantly, this quantitative analysis also examines what role the dynamic match shock (*i.e.*, comparative advantage) plays in the evolution of wages and mobility.

5.1 Computation

Accounting for both the dynamic match and sectoral shocks in the presence of the sector-specific skill premium implies a computationally intensive task. Moreover, one must keep track of wages for each worker by their sectoral tenure. Thus, both the solution and simulation imply a stochastic dynamic problem with a large state space.

5.1.1 Numerical solution

The model is solved by discretizing the state space along x and z. The sectoral technology shock, z, is approximated by a three-state Markov chain. A relatively fine grid for x is necessary to generate the observed level of mobility and the wage gap between movers and stayers. For this reason, the stochastic process for x is approximated by a 51-state Markov chain. The Markov chains are constructed using the finite-state process of Rouwenhorst (1995).[11] Then, using value function iteration, we find the decision rule in equation (14) for each sector j and for each discrete value of x, z and h.

5.1.2 Simulation

To simulate the model, we draw the sequence of the three-state shock, z, for $T = 2000$ periods while keeping track of the distribution of heterogeneous agents over (j, h, x, τ).

[11] Galindev and Lkhagvasuren (2010) show that the method of Rouwenhorst (1995) outperforms the other commonly used discretization methods for highly persistent AR(1) shocks.

The first 500 periods are discarded, and T is set so that increasing it does not affect the moments. For the initial measures, $\mu_{j,0}(h, x, \tau)$, $j \in \{0, 1\}$, we consider the case that all workers are unskilled (*i.e.*, $h = 0$) and distributed equally between the two sectors.

To measure life-cycle income and individual-level mobility, we consider the wages and mobility decisions of 50,000 individuals. The other moments are measured by normalizing the total number of workers to 1. It should be emphasized that simulating the wage and mobility of these workers is not that computationally demanding per se. However, keeping the record of sectoral history and past wages of each worker and using this information for every iteration of the calibration procedure (finding the minimum distance between the empirical and targeted moments) imposes the main computational demand.

5.2 Calibration details

Several of the model parameters can be set directly from the data. The remaining parameters are calibrated to match certain moments in the data.

5.2.1 Parameters calibrated directly from data

The time period is one year. We set $\beta = 1/1.04$, consistent with an annual interest rate of 4 percent. The probability of leaving the labor market (or retiring), δ, is set to 0.025, implying an expected working lifetime of 40 years. The probability of becoming skilled, p, is set by following Kambourov and Manovskii (2009a). Specifically, we observe that the positive slope of the wage-tenure profile decreases at tenure levels of 8 to 12 years. Accordingly, we set $p = 1/10$, implying an average duration of 10 years to become skilled in a particular sector. Below we analyze how the main predictions of the model respond to the parameter p.

As mentioned earlier, the constant term of the wage function in equations (7) and (8) is introduced merely to normalize the average wage to 1. In the benchmark model, $\omega = 0.7422$. To measure the sector-specific productivity shock z_t, we use annual per-worker output from 1987 through 2012, tabulated by the Bureau of Labor Statistics (BLS). Analogous to Blanchard and Katz (1992), we measure z_t as the difference between per-worker output in the Manufacturing sector relative to per-worker productivity of the Non-Farm Business sector of the U.S., measured in log-differences. We take the standard deviation and annual autocorrelation of this relative productivity from the trend (HP filtered with smoothing parameter 100). This yields $s = 0.0068$ (standard deviation) and $r = 0.4236$ (persistence). Sensitivity of these numbers to the sample period is discussed in Appendix B.11.

5.2.2 Parameters calibrated by targeting moments

This leaves the parameters governing the match shock, ρ and σ, and the skill premium π. We calibrate these parameters by targeting three moments in data.

First, we target the annual gross mobility rate of 6.78%, which is measured in Table A.2. The second targeted moment is volatility of sectoral employment. For the volatility of sectoral employment, similarly to the case of sectoral output above, we measure sectoral employment using the Manufacturing sector relative to aggregate (non-farm business) employment. We take the log of Manufacturing employment minus the log of aggregate employment. As with per-worker output above, both employment series are tabulated by the BLS. We use annual data from 1987 to 2012 and calculate an unconditional standard deviation of Manufacturing employment (from its HP trend with the smoothing parameter 100) of 0.0059 (0.59%).

The third moment we target is repeat mobility. Repeat mobility is the probability that a worker moves, conditional on having moved in the previous period. From the PSID data, repeat mobility is 27 percent; *i.e.*, approximately one quarter of current movers move again in one period. Note that repeat mobility captures a certain feature of the distribution of workers by sectoral tenure: the number of workers with two years of tenure is 27% lower than the number of workers with one year of tenure.

The main intuition behind our calibration is the following. As mentioned earlier, overall mobility declines with the skill premium π. Also, a more persistent match shock implies a smaller probability of repeat mobility. Finally, a higher dispersion of the match shock implies lower net mobility, which in turn implies a lower volatility of sectoral employment. This is because a higher dispersion of the match shock implies fewer workers on the margin between moving and staying.[12] In fact, we show in Section 6.4 that reducing σ (the standard deviation of the match shock) raises the density of these marginal workers, making sectoral employment more volatile. The above data targets imply $\sigma = 0.1112$, $\rho = 0.5440$, and $\pi = 0.3137$. Table 2 displays the benchmark parametrization.

[12] In Figure B.1, it can be seen that there are fewer workers along the Indifference line when the dispersion of the x shock, σ, is higher.

Table 2: Benchmark Parameters

Parameters	*Values*	*Description*
Using data and normalization		
β	0.9615	time discount factor
δ	0.0250	probability of retirement
ω	0.7422	the constant term of the wage function
r	0.4236	persistence of the sector shock
s	0.0068	standard deviation of the sector shock
p	0.0100	probability of becoming skilled
Targeting moments		
π	0.3137	skill premium
ρ	0.5440	persistence of the match shock
σ	0.1112	standard deviation of the match shock

Notes: The table summarizes the key parameters of the benchmark model. The constant term of the wage function, ω, is chosen by normalizing the mean wage to 1.

5.3 Results

Table 3 presents the main results. First, the benchmark model performs well capturing the main patterns of mobility. Specifically, it generates the observed level of gross, net and repeat mobility, the main targets in our calibration.

To evaluate the model's performance, Table 3 also displays how well the model matches several un-targeted moments. Specifically, we consider movers' wage at their origin and the destination, average wage growth among recent movers, and the correlation of lifetime earnings and mobility. It is worth noting that all of the un-targeted moments are wage related, while the targeted moments are exclusively mobility related. In this regard, the model's success in capturing the un-targeted moments is promising.

5.3.1 The mover-stayer wage gap

First, consider the model's performance matching movers' wage at their origin. From Table 3, the model slightly overpredicts this wage gap. According to the data, 1 year before moving, movers have average wages 18.02% below the average wage in the origin industry. Our model predicts a wage gap of 19.60%. Next, the model also

Table 3: Main Predictions

Moments	*Data*	*Bench-mark* $p{=}\frac{1}{10}$	*Faster skill accum.* $p{=}\frac{1}{8}$	*Slower skill accum.* $p{=}\frac{1}{12}$
Moments targeted in the benchmark model				
mobility	0.0678	0.0683	0.0580	0.0778
volatility of sectoral employment	0.0059	0.0059	0.0055	0.0064
repeat mobility	0.2729	0.2247	0.2123	0.2332
Key predictions				
movers' mean wage at the origin	-0.1803	-0.1960	-0.2045	-0.1879
movers' mean wage at the destination	-0.1786	-0.1794	-0.1877	-0.1715
wage growth among recent movers	0.0259	0.0263	0.0296	0.0237
corr. of lifetime earnings and mobility	-0.1523	-0.1685	-0.1649	-0.1681

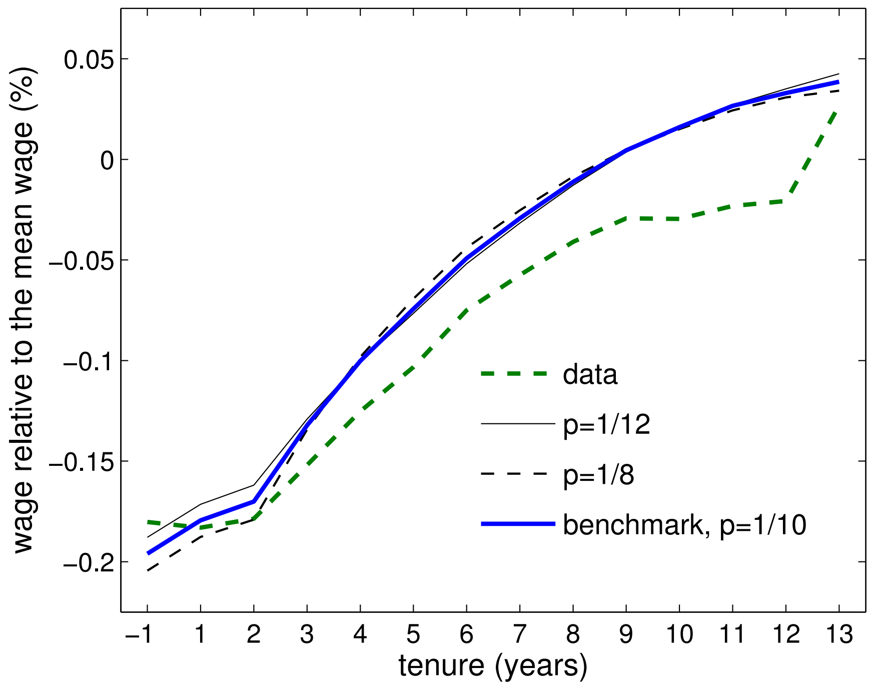
Notes: The columns denoted by *Data* and *Benchmark* summarizes moments measured from the PSID data and the benchmark models, respectively. In the benchmark model, it takes on average 10 years for workers to become skilled in a sector, *i.e.*, $p = \frac{1}{10}$. In the last two columns of the table, we examine the impact of this parameter, where the model is simulated for different values of p, holding the other parameters at their benchmark values.

slightly overpredicts the mover-stayer wage gap in the destination sector. In the data, movers' have average wages 17.86% below the average in the destination sector, while the model predicts a wage gap of 17.94%.

5.3.2 The wage-tenure profile

In Figure 5, we compare the wage-tenure profile in the data to that generated by the model. Here, we plot the evolution of wages, starting one year before the move, and following wages for 12 years after. The figure shows that the wage-tenure profile in the model lies above that in data. So, our model slightly overpredicts wage growth among recent movers. Specifically, as Table 3 shows, in the PSID data, workers experience annual wage growth of 2.59% during the first five years following mobility, while the model predicts annual wage growth of 2.63%. Nevertheless, the model is able to capture the main pattern on the wage-tenure profile.

Figure 5: The Wage-Tenure Profile



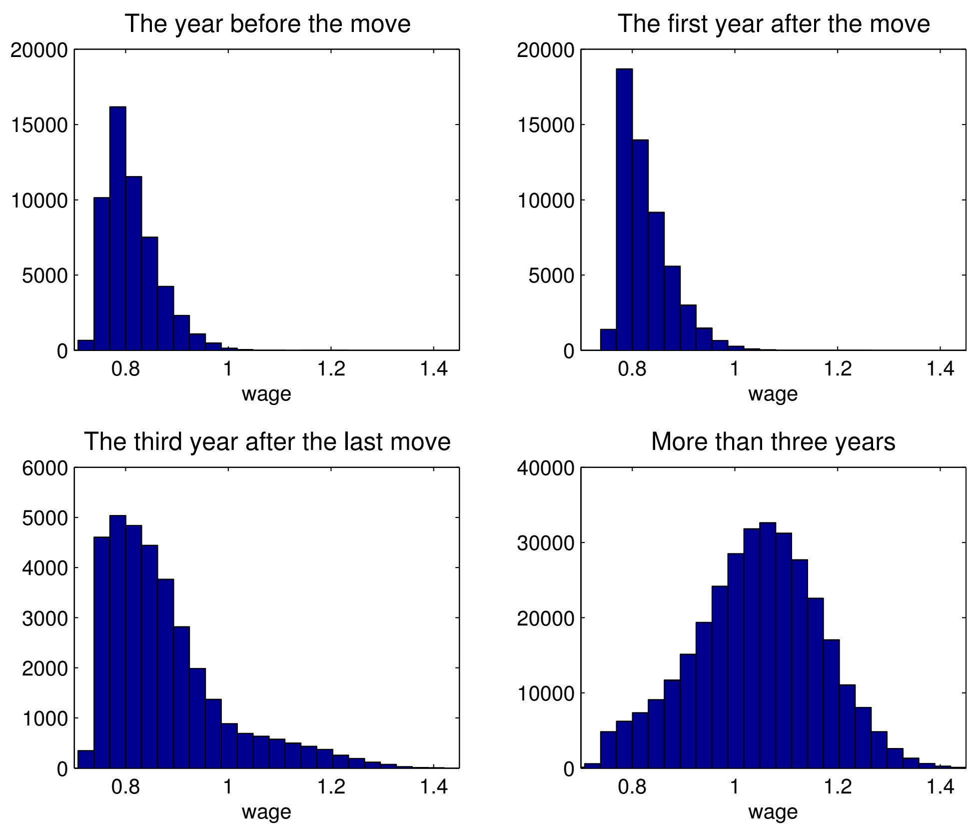
Notes: This figure plots how well the model captures the evolution of wages for recent movers and the effect of the parameter p on this wage-tenure profile. Tenure of 1refers to the first year after mobility. Tenure of -1 refers to the year before mobility. The empirical wage profile labeled "data" is the specification where industry, year, age and education effects are controlled for (*i.e.*, the thick, dashed curve shown in Figure 2). The profiled labeled "benchmark" is generated by the benchmark model. The other two profiles are generated by the model for different values of p, the probability of becoming skilled (also see Propositions 1 to 3 and Table 3).

5.3.3 Evolution of the wage distribution

Figure 1 shows evidence supporting different wage distributions for movers and stayers. To examine the model's predictions on this dimension, and to illustrate how wages evolve for movers in the model, we plot the wage distribution for different tenure levels in Figure 6. It shows that movers are drawn more from the lower tail of the wage distribution both before and after the move. As workers' tenure in the sector increases, their wages are drawn from a distribution with a higher mean.

This occurs for two reasons. First, as workers stay longer in a sector, they become more skilled, which can be seen from the evolution of the right-tail of the wage

Figure 6: The Wage Distribution and Tenure in the Benchmark Model



Notes: The figure plots the wage distribution in the benchmark model. It shows that movers tend to be drawn more from the lower end of the wage distribution both before and after mobility, consistent with the empirical facts shown in Figure 1 and the analytical results contained in Propositions 1 and 2. The mean wage in the entire economy is 1. This figure also shows how wages evolve for recent movers. Among recent movers, those who stay longer in their current sector are those who draw, on average, better subsequent shocks (also see Proposition 3 and Figure 3) and become more skilled. The distributions in the figure are based on 800,000 observations.

distribution among recent movers shown in the lower-left panel of Figure 6. Second, by referring to Proposition 3 and by looking at the left tail of the distributions in Figure 6, those who remain in a sector longer tend to have better match shocks while movers are drawn more from the lower end of the wage distribution (also see Figure 3).

5.3.4 Lifetime earnings and mobility

The empirical analysis in Section 2 introduced a novel fact: lifetime income and mobility are negatively correlated. In Table 3, we show that the model captures the negative correlation between lifetime income and mobility measured in Table 1.[13] The comparison is made using the correlation between indexes $\mathcal{M}^{\mathrm{b}}$ and $\mathcal{E}^{\mathrm{b}}$ described in Section 2. According to the data, the correlation is -0.1523 compared to a model generated value of -0.1685.

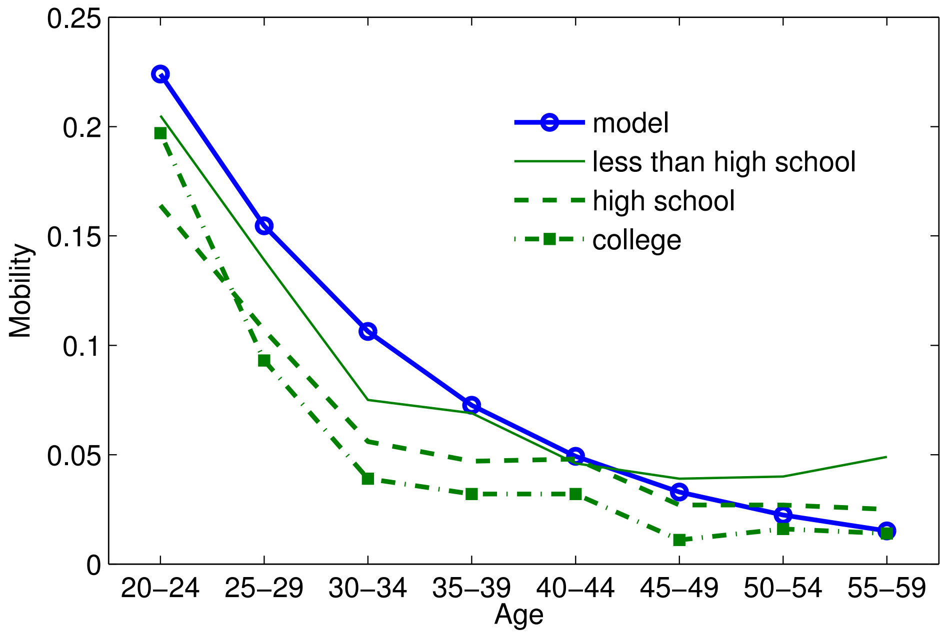
There are two main forces behind this negative correlation. First, workers with less mobility have longer tenure in a sector. In contrast, a worker changing industries several times over their lifetime are less likely to realize the wage gains from tenure. Second, as Proposition 3 and Figure 6 show, workers changing sectors more frequently on average have a lower quality match shock at both the origin and the destination. Since the shock is persistent, this poor match quality persists; as a result, they suffer lower lifetime income. Below in Section 7, we show that in the absence of skill accumulation, a persistent match shock alone can generate a substantial negative correlation between lifetime income and mobility.

5.3.5 Calculating the correlation of lifetime earnings and mobility

As equation (6) shows, the labor income shocks $e_{0,t}$ and $e_{1,t}$ have a common component denoted by y_t (also, see Figure B.1 in Appendix B.1). As mentioned before, this common component is omitted in the model as it does not affect mobility, the mover-stayer wage gap, and the wage-tenure profile. However, when the y_t component is omitted in the model, the variance of lifetime earnings generated by the model is lower than what it should be if the model were simulated using the labor income shocks $e_{0,t}$ and $e_{1,t}$ (as opposed to x and $-x$). Therefore, in order to compare the correlation between lifetime earnings and mobility generated by the model with that in data, one needs to take into account the earnings dispersion driven by the y_t component. For this reason, when calculating the correlation using the simulated data, we allow for

[13] If we allow workers to exert effort that affects skills, the persistence of the two income shocks (x and z) could substantially raise the correlation between lifetime earnings and mobility.

Figure 7: Mobility and Labor Market Experience



Notes: This figure shows that the model captures the negative relationship between mobility and labor market experience. The curve with circles (blue curve) shows the mobility rate in the model, assuming that a worker enters the labor market at the age of 20 years. The average labour market experience in the model is 40 years. The simulated data contain 20,000 individuals. The other curves (green) show the mobility rate among different age and educational groups in PSID.

the impact of the y shock by considering the overall dispersion of lifetime earnings in PSID data. The details are described in Appendix B.10.

5.3.6 Additional evidence: mobility by labor market experience

In Figure 7, we plot mobility as a function of age, as calculated from PSID. The figure shows that younger workers are more likely to move relative to older workers (also, see Table A.2). Although we do not target any age-mobility moment in the model, we can look at how mobility changes with overall labor market experience. As Figure 7 shows the model performs well in producing the negative relationship between mobility and labor market experience.

In the model, the stronger a worker's comparative advantage in their current sector becomes, the less likely they are to move. From Figures 5 and 6, "older" workers are more likely to have a comparative advantage strong enough to prevent mobility. Thus, a skill premium and a persistent match shock, which are the key forces driving the evolution of the wage distribution shown in Figure 6, also account for the negative relationship between mobility and age.

5.3.7 Skill accumulation speed

In the benchmark model, we consider 0.1 for p, the probability of becoming skilled in the new sector. We also consider different values for this parameter. To understand why the value of this parameter is important, imagine that match shocks did not exist; that is, the only potential source of wage growth is the skill premium, π. In such a world, the slope of the wage-tenure profile generated by the model is entirely dependent on π and p. A lower value of p implies workers become skilled faster, increasing the slope of the wage-tenure profile early in the spell, while flattening it later in the spell. Surprisingly, as Figure 5 and Table 3 show, in the model, higher ($p = 1/8$) or lower ($p = 1/12$) p has a little effect on the slope of the wage-tenure profile, suggesting that the mobility decision driven by the dynamic match shock is important for wage growth. We explore this with further experiments below.

6 Numerical experiments

To this point, we have established important links between wages, tenure, mobility, and lifetime income and demonstrated that the dynamic model captures these facts. This section performs several numerical experiments disentangling the different effects driving mobility and wages.

6.1 Evaluating the role of the sectoral shock

We start our numerical experiments by examining the importance of the sectoral shock. In particular, we simulate the model by turning off the sectoral shock, *i.e.*, by setting $z = 0$. Table 4 summarizes the quantitative predictions and compares them to those of the benchmark model. In the absence of the sectoral shock, the individual-level relationship between mobility and wages remain almost identical to the baseline case. Of course the model does not generate volatility in sectoral employment, but all other moments are virtually unchanged in this case. This suggests that excess mobility driven by a persistent match shock remains key to understanding the patterns of wages and mobility. While incorporating the sectoral shock helps us identify and

Table 4: Impact of the Sectoral Shock

Moments	*Benchmark model*	*No sectoral shock* (z=0)
mobility	0.0683	0.0683
volatility of sectoral employment	0.0059	0
repeat mobility	0.2247	0.2247
the mean wage of movers at the origin	-0.1960	-0.1963
the mean wage of movers at the destination	-0.1794	-0.1798
annual wage growth among recent movers	0.0263	0.0263
correl. of lifetime earnings and mobility	-0.1685	-0.1693

Notes: This table shows that in the absence of the sectoral shock, the individual-level relationship between mobility and wages remain almost identical to the baseline case. The column denoted by *Benchmark model* summarizes the predictions of the benchmark model. The column *No sectoral shock* corresponds to the predictions of the model where the sectoral shock is turned off (*i.e.*, set $z = 0$ in the benchmark model.)

calibrate the dispersion of the match shock, σ, it does not drive our main results on the wage-mobility and wage-tenure relationships. This is in contrast to much of the literature that has focused on net mobility driven by sector-wide shocks.

6.2 Implications on the number of sectors

An important implication of the last experiment is that one can account for the key features of wage and mobility data without introducing sector-wide shocks. Then, under directed mobility, the model can be recast as an economy with $N > 2$ sectors. Specifically, one can interpret x as the worker's match shock in the current sector, and $-x$ as the best of the $N-1$ match shocks from the remaining $N-1$ sectors and directly extend the main quantitative results to an economy with N sectors.

6.3 Quantifying the impact of mobility

In Table 5, we conduct two counterfactual experiments to quantify the impact of mobility and match shocks. The first experiment is to prohibit mobility, which provides a measure for the overall impact of mobility. Table 5 shows that when mobility is pro-

Table 5: The Mean Wage

the benchmark model	100.00%
when mobility is prohibited in the benchmark model	99.30%
mobility as in the benchmark model, but no match shock (*i.e.,* $x = 0$)	97.54%

Notes: This table compares the mean wage in the benchmark model with that in other restricted versions of the model. Mobility raises overall labor income by 0.7% (=100-99.3). The match shock accounts for 2.46% (=100-97.54) of labour income. See Section 6.3 for a further discussion.

hibited in the benchmark model, overall labor income declines by 0.7% (=100-99.3). That is, mobility raises overall labor income by 0.7%.

The second experiment asks what happens when the effect of the match shock on wages is zero (*i.e.,* $x = 0$), but mobility remains as it is in the benchmark model. The results of this experiment show that the match shock accounts for 2.5% (=100-97.5) of labour income. Using the numbers in Table 5, one can say that the net gain from mobility is 1.8% ($= 2.5\% - 0.7\%$) of labor income.

These numbers are much lower than the impact of mobility measured by Jovanovic and Moffitt (1990). Using a two-period model, they find that sectoral mobility raises income by 8.5 to 13 percent. The main reason behind this large difference between their numbers and ours is the nature of idiosyncratic match productivity. In Jovanovic and Moffitt (1990), match productivity in the initial sector remains constant. Therefore, if a worker is not allowed to move, the initial bad productivity affects future earnings permanently. In contrast, in our model, match productivity evolves over time and the impact of the initial match shock diminishes over time. This suggests that evolution of the match shock might be an essential element for quantifying the impact of mobility.

6.4 High net mobility

Next, we consider a counterfactual case where mobility is driven primarily by the sectoral shock. Specifically we set the dispersion of the match shock σ to one fifth of its benchmark value. This means that the role of the sectoral shock is higher when compared with the benchmark model. Column B of Table 6 shows the results of this alternative parametrization. The key issue is that it generates implausibly high volatility in sectoral employment. Specifically, employment is five times more volatile than that in data (0.0294 versus 0.0059). This further underscores the important role played by the dynamic worker-sector match shock. Given observed volatility in sectoral productivity and employment, sectoral shocks (*i.e.*, net mobility) alone are insufficient to explain the overall observed pattern of mobility.

7 Re-calibration in the absence of skill premium

The dynamic worker-sector match shock represents a key innovation of this paper. In Section 4, we have shown analytically that a model featuring only the match shock, with no skill premium, can *qualitatively* capture the key patterns of the wage-mobility relationship. Here, we analyze the model numerically in the absence of skill premium and show that such a model also performs well in capturing the key facts *quantitatively.* These facts include the wage growth of recent movers, the negative wage gap between movers and stayers, the negative correlation between lifetime income and individual-level mobility.

We first set the skill premium to zero, *i.e.*, $\pi = 0$, and then calibrate the parameters of the match shock (ρ and σ) by targeting gross mobility and employment volatility (or, equivalently, net mobility).

The last column (column C) of Table 6 presents the key moments and the parameter values associated with this re-calibration. Similarly to our benchmark calibration, this new calibration matches several key un-targeted moments from the data. For example, as shown in Figure 8, the re-calibrated model replicates the main characteristics of the wage-tenure relationship. It is remarkable that even in the absence of the skill premium, the model is able to generate both the negative wage-gap between movers and stayers, and especially the wage growth among recent movers. One dimension along which the model performs relatively poorly, at least quantitatively, is the pre-move wage gap. Now, the pre-move wage gap is -0.28 as opposed to -0.18 in data.

The last row of Table 6 also shows that the negative correlation between mobility and lifetime earnings in the model are highly comparable with its empirical counterparts. So, the re-calibrated model does well replicating the key features of the wage-mobility and wage-tenure relationships.

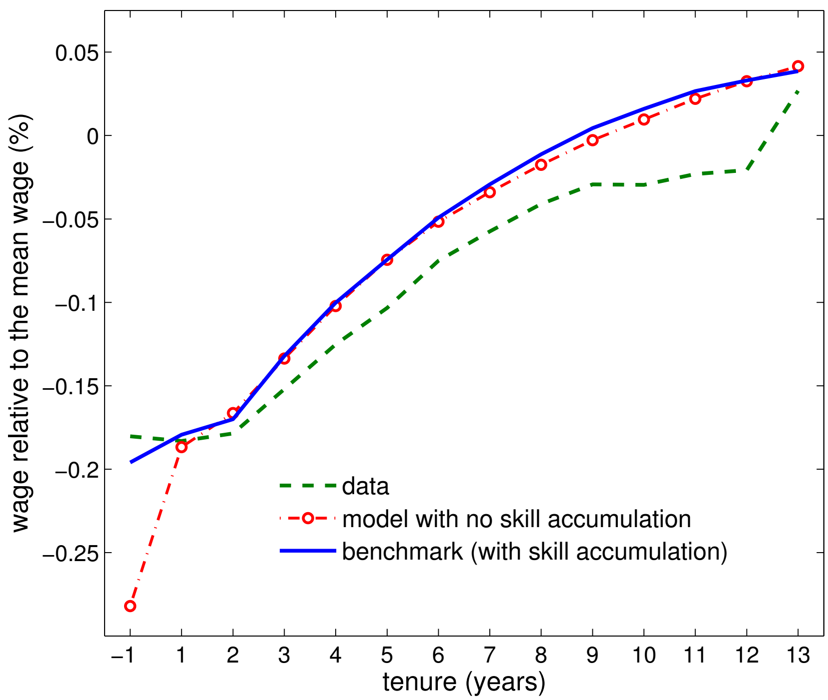
As can be seen from Figure 9 and the theoretical results in Section 4.2, persistence

Table 6: Re-Calibration

	A	B	C
	data	*low dispersion of the match shock* $\sigma = 0.0221$	*no skill premium* $\pi = 0$
Key parameters			
the persistence of the match shock, ρ		0.5421	0.9352
dispersion of the match shock, σ		0.0221	0.4488
the skill premium, π		0.3288	0
Moments			
mobility	0.0678	0.0689	0.0679
repeat mobility	0.2729	0.2249	0.0884
volatility of sectoral employment	0.0059	0.0294	0.0059
wage of movers at the origin	-0.1803	-0.2361	-0.2821
movers wage at the destination	-0.1786	-0.2328	-0.1868
annual wage growth among recent movers	0.0259	0.0293	0.0281
correlation of lifetime earnings and mobility	-0.1523	-0.2127	-0.2007

Notes: Column A summarizes the empirical moments. Column B shows the key parameters and predictions of the model when the dispersion of the match shock is set to one fifth of its benchmark value (*i.e.*, $\sigma = 0.0221$), and the parameters π and ρ are re-calibrated by targeting gross mobility and repeat mobility. Column C shows the results of the re-calibration that omits sector specific human capital (*i.e.*, $\pi = 0$), where the parameters of the match shock, σ and ρ, are re-calibrated by targeting gross and net mobility in the data while setting the skill premium parameter π to zero. The rest of the parameters, except for ω, are as in the benchmark model. The constant term of the wage function under the calibration in the last column is $\omega = 0.6422$.

Figure 8: The Wage-Tenure Profile in the Model With No Skill Premium

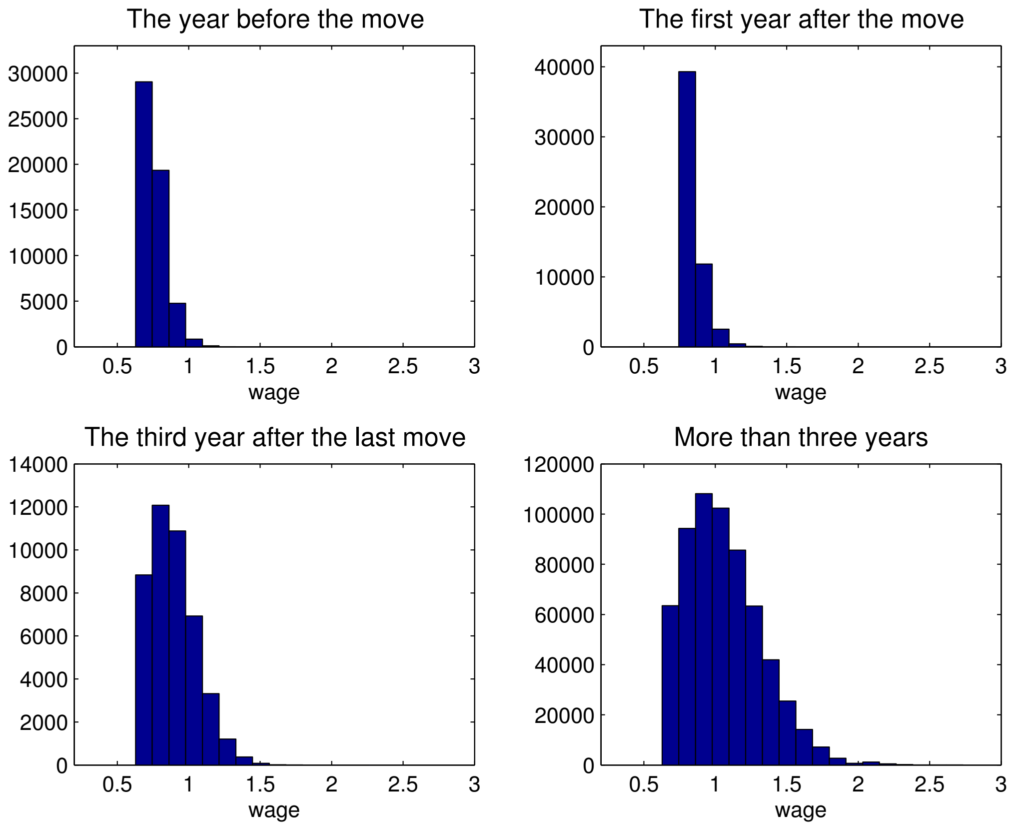


Notes: This figure plots the wage-tenure profile in a model with only dynamic match shocks and no skill premium, and compares the profile with those in PSID data and the benchmark model. Tenure of 1 refers to the first year after mobility. Tenure of -1 refers to the year before mobility. The details of this re-calibrated model are contained in Section 7.

in the match shock implies that those who find a suitable match are more likely to have a better match shock and thus stay in the current sector. This dynamic selection effect is able to generate the negative wage gap between movers and stayers, the positive wage-tenure relationship, and the negative correlation between mobility and lifetime earnings.

There does exist, however, an important difference between the benchmark model and the model with no skill premium. Specifically, repeat mobility is substantially low in the model with no skill premium. This points to the importance of the skill premium considered in the benchmark model. The skill premium π makes workers less sensitive to x; *i.e.*, it took a larger x shock for a skilled worker to move, relative to an unskilled worker. When $\pi = 0$, this implicit moving cost is absent. Importantly,

Figure 9: The Wage Distribution in the Model With No Skill Premium



Notes: The figure plots the wage distribution generated by the model when it is re-calibrated in the absence of a sector-specific skill premium. The distributions in the figure are based on 800,000 observations. It shows that the movers tend to be drawn more from the lower end of the wage distribution both before and after mobility, which is consistent with the empirical facts shown in Figure 1 and the analytical results contained in Propositions 1 and 2. Among recent movers, those who stay longer in their current sector are those who draw, on average, better subsequent shocks, which is consistent with Proposition 3 and Figure 3.

however, because the x shock is more persistent with $\pi = 0$, it becomes very unlikely for a worker to receive a shock moving them from sector 0 to sector 1 in period t, and another shock moving them back from sector 1 to sector 0 (*i.e.*, repeat mobility). So, not only overall mobility, but also the pattern of how workers are distributed by mobility and thus by sectoral tenure is important for understanding the nature of a labor income shock.

Nevertheless, the results show that a persistent worker-sector match shock is consistent with the main features of the wage-mobility relationship.

8 Conclusion

According to PSID data, movers tend to have a lower wage than non-movers both prior to, and after the move. The wage increases with sectoral tenure. Lifetime earnings is negatively related to sectoral mobility. A sector experiences simultaneous inflows and outflows of workers while the corresponding net flows are very small relative to the gross flows. Labor mobility decreases with labor market experience. Recent movers are more likely to move again. These facts can be accounted for by a highly parsimonious model with persistent worker-sector match productivity.

In the model, workers move to sectors that they are better matched with. This match quality persists: those who are better matched in their current sector tend to remain better matched and become skilled there. Unlike in many existing dynamic multi-sector models, in our model, mobility is directed in that a worker knows her match quality across sectors before switching sectors. Both the analytical and numerical results in the paper show that when labor mobility is directed by dynamic match productivity, a simple multi-sector model goes a long way to account for many features of sectoral mobility and wages, including the novel facts established in the paper.

According to the model, the impact of net mobility driven by a sector-wide shock on wages is relatively small, while excess mobility driven by a worker-sector match productivity shock plays the key role for wage growth and lifetime earnings. Sectoral mobility raises overall labor income by 0.7%, and the sector-specific match shock accounts for 2.5% of labor income.

The results in the paper show that one can generate a substantial wage gap between movers and stayers either by using a highly persistent match shock in the absence of the skill premium or by using a combination of a substantial skill premium and a transitory match shock. However, these two approaches have very different implications on the individual-level mobility pattern. Therefore, in order to understand the role of mobility for wages, one needs to consider endogenous mobility, an endogenous distribution of workers by sectoral tenure, and the implied wage-tenure relationship. The model developed in the paper offers a parsimonious framework to accomplish this.

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A Data appendix

This appendix provides additional details on mobility and wages using the main PSID sample.

A.1 Mobility and wages

Table A.1 presents the results of the OLS regressions of the log real hourly wage on a dummy for mobility (across sectors), along with various combinations of controls. Controlled effects include full sets of dummies for age, education, year, state, current and previous industries and sector tenure. These results indicate that mobility is associated with significantly lower wages. Moreover, comparing the columns, one can see that both sector tenure and individual-specific unobserved effects are important for the wage-gap between movers and stayers (also, see Figures 1 and 2 for further details). Table A.2 summarizes labor mobility across industries by age and education. Table A.3 shows the evolution of the quantile of wages by sector tenure. Specifically, it reports the quantile values associated with Figure 2.

A.2 Correlation between mobility and earnings

Table A.4 shows that correlation between mobility and earnings in the main PSID sample is weaker than that in the Retrospective Occupation-Industry Supplemental Data Files. This is because the main PSID sample produces less accurate estimates of mobility as pointed out by Kambourov and Manovskii (2008).

Table A.1: Wage Regressions

Variables	*Specifications*		
mobility dummy	-0.391 (0.019)	-0.233 (0.016)	-0.153 (0.026)
age, education, year, state, sectors		✓	✓
race		✓	✓
industry tenure			✓
R-squared	0.029	0.389	0.401

Notes: The table shows the results of the wage regressions with different specifications. Standard errors are in parenthesis. The sample consists of 25310 year-person observations. See Figures 1 and 2 for further details.

Table A.2: Mobility by Age and Education

Age	*Educational attainment (grades)*					
	≤4	5-7	8-11	12-15	16≤	all
20-24	0.000	0.188	0.207	0.164	0.197	0.178
	3	32	646	1,562	239	2,482
25-29	0.000	0.193	0.133	0.107	0.093	0.110
	3	88	753	2,422	894	4,160
30-34	0.200	0.119	0.065	0.056	0.039	0.057
	10	134	751	1,687	775	3,357
35-39	0.077	0.087	0.064	0.047	0.032	0.052
	65	207	846	1,286	534	2,938
40-44	0.054	0.041	0.046	0.048	0.032	0.044
	129	268	917	1,228	498	3,040
45-49	0.048	0.049	0.034	0.027	0.011	0.030
	166	348	903	1,163	471	3,051
50-54	0.041	0.070	0.026	0.027	0.016	0.032
	219	356	800	922	368	2,665
55-59	0.039	0.064	0.045	0.025	0.014	0.037
	206	298	606	673	220	2,003
60-65	0.041	0.057	0.061	0.047	0.007	0.050
	145	246	558	529	136	1,614
all	0.048	0.072	0.073	0.072	0.050	**0.068**
	946	1,977	6,780	11,472	4,135	**25,310**

Notes: Each age-and-education cell has two entries: the mobility rate (top) and the number of observations (bottom). The sample consists of 25310 year-person observations. The description of the variables are provided in Section 2.1. Also, see Figure 7 for a related discussion.

Table A.3: Wage Quantiles by Tenure

Years	*Control variables*				*N*
	industry year	industry year age	industry year education	industry year age education	
-1	0.352 (0.270)	0.399 (0.284)	0.352 (0.274)	0.439 (0.295)	1,715
1	0.335 (0.266)	0.374 (0.283)	0.338 (0.267)	0.412 (0.292)	1,715
2	0.369 (0.264)	0.395 (0.282)	0.361 (0.259)	0.425 (0.290)	1,075
3	0.405 (0.26)	0.417 (0.277)	0.403 (0.260)	0.451 (0.285)	767
4	0.420 (0.272)	0.418 (0.286)	0.412 (0.268)	0.447 (0.290)	594
5	0.463 (0.274)	0.456 (0.286)	0.458 (0.269)	0.478 (0.291)	458
6	0.472 (0.278)	0.454 (0.287)	0.469 (0.271)	0.481 (0.286)	371
7	0.481 (0.278)	0.453 (0.285)	0.487 (0.270)	0.500 (0.286)	278
8	0.513 (0.267)	0.471 (0.276)	0.524 (0.257)	0.512 (0.273)	216
9	0.509 (0.278)	0.463 (0.282)	0.522 (0.268)	0.508 (0.275)	157
10	0.529 (0.269)	0.475 (0.281)	0.551 (0.260)	0.515 (0.280)	104
11	0.510 (0.288)	0.465 (0.291)	0.539 (0.274)	0.501 (0.291)	65
12	0.535 (0.270)	0.485 (0.267)	0.589 (0.210)	0.584 (0.219)	26

Notes: The values at -1 refer to the year before mobility. Standard deviations are in parenthesis. *N* denotes the number of observations.

Table A.4: Comparing Different Samples

	Restrospective Files 1968-1980	Retrospective Files and PSID 1968-1997
corr($\mathcal{M}^{\mathrm{a}}, \mathcal{E}^{\mathrm{b}}$)	-0.141	-0.105
corr($\mathcal{M}^{\mathrm{b}}, \mathcal{E}^{\mathrm{b}}$)	-0.152	-0.104
corr($\mathcal{M}^{\mathrm{c}}, \mathcal{E}^{\mathrm{b}}$)	-0.168	-0.106
corr($\mathcal{M}^{\mathrm{d}}, \mathcal{E}^{\mathrm{b}}$)	-0.172	-0.108

Notes: This table shows how the correlation of lifetime earnings (measured by $\mathcal{E}^{\mathrm{b}}$) and various measures of individual-level mobility ($\mathcal{M}$) differ between the samples. Specifically, it shows that including inaccurate measure of mobility in the main PSID sample (also see Kambourov and Manovskii, 2008) results in a weaker correlation between mobility and earnings. The above measures of mobility and lifetime earnings are defined in Section 2.

B Model appendix

This section contains further details of the model and the proofs of the analytical results in the paper. It also demonstrates the role of the key elements in the model and provides some robustness checks for the baseline parameterization.

B.1 The distribution of the match shocks

Figure B.1 illustrates the distribution of the match shocks across workers. A worker's pair of the match shocks (e_0, e_1) is described by a point in the graph. The iso-probability contours reflect the bivariate distribution of the match shocks. To simplify the discussion, suppose for now that there is no sectoral shock, *i.e.*, $z = 0$.

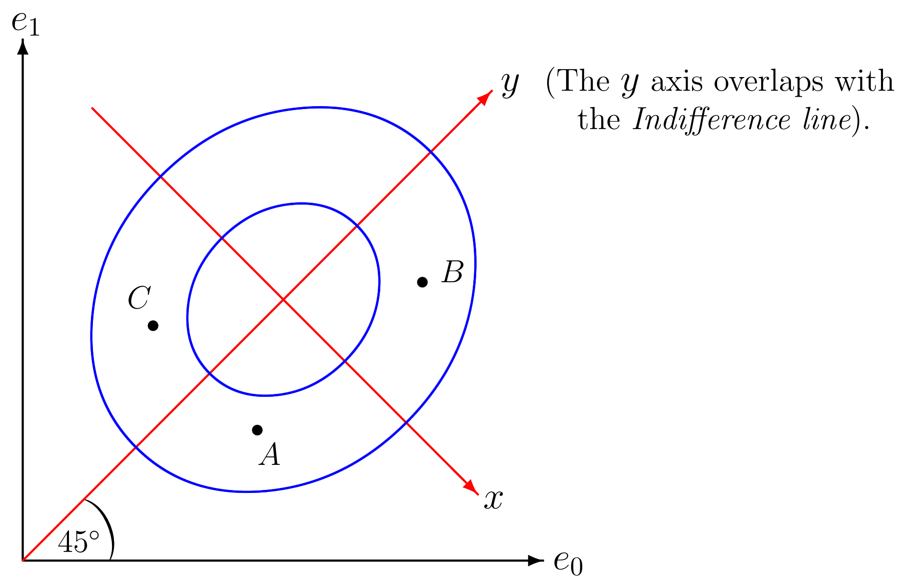
B.1.1 Indifference line

Consider an unskilled worker. If the two shocks are the same, the worker will be indifferent between the two sectors. In other words, workers "lying" along the line $e_0 = e_1$, are indifferent between the two sectors. This line is referred to as the Indifference line. If the worker's pair of shocks are given by a point above the Indifference line, the person prefers sector 1. Conversely, if they are at a point below the line, the person prefers sector 0.

B.1.2 Which component of the shock is relevant for mobility?

Suppose that the worker is in sector 0 at time t, and let her pair of match shocks be given by point A (at time t). Since the point lies below the Indifference line, at time t the person has a better match shock in sector 0 than in sector 1: $e_0 > e_1$. Now, suppose that at the beginning of $t+1$, the worker is hit by new shocks given by point B. In this case, the person does not move, as the match quality remains below the

Figure B.1: Decomposing the Match Shocks



Notes: The figure illustrates the distribution of the match shocks among *unskilled workers* when there is no sectoral shock (*i.e.*, $z = 0$). A pair of the match shocks (e_0, e_1) of a person is described by a point in the graph. For example, at point A, e_0 is higher than e_1, while at point C, e_1 is higher than e_0. The iso-probability contours reflect the bivariate distribution of the match shocks.

The line at which workers are indifferent between the sectors is referred to as the *Indifference line*. When $z = 0$, this line is given by the 45° line. Given the decomposition in equation (6), y shock is along the Indifference line, while the x shock is orthogonal to the Indifference line. The x shock of a particular person is positive if the worker is below the Indifference line (for a worker at A or B), while it is negative if the person is above the Indifference line (for a worker at C). See Appendix B.1 for a further discussion.

Indifference line. If instead the new shock is given by point C, the person may decide to move. Therefore, any component of the shock that shifts match productivity along the Indifference line is not relevant for mobility. It is important to keep this in mind when we decompose the match shocks in Appendix B.1.

B.1.3 The sectoral shock and skill premium

Figure B.1 illustrates the case where there is no skill premium ($\pi = 0$) and no sector-wide shock ($z = 0$). Consistent with equation (14), the Indifference line shifts with the sectoral shock, z. For example, if sector 1 is hit by a positive z shock, the Indifference line shifts downward. Moreover, the Indifference line is different between skilled versus unskilled workers. For example, for a skilled worker of sector 0, the Indifference line should be above the one depicted in Figure B.1. See McLaughlin and Bils (2001) for a further discussion on how the Indifference line in the standard Roy (1951) model responds to the z shock, when the match productivity is permanent.

B.2 The mobility decision and the match shock

Figures B.2 and B.3 help understand how x affects the mobility decision. In Figure B.2 we plot the wage as a function of the match shock, x, for both unskilled and skilled workers.[14] For positive values of x, the worker is more productive in sector 0 relative to sector 1, which in turn affects the mobility decision. This is evident in Figure B.3 where we plot the value of remaining in sector 0, $S_0(\cdot)$, and of moving to sector 1, $M_0(\cdot)$.

At any point in time (or for each value of z), for each skill level h, there exists a critical value of the match shock, $\hat{x}_0(h, z)$, where a worker prefers to move from sector 0 to sector 1 if the match shock of the person is below the critical value, *i.e.*,

[14] In both Figures B.2 and B.3 we simplify by setting $z = 0$. This abstraction is done to simplify the intuition, which carries over for a non-zero z.

$x < \hat{x}_0(h, z)$. Among workers in sector 0, this critical value is higher for an unskilled worker than for a skilled worker: $\hat{x}_0(1, z) < \hat{x}_0(0, z)$ (see Figure B.3). Therefore, a skilled worker is less likely to change sectors, all else equal. Indeed, the skill premium, π, acts as an implicit moving cost. In our quantitative exercises below, we further highlight the role of π in replicating observed mobility patterns.

B.3 Sectoral dynamics

Let $\nu_{j,t}(h, x)$ denote the measure of workers after the realization of the match shocks in period t:

$$\nu_{j,t}(h, x, \tau) = \int \mu_{j,t-1}(h, \tilde{x}, \tau) \frac{\partial F(x \mid \tilde{x})}{\partial \tilde{x}} d\tilde{x}. \tag{B.1}$$

Then, denote the total number of workers moving from sector j to $1-j$ in period t by $m_{1-j,t}$. This is given by

$$m_{1-j,t}(x) = (1-\delta) \sum_h \sum_\tau (1 - \Omega_j(h, x, z_t)) \nu_{j,t}(h, x, \tau), \tag{B.2}$$

where z_t is the sectoral shock at time t. At the end of the current period, these movers will have worked for one period at their destination; therefore, the measure of workers in sector j with $(h, x, \tau) = (0, x, 1)$ is equivalent to the measure of workers that move to sector j in period t. That is,

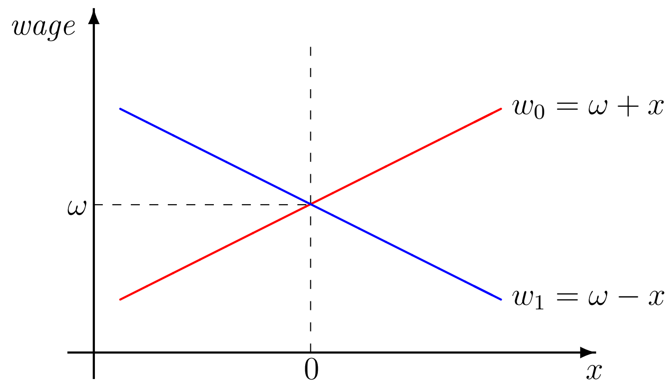
$$\mu_{j,t}(0, x, 1) = m_{j,t}(x). \tag{B.3}$$

For stayers, sectoral dynamics are captured by the following two equations. The measure of unskilled workers in sector j at the end of period t with tenure $\tau + 1$ is given by

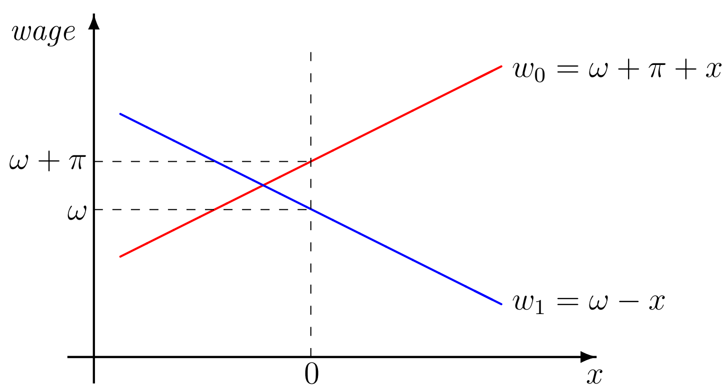
$$\mu_{j,t}(0, x, \tau + 1) = (1-\delta)(1-p)\Omega_j(0, x, z_t)\nu_{j,t}(0, x, \tau), \tag{B.4}$$

Figure B.2: The Match-Specific Wage

Panel A. An Unskilled Worker of Sector 0



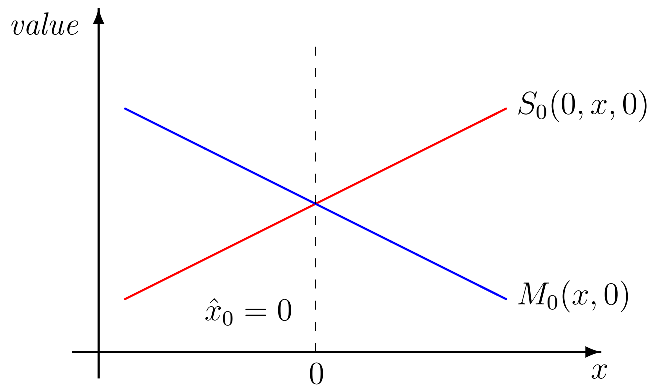
Panel B. A Skilled Worker of Sector 0



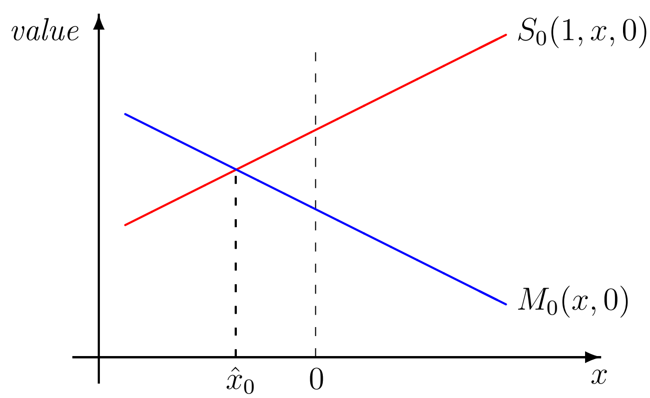
Notes: The figure shows the wages when the sector-wide shock z is zero (*i.e.*, $z = 0$). w_0 is the wage of a worker staying in sector 0 when his or her match-specific productivity for the sector is x. w_1 is the wage of a worker moving to sector 1, when his or her match productivity for sector 0 is x. (Also, see Figure B.3.)

Figure B.3: The Mobility Decision

Panel A. An Unskilled Worker of Sector 0



Panel B. A Skilled Worker of Sector 0



Notes: The figure shows who moves and who stays behind when the sector-wide shock z is zero (*i.e.*, $z = 0$). S_0 is the value to a worker of staying in sector 0 when his or her match-specific productivity for the sector is x. M_0 is the value of leaving sector 0. Among those in sector zero, those hit by a match-specific shock below $\hat{x}_0$ will move across sectors. For unskilled workers, when $z = 0$, $\hat{x}_0 = 0$. Among skilled workers, when $z = 0$, $\hat{x}_0 < 0$. (Also, see Figure B.2.) Using the figures and recalling the persistence of the match shock, it can be seen that, on average, a) movers are those with worse match shocks and b) skilled workers move less.

which is the probability of surviving and remaining unskilled (the $(1-\delta)(1-p)$) term) multiplied by the measure of unskilled workers with tenure τ (entering period t) who receive the match shock x during period t (the $\nu_{j,t}(0,x,\tau)$ term) and who decide to stay in sector j (the $\Omega_j(0,x,z_t)$ term). Similarly, for the measure of skilled workers,

$$\mu_{j,t}(1,x,\tau+1) = (1-\delta)\left(\Omega_j(1,x,z_t)\nu_{j,t}(1,x,\tau) + p\Omega_j(0,x,z_t)\nu_{j,t}(0,x,\tau)\right), \quad \text{(B.5)}$$

where $\tau \in \{1, 2, \cdots\}$.

B.4 Definition of the equilibrium

An equilibrium consists of a set of value functions $\{U_j,\ S_j,\ M_j\}$, a decision rule Ω_j, a sequence of the sectoral technology shock $\{z_t\}_{t=1}^{T}$ for an integer $T > 0$, and the sequence of numbers of workers $\{\mu_{j,t}(0,x,\tau),\ \mu_{j,t}(1,x,\tau)\}_{t=0}^{T}$ for any j, τ and x such that

1. stayer: given U_j, the value function $S_j(h,x,z)$ solves equations (10) and (11);
2. mover: given S_j and M_j for each j, the decision rule $\Omega_j(h,x,z)$ and the value function $U_j(x)$ solve equation (13); and
3. consistency of the law of motion: for each (x,j,τ), $\{\mu_{j,t}(0,x,\tau),\ \mu_{j,t}(1,x,\tau)\}_{t=1}^{T}$ satisfy equations (15) and (16), and (B.1) through (B.5), subject to the sequence of the sectoral technology shock $\{z_t\}_{t=1}^{T}$ and the initial measure $\{\mu_{0,0}(h,x,\tau),\ \mu_{1,0}(h,x,\tau)\}$.

B.5 Simulating the simple economy in Section 4.2

1. For each worker, the match shock for period 1 is drawn from the normal distribution $\mathcal{N}(0,\sigma^2)$ independently. If the shock is positive or equal to zero, assign the worker to sector 0, otherwise to sector 1.

2. For each person, generate the subsequent match shocks according to the AR(1) process in equation (9), where ϵ_t is drawn from $\mathcal{N}(0, (1-\rho^2)\sigma^2)$ independently.

3. For a given worker, if the sign of the match shock changes between t and $t+1$, move the worker across sectors. Otherwise, keep the worker in the same sector. (If $x_{t+1} = 0$, consider it as the same sign as x_t.)

B.6 Proof of Claim 1

1. For those who just moved from sector 1 to sector 0, the mean wage in sector 0 is given by $\mathbf{E}(\epsilon_t \mid \epsilon_{t-1} \leq 0 \text{ and } \epsilon_t > 0)$. For those who stayed in sector zero during $t-1$ and t, their mean wage at t is given by $\mathbf{E}(\epsilon_t \mid \epsilon_{t-1} \geq 0 \text{ and } \epsilon_t \geq 0)$. Since, ϵ_t is independently distributed for all t, these two values are equal to the mean wage of sector 0, $\mathbf{E}(\epsilon_t \mid \epsilon_t \geq 0)$. Then, using equation (7), the mean wage is the same between the newcomers and incumbents.

2. For those who will move from sector 0 to sector 1 in the next period, the mean value of their current sectoral shocks is given by $\mathbf{E}(\epsilon_t \mid \epsilon_{t+1} < 0 \text{ and } \epsilon_t \geq 0)$. Similarly, the mean value of the current sectoral shocks of those who will stay in sector 0 is given by $\mathbf{E}(\epsilon_t \mid \epsilon_{t+1} \geq 0 \text{ and } \epsilon_t \geq 0)$. Since, ϵ_t is independently distributed for all t, these two values are equal to the mean wage of sector 0, $\mathbf{E}(\epsilon_t \mid \epsilon_t \geq 0)$. So, the average wage of the future movers is equal to the average wage of the future stayers.

3. Using the results of, it can be seen that for those with τ years of sectoral tenure, the mean wage is independent of tau and given by $\mathbf{E}(\epsilon_t \mid \epsilon_t \geq 0)$. So, the average wage remains constant with sectoral tenure.

4. Regardless of the mobility status and sectoral tenure, the mean value of the match shocks at each level of sectoral tenure is given by $\mathbf{E}\big(\epsilon_t \mid \epsilon_t \geq 0\big)$. Thus, the number of moves a person makes is uncorrelated with overall earnings, over a finite number of periods.

B.7 Proof of Proposition 1

Consider the workers who worked in sector 0 during periods t and $t+1$. For these workers, it is the case that $x_t \geq 0$ and $x_{t+1} = \rho x_t + \epsilon_{t+1} \geq 0$. Then, the mean shock of these incumbent workers of sector 0 in period $t+1$ is given by the following conditional mean:

$$\overline{x}_0^{\text{Old}} = \mathbf{E}\big(\rho x_t + \epsilon_{t+1} \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t \geq 0\big).$$

Because $\rho > 0$, $\sigma > 0$ and ϵ_t-s are *i.i.d*,

$$\begin{aligned} \overline{x}_0^{\text{Old}} &= \mathbf{E}\big(\rho x_t + \epsilon_{t+1} \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t \geq 0\big) \\ &> \mathbf{E}\big(\rho x_t + \epsilon_{t+1} \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t = 0\big) \\ &= \mathbf{E}\big(\epsilon_{t+1} \mid \epsilon_{t+1} \geq 0\big). \end{aligned} \tag{B.6}$$

On the other hand, for those who worked in sector 1 during period t and in sector 0 during period $t+1$, it is the case that $x_t \leq 0$ and $x_{t+1} = \rho x_t + \epsilon_{t+1} > 0$. Then, the mean shock of these newcomers of sector 0 in period $t+1$ is given by the following conditional mean:

$$\overline{x}_0^{\text{New}} = \mathbf{E}\big(\rho x_t + \epsilon_{t+1} \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t < 0\big).$$

As before, because $\rho > 0$, $\sigma > 0$ and ϵ_t-s are $i.i.d$,

$$\begin{aligned} \overline{x}_0^{\text{New}} &= \mathbf{E}\big(\rho x_t + \epsilon_{t+1} \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t < 0\big) \\ &< \mathbf{E}\big(\rho x_t + \epsilon_{t+1} \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t = 0\big) \\ &= \mathbf{E}\big(\epsilon_{t+1} \mid \epsilon_{t+1} \geq 0\big). \end{aligned} \tag{B.7}$$

Combining inequalities (B.6) and (B.7), we get

$$\overline{x}_0^{\text{New}} < \overline{x}_0^{\text{Old}}.$$

The latter means that, on average, the incumbent workers have a better shock than the newcomers. Further, using equation (7), on average, the incumbent workers have a higher wage than the newcomers.

B.8 Proof of Proposition 2

For those who will move from sector 0 to sector 1 at the beginning of period $t+1$ (future movers), the mean value of their current sectoral shocks (at period t) is given by

$$\overline{x}_0^{\text{FM}} = \mathbf{E}\big(x_t \mid \rho x_t + \epsilon_{t+1} < 0 \text{ and } x_t \geq 0\big).$$

Similarly, the mean value of the current sectoral shocks of those who will stay in sector 0 during periods t and $t+1$ (future stayers) is given by

$$\overline{x}_0^{\text{FS}} = \mathbf{E}\big(x_t \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t \geq 0\big).$$

Since $\rho > 0$ and $\sigma > 0$, for the future movers,

$$\overline{x}_0^{\text{FM}} = \mathbf{E}\big(x_t \mid \rho x_t + \epsilon_{t+1} < 0 \text{ and } x_t \geq 0\big) < \mathbf{E}\big(x_t \mid x_t \geq 0\big) \tag{B.8}$$

and for the future stayers,

$$\overline{x}_0^{\text{FS}} = \mathbf{E}\big(x_t \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t \geq 0\big) > \mathbf{E}\big(x_t \mid x_t \geq 0\big). \tag{B.9}$$

So, we have

$$\overline{x}_0^{\text{FM}} < \mathbf{E}\big(x_t \mid x_t \geq 0\big) < \overline{x}_1^{\text{FS}}.$$

Then, using equation (7), the average wage of future stayers is higher than that of future movers.

B.9 Proof of Proposition 3

Using equation (9), for any integer $k \in \{0, 1, \cdots, \tau - 1\}$, one can write

$$x_{t-k} = \rho^{\tau-k} x_{t-\tau} + e_{t,\tau,k},$$

where

$$e_{t,\tau,k} = \sum_{\ell=k}^{\tau-1} \rho^{\ell-k} \epsilon_{t-\ell}.$$

On the other hand, given any large finite number T, for those with tenure of $\tau < T$ years in sector 0 at the end of period t, it must be the case that $x_t \geq 0$, $x_{t-1} \geq 0$, $\cdots$, $x_{t-\tau+1} \geq 0$, $x_{t-\tau} > 0$ and $x_{t-\tau-1} \leq 0$. The mean value of their shocks at time t is given by

$$\overline{x}_0(\tau) = \mathbf{E}\big(\rho^\tau x_{t-\tau} + e_{t,\tau,0} \mid \rho^{\tau-k} x_{t-\tau} + e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau - 1, x_{t-\tau} > 0, x_{t-\tau-1} \leq 0\big).$$

Since $\rho > 0$ and the conditional mean increases with $x_{t-\tau}$, we can write that

$$\overline{x}_0(\tau) > \mathbf{E}\big(e_{t,\tau,0} \mid \rho^{\tau-k} x_{t-\tau} + e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau - 1, x_{t-\tau-1} \leq 0\big).$$

Because ϵ_t-s are $i.i.d$, it follows that

$$\begin{aligned} \overline{x}_0(\tau) &> \mathbf{E}\big(e_{t,\tau,0} \mid e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau-1, x_{t-\tau-1} \leq 0\big) \\ &= \mathbf{E}\big(e_{t,\tau,0} \mid e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau-1\big). \end{aligned} \tag{B.10}$$

Similarly, the mean shock of those who have worked for exactly $\tau - 1$ periods in sector 0 is given by

$$\overline{x}_0(\tau-1) = \mathbf{E}\big(\rho^{\tau} x_{t-\tau} + e_{t,\tau,0} \mid \rho^{\tau-k} x_{t-\tau} + e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau-2, x_{t-\tau+1} > 0, x_{t-\tau} \leq 0\big).$$

Then, since $\rho > 0$ and the conditional mean increases with $x_{t-\tau}$,

$$\overline{x}_0(\tau-1) < \mathbf{E}\big(e_{t,\tau,0} \mid \rho^{\tau-k} x_{t-\tau} + e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau-2, x_{t-\tau+1} > 0, x_{t-\tau} = 0\big).$$

Furthermore, because ϵ_t-s are $i.i.d$,

$$\begin{aligned} \overline{x}_0(\tau-1) &< \mathbf{E}\big(e_{t,\tau,0} \mid e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau-2, \epsilon_{t-\tau+1} > 0\big) \\ &= \mathbf{E}\big(e_{t,\tau,0} \mid e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau-1\big). \end{aligned} \tag{B.11}$$

Using inequalities (B.10) and (B.11),

$$\overline{x}_0(\tau-1) < \overline{x}_0(\tau)$$

for $\tau < T$. Then, using equation (7), over any finite range of sectoral tenure, the average wage increases with sectoral tenure.

B.10 Lifetime earnings and mobility

Given the depcomposition in Appendix B.1 and Figure B.1, the model is simulated while omitting the y shock. Subsequently, the lifetime earnings simulated by the model have less variation. Therefore, in order to compare the correlation between lifetime earnings and mobility in the model with that in the data, one has to take into account the overall earnings dispersion in the data. Here, we describe how we adjust the correlation in the simulated data by using overall earnings dispersion in PSID.

Let $\mathcal{E}_i^x$ and $\mathcal{M}_i$ denote the lifetime earnings and overall mobility of person i in the model. Let $\mathcal{E}_i^y$ be the component of the lifetime earnings based on the y shock and $\mathcal{E}_i$ be the total lifetime earnings of person i: $\mathcal{E}_i = \mathcal{E}_i^x + \mathcal{E}_i^y$. Then, the correlation of the total lifetime earnings and mobility can be written as follows:

$$\text{corr}(\mathcal{E}_i, \mathcal{M}_i) = \frac{E(\mathcal{E}_i^x + \mathcal{E}_i^y, \mathcal{M}_i) - E(\mathcal{E}_i^x + \mathcal{E}_i^y)E(\mathcal{M}_i)}{\text{std}(\mathcal{E}_i^x + \mathcal{E}_i^y)\text{std}(\mathcal{M}_i)} \tag{B.12}$$

Given the independence of $\mathcal{E}_i^y$ and $\mathcal{M}_i$,

$$\begin{aligned} \text{corr}(\mathcal{E}_i^x + \mathcal{E}_i^y, \mathcal{M}_i) &= \frac{E(\mathcal{E}_i^x, m_i) - E(\mathcal{E}_i^x)E(m_i)}{\text{std}(\mathcal{E}_i^x)\text{std}(\mathcal{M}_i)} \frac{\text{std}(\mathcal{E}_i^x)}{\text{std}(\mathcal{E}_i^x + \mathcal{E}_i^y)} && \text{(B.13)} \\ &= \text{corr}(\mathcal{E}_i^x, \mathcal{M}_i)\frac{\text{std}(\mathcal{E}_i^x)}{\text{std}(\mathcal{E}_i^x + \mathcal{E}_i^y)}. && \text{(B.14)} \end{aligned}$$

In the model, $\text{corr}(\mathcal{E}_i^x, \mathcal{M}_i) = -0.655$ and $\text{std}(\mathcal{E}_i^x) = 0.086$. Using the PSID data, $\text{std}(\mathcal{E}_i^x + \mathcal{E}_i^y) = 0.368$. Inserting these numbers into the above equation, we get $\text{corr}(\mathcal{E}_i, \mathcal{M}_i) = -0.152$.

B.11 Robustness: sectoral-level productivity

In the model, the sectoral shock is governed by the following process:

$$z_{t+1} = r z_t + u_t,$$

where $\mathrm{Var}(z_t) = s^2$. In the quantitative analysis of Section 5, the persistence and standard deviation of this sectoral shock, r and s, are calibrated using the relative productivity series of the manufacturing sector of 1987 to 2012. One could argue that the length of this productivity series may not be sufficient enough to precisely measure the volatility of the sectoral shock.

To address this issue, we re-scale the two parameters using much longer aggregate productivity data of 1947 to 2012. For this purpose, let $\rho^{\mathrm{L}}_{\mathrm{agg}}$ and $\sigma^{\mathrm{L}}_{\mathrm{agg}}$ be the persistence and standard deviation of aggregate US productivity in the longer sample (1947 to 2012). Also, let $\rho^{\mathrm{S}}_{\mathrm{agg}}$ and $\sigma^{\mathrm{S}}_{\mathrm{agg}}$ be the persistence and standard deviation of aggregate US productivity in the shorter sample (1987 to 2012). Then, one can consider the following re-scaling:

$$\begin{cases} r^{\mathrm{L}} = r^{\mathrm{S}} \times \dfrac{\rho^{\mathrm{L}}_{\mathrm{agg}}}{\rho^{\mathrm{S}}_{\mathrm{agg}}}, \\ s^{\mathrm{L}} = s^{\mathrm{S}} \times \dfrac{\sigma^{\mathrm{L}}_{\mathrm{agg}}}{\sigma^{\mathrm{S}}_{\mathrm{agg}}}, \end{cases} \tag{B.15}$$

where r^{S} and s^{S} are the persistence and standard deviation of relative productivity of the manufacturing sector from 1987 to 2012. These equations imply a slightly more volatile, but less persistent, shock where $r^{\mathrm{L}} = 0.3683$ and $s^{\mathrm{L}} = 0.0090$.

Table B.1 summarizes the key predictions of the model with these two parameters of the sector shock. The results indicate that considering a slightly more volatile sectoral shock does not have much impact on the wage-mobility relationship. However,

Table B.1: Predictions under Re-scaled Sectoral Shock

	Data	*Benchmark model*	*A more volatile sectoral shock*
Key parameters			
persistence of the sectoral shock		0.4236	0.3683
st.dev. of the sectoral shock		0.0068	0.0090
Moments			
mobility	0.0678	0.0683	0.0683
volatility of sectoral employment	0.0059	0.0059	0.0078
repeat mobility	0.2729	0.2247	0.2247
movers' mean wage at their origin	-0.1803	-0.1960	-0.1955
movers' mean wage at destination	-0.1786	-0.1794	-0.1789
annual wage growth of recent movers	0.0259	0.0263	0.0262
corr. of lifetime earnings and mobility	-0.1523	-0.1685	-0.1680

Notes: The table summarizes predictions of the benchmark model with that of the model with a more volatile sectoral shock.

most of its impact is on sectoral employment. This is simply because a more volatile sectoral shock raises net mobility and thus raises sectoral employment.