The Role of a *Persistent* Match Shock in Sectoral Mobility, the Wage-Tenure Profile, and Lifetime Earnings[*]

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Abstract

Sectors experience simultaneous inflows and outflows of workers that are much larger than the corresponding net flows. A common approach to generate large gross flows in otherwise standard multi-sector models is to consider a stochastic shock specific to a worker-sector match. This paper develops a stochastic Roy model of sectoral mobility featuring a *dynamic match shock*. Both the analytical and quantitative results show that the persistence of a match shock is important not only for large gross flows and individual-level mobility patterns, but also for the wage-tenure profile and lifetime earnings. In the model, movers tend to have a lower wage than non-movers both prior to, and after the move. Wages grow with sectoral tenure. Those who move more frequently have lower lifetime earnings. More recent movers are more likely to move again. Mobility decreases with labor market experience. These model predictions are consistent with PSID data, but generated by a remarkably simple, persistent match shock. There is a strong trade-off between the skill premium and the persistence of the match shock in generating the negative mover-stayer wage gap and the positive wage-tenure profile observed in data. Ignoring the persistence of a match shock, when in fact it is present, introduces an upward bias into the estimates of the specific skill premium.

Keywords: stochastic multi-sector model, Roy model, match shock, labor mobility, sectoral mismatch, labor income shock, lifetime earnings, return to tenure, directed mobility

JEL Codes: E24, J31, J24, J62

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1 Introduction

It is well established that sectors (industries and occupations) experience simultaneous inflows and outflows of workers that are much larger than the corresponding net flows (*e.g.,* Jovanovic and Moffitt, 1990; Davis, Haltwanger, and Schuh, 1996; and Moscarini, 2001). Using Panel Study of Income Dynamics (PSID) and Bureau of Labor Statistics (BLS) data, we show that net mobility accounts for only one tenth of overall mobility across sectors. Jovanovic and Moffitt (1990) argue that accounting for such large gross flows requires an idiosyncratic shock specific to a worker-sector match, as sector-wide effects such as those in the Lucas and Prescott (1974) model cannot generate simultaneous inflows and outflows. Accordingly, a match quality shock has become a key element in dynamic multi-sector models.

The literature has also established a substantial heterogeneity in underlying mobility rates among workers. For example, in PSID, those who switched sectors in the recent past are five times more mobile than the rest of the labor force (see Table 3). Farber (1994) argues that this strong persistence in the individual-level mobility rates, along with the negative relationship between mobility and labor market experience, is consistent with evolving match quality. Central to the argument is persistent match quality. However, little is known about the underlying mechanisms through which persistent match quality affects the relationship between wages and mobility. Although there is a growing literature that links large gross flows to data on individual wage dynamics, the focus has been on *transitory* worker-sector shocks.[1]

In this paper, we explore the role of a *persistent* match shock in the wage-mobility relationship. Using a dynamic, multi-sector model, we argue that when the match shock is sufficiently persistent, it has implications far beyond generating large gross flows. Both the analytical and quantitative results in the paper show that a persistent match shock can account for the key features of the relationship between individual-level wage dynamics and mobility across industries and occupations, including the new facts established in this paper. For example, when the match shock is sufficiently persistent, the well-known positive relationship between the wages and sectoral tenure[2] arises naturally, even in the absence of sector-specific skill accumulation. At the same time, the model replicates key data patterns of mobility, such as strong heterogeneity in the individual-level mobility rates and the negative relationship

[1] Miller (1984), Flinn (1986), and Jovanovic and Moffitt (1990) are among the first to consider a transitory match-specific shock to analyze worker flows across sectors and job types. A sample of recent studies featuring a match-specific shock includes Coen-Pirani (2010), Lkhagvasuren (2012), Papageorgiou (2014), Groes, Kircher, and Manovskii (2015), Carrillo-Tudela and Visschers (2014), and Pilossoph (2014).

[2] Sectoral tenure refers to the number of years that a worker has been in their current employment sector. The positive wage-tenure profiles are well established by a number of authors, such as Altonji and Shakotko (1987) and Topel (1991). Figure 2 plots the positive relationship between wages and sectoral tenure.

between mobility and labor market experience.

To our knowledge, this paper is the first attempt to link individuals' mobility to wage-tenure profiles and lifetime earnings in a stochastic multi-sector economy. This is important for the following simple reasons. First, sectoral tenure is the flip side of sectoral mobility; longer tenure means less mobility. Thus, the wage-tenure relationship is inherently the wage-mobility relationship. In fact, the empirical literature has long considered the wage-tenure relationship (*e.g.,* Altonji and Shakotko (1987) and Topel (1991)), providing a simple, yet robust foundation for our analysis. Second, if sectoral mobility is driven by a persistent income shock, the number of moves a person makes over her lifetime could be intimately related to her overall level of lifetime earnings. Indeed, our empirical analysis provides new evidence to support this argument.

The key innovation of our model is the combination of a persistent match shock and directed mobility. Directed mobility refers to the assumption that a worker knows their wage at the destination sector *before* the move occurs. Although inherent in the standard models of both sectoral selection (*e.g.*, Roy (1951)) and sectoral dynamics (*e.g.,* Lucas and Prescott (1974)), directed mobility is computationally disadvantageous in stochastic multi-sector models, as it requires a large state space for the dynamic programming problem. Consequently, many recent studies maintain an alternative assumption that the wage of a mover at the destination must be drawn *after* the move occurs.[3] The latter assumption, commonly referred to as random search, is ill-suited to the specific purpose of our paper, as it is equivalent to specifying an exogenous match distribution for movers at their destination, thus creating an exogenous wage gap between movers and stayers by construction. In contrast, with directed mobility, it is unnecessary to impose an exogenous distribution for movers, and different wage distributions for stayers and movers can arise endogenously. This is important, especially when one focuses on the joint determination of mobility and wages. Also, when considering wage data of annual frequency, random search appears to be an untenable assumption since it means that a worker must wait for a year to be able to revise her sectoral choice. For these reasons, we consider directed mobility.

In addition to the persistent match shock, there are two other factors that influence wages and mobility in our benchmark model. First, as in Rogerson (2005) and Kambourov and Manovskii (2009), workers may acquire a sector-specific skill premium. Second, similarly to the islands model of Lucas and Prescott (1974), workers within a particular sector are subject to a common productivity shock. This sector-wide shock creates net mobility, while the persistent match shock generates simultaneous inflows and outflows.

To illustrate the power of the persistent idiosyncratic match shock to explain the individual-

[3] See, for example, Alvarez and Veracierto (2000), Kambourov and Manovskii (2009), Coen-Pirani (2010) and Lkhagvasuren (2012).

level wage-mobility relationships, we establish a set of baseline facts using PSID data. Those who will leave their current sector over the next year earn substantially less than those who will stay in the same sector. Once these movers arrive at a new sector, on average, their wages are much lower than the wages of the incumbents of the new sector. This pre- and post-move wage gap is explored in earlier work of Jovanovic and Moffitt (1990) and McLaughlin and Bils (2001). The PSID data allow us to extend these two-period, pre- and post-move wage comparisons by tracing the movers' wages over their lifetime.

Our empirical analysis yields two novel results. First, the average wage of new arrivals steadily increases with sectoral tenure, but remains below the mean wage of the sector for nearly ten years (see Figure 2). Second, individuals changing sectors more frequently tend to have significantly lower lifetime earnings. These patterns are robust for mobility across occupations and industries.

Using a simplified version of the model, we first show analytically that a persistent match shock is consistent with the key features of mobility and wages. When the match shock is persistent, those who have better match quality are more likely to have better match quality in the next period and thus to stay in their current sector. In contrast, those with lower match quality are more likely to be hit by an adverse shock and thus move across sectors. This dynamic selection effect generates the negative wage gap between movers and stayers, the positive wage-tenure relationship, and the negative correlation between mobility and lifetime earnings.

To quantitatively explore these results further, we then calibrate the benchmark model to PSID and the BLS data. Although we calibrate the model to mobility data only, the model is able to generate key wage-mobility relationships observed in data, including those between mobility, the wage level, wage growth, sectoral tenure, and lifetime earnings. Strikingly, these data features are generated by a simple AR(1) match shock.

To further illustrate the role played by evolving worker-sector comparative advantage, we consider a version of the model where the sector-specific skill premium is absent. When the match shock is highly persistent, the model is still capable of capturing the key features of the data, in the absence of a sector-specific skill premium. When the persistence of the match shock is overly high, however, the model is unable to capture the strong heterogeneity in mobility rates among workers emphasized by Farber (1994). Nevertheless, the model with no skill premium shows that directed mobility driven by dynamic match productivity goes a long way to account for the key data patterns, including the novel facts on lifetime earnings and wage-tenure profiles.

The notion of match-specific permanent productivity can be traced back to Roy's (1951) seminal model of sectoral selection and wage distribution (see Heckman and Taber (2008) for various extensions of Roy's model). More recent literature on sectoral dynamics considers evolving match productivity. For example, Lkhagvasuren (2012) and Carrillo-Tudela and Visschers (2014) introduce match-specific productivity shocks to the Lucas and Prescott (1974) islands model.[4] Our paper differs from these studies in two ways. First, these models focus mainly on sectoral and aggregate level effects and do not consider the individual-level relationship between mobility and wages, which we focus on exclusively. Second, as mentioned earlier, mobility is *directed* in our model while many of the models built on the Lucas and Prescott (1974) islands model treat mobility as random search.

In recent work, Papageorgiou (2014) and Groes et al. (2015) argue that workers learn their ability over time and the process of this learning is important for understanding the relationship between occupational mobility and wages. The former considers comparative advantage, while the latter focuses on absolute advantage. In these models, however, match-specific productivity is subject to a transitory shock, while in our model it is subject to a persistent shock. Our model is closer to Papageorgiou (2014), as it also features comparative advantage and sector-specific skill accumulation. In the Appendix, we consider a setting with dynamic absolute advantage and show that a persistent match shock might also be quantitatively relevant for certain features of data considered by Groes et al. (2015). This is not to say that a persistent match shock accounts for every feature of sectoral mobility and wages. Instead, the persistence of the match shock might be important for both comparative and absolute advantage. Specifically, we show that ignoring the persistence of the match shock introduces a large upward bias in the return to the specific skills, a common element in learning models considered by recent work.

It is worth emphasizing that robustness to data frequency should be an important concern for any multi-sector analysis. Since mobility is a flow variable, the number of people moving across sectors increases with the length of a period. Thus, the forces influencing a worker's mobility decision should be tightly linked to the period length and the time discount factor. With a transitory shock, it is difficult, if not impossible, to establish such a link and therefore interpret the magnitude of the income shock. But, with a persistent match shock, one can simply raise the persistence of the income shock when using higher frequency data.

Given the emphasis on a persistent match shock, our paper is closely related to the literature on labor income dynamics. Recent studies find that individual labor income is subject to a *persistent* shock (*e.g.*, Guvenen, 2009 and Hryshko, 2012). Our quantitative results suggest that a substantial part of this persistent income process might be sector-specific. Unlike

[4]It should be noted that a match-specific productivity shock is not the only way to generate large gross flows. For example, earlier work of Coen-Pirani (2010) analyzes large gross migration across the U.S. by using a persistent preference shock specific to a worker-location match, while Pilossoph (2014) considers a transitory taste shock specific to a worker-sector match to analyze the impact of sectoral mobility on aggregate unemployment.

much of the literature that uses an exogenous process to explore the nature of individual labor income, however, our model features joint determination of sectoral mobility and labor income. Therefore, in our model, the persistent income process is endogenous and influenced by mobility.

The remainder of the paper is organized as follows. Section 2 details the facts characterizing the wage-mobility relationship. Sections 3 and 4 describe the model and its key elements. Section 5 considers a highly simplified version of the model, and presents analytical results on the impact of the dynamic match shock. Section 6 describes the calibration and the main quantitative results. Section 7 performs a set of numerical experiments to disentangle the effects driving the key features of the wage-mobility relationship. Section 8 considers a re-calibrated version of the model where the skill premium is omitted. Section 9 concludes.

2 Empirical analysis

In the Section 3 model and in Roy-type models, the wages of movers relative to that of the other workers in the sector represent the key variable. Consistently, our empirical analysis starts by examining the relative mover-stayer wage gap at both the origin and destination, and the evolution of this wage gap following mobility. Then, we present a novel finding linking lifetime earnings to mobility.

The analysis in this section uses data from the Panel Survey of Income Dynamics (PSID) of 1968-1997, with some portions of the analysis restricted to the Retrospective Occupation-Industry Supplemental Data Files, released in 1999. Our sample of the Retrospective Files consists of 3,057 male household heads aged 20-65, totaling 28,443 person-year observations.

Our main empirical findings concern mobility across both industries and occupations. However, the quantitative analysis of the model presented in Section 6 focuses on mobility and wages across broadly defined sectors, including agriculture, manufacturing and mining (hereafter manufacturing), public sector and services.[5] (Tables A.1 and A.2 in the Appendix contain the list of industries and occupations considered in our empirical analysis.)

2.1 Mobility

Sectoral mobility occurs if an individual switches sectors or occupations between two consecutive years. As mentioned earlier, gross flows are much larger than the corresponding net flows. For example, according to our estimation below, net mobility accounts for less than

[5]Appendix A.2 explains the data limitation reasons for why the model is calibrated using data on broadly defined sectors. Nevertheless, using large administrative panel data that are much larger than PSID, one can consider mobility across much finer industries, occupations, or their combinations, while applying the match productivity process considered in this paper.

1% of total sectoral employment. Gross mobility, however, is relatively large, constituting 7% of employment. For example, in the 1968-1980 Retrospective Files, 541 manufacturing workers moved to the service sector, and 534 service sector workers moved to manufacturing. Tables A.1 and A.2 in the Appendix also show that gross mobility is large across the other sectors and across occupations. So, the individual-level effects, reflected in gross mobility, are potentially more important for the wage-mobility relationship.

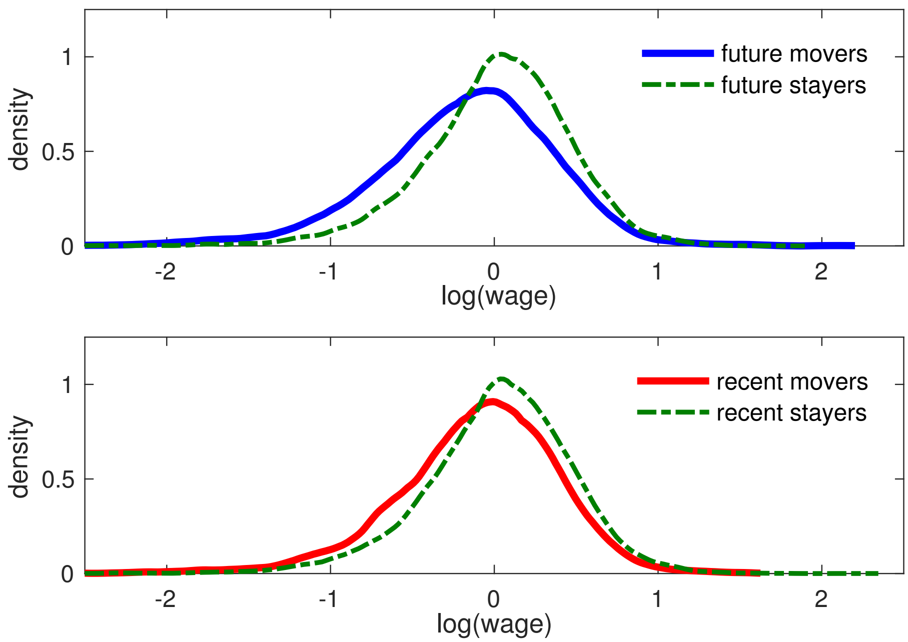
2.2 The mover-stayer wage gap

Earlier work from Jovanovic and Moffitt (1990) and McLaughlin and Bils (2001) establishes the following two facts: (*i*) workers who are about to change sectors have lower wages relative to those who will stay in the sector and (*ii*) once they do change sectors, they also have lower wages relative to the incumbents. For example, consider workers moving from service to manufacturing between time $t-1$ and t. Their wage before changing industries is 74% of the average wage among those remaining in the service sector. Once these movers arrive at manufacturing, on average their wage is 70% of the manufacturing incumbents'. The pattern also holds for the reverse flow. Specifically, the average wage of those who will move from the manufacturing sector to the service sector is 75% of the average wage among workers who will remain in the manufacturing sector. Once the movers arrive at the service sector, on average their wage is 74% of the service sector incumbents'. In the Appendix, namely in Tables A.3 to A.6, we measure the mover-stayer wage gap across both sectors and occupations while controlling for worker characteristics, sector-wide effects and unemployment spells. The pattern remains robust (see, also Appendix B.11).

2.3 The wage distribution among movers versus stayers

In our model below, the wage distributions differ substantially between movers and stayers. For comparison, we look at the wage distributions of movers and stayers in the PSID. In Figure 1, we plot the wage distributions of movers and stayers before and after the move, respectively. The distributions in Figure 1 suggest that movers and stayers might have different wage distributions. To validate this, we perform a formal statistical test of the equality of the two distributions. Given the small size of the PSID sample, we use the finite-sample, nonparametric, two-sample Kolmogorov-Smirnov test. According to the test, the wage distribution of movers is different from the wage distribution of stayers, both before and after mobility, with highly significant p-values of 0.001. Thus, movers are indeed drawn significantly more from the lower half of the wage distribution. Below, we show that the combination of a persistent match shock and directed mobility introduced in this paper offers a natural explanation for

Figure 1: Wage Distribution Among Movers and Stayers



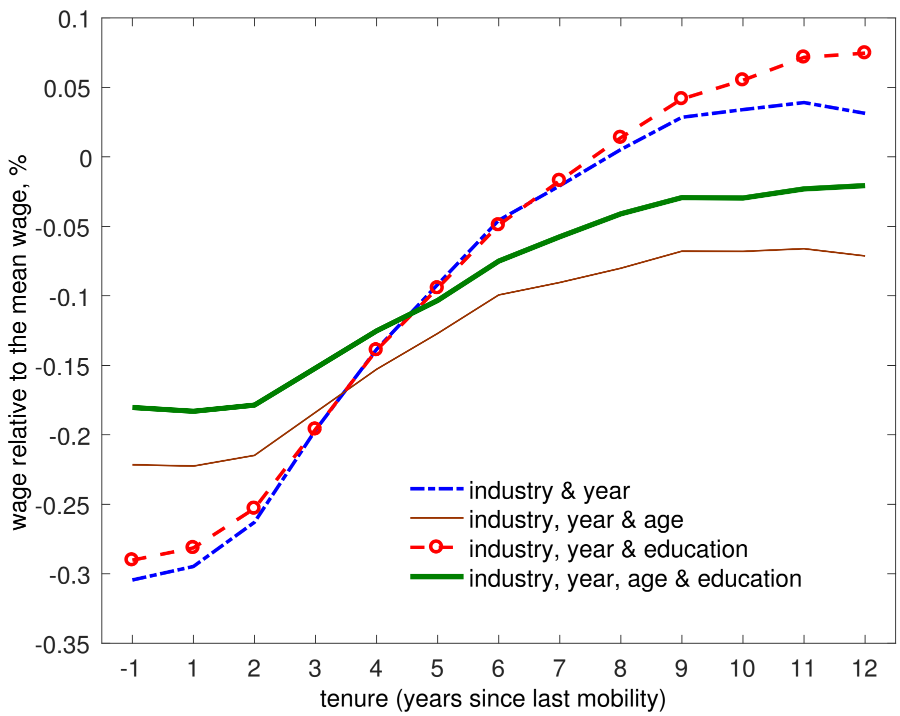
Notes: The figure compares the log-wage distributions of industry movers and stayers in PSID. The log-wages are measured as the residuals of a regression of the real log wages on year, industry, age group, education group dummies, and their interactions. The upper panel displays the kernel density estimates of the residual log-wage distributions among those who will be working in a different industry in the next year (*future movers*) and those who will be working in the same industry in the next year (*future stayers*). The lower panel displays the kernel density estimates of the residual log-wage distributions among those who moved within the last year (*recent movers*) and those who did not change their industry within the last year (*recent stayers*). According to the finite sample, nonparametric Kolmogorov-Smirnov test, the distributions within each panel are different at a significance level of 0.001. So, movers are drawn significantly more from the lower half of the wage distribution. See also Section 2.2 and McLaughlin and Bils (2001).

why these different wage distributions arise endogenously when mobility and wages are jointly determined.

2.4 The wage-tenure profile

Sectoral tenure is the flip side of sectoral mobility. Less mobility implies longer tenure; therefore, the evolution of wages for movers at their new sector is inherently the wage-mobility relationship. To examine the evolution of wages after a move, we plot the log hourly wage

Figure 2: Wage-Tenure Profiles in PSID



Notes: This figure plots the mean log hourly wage difference between movers and stayers as a function of sectoral tenure. A tenure of 1 means the first year after mobility. A tenure of -1 refers to the year before mobility. The difference is measured within the specific group to which the individual belongs. The profile labeled "industry & year" plots the average wage difference within industry-year cells. The other profiles control for different effects, including age and education. Each profile is smoothed using local polynomials. See also Neal (1995) and McLaughlin and Bils (2001).

difference between movers and stayers as a function of sectoral tenure, starting one year before the move occurs and following a worker for more than ten years in the new sector. Figure 2 plots this wage-tenure profile. It is evident that a worker's wage is below the group average prior to and after the move. Importantly, wages increase with tenure in the new sector, but do so relatively slowly. Wages remain below the average wage of identical workers (in observable characteristics) for about ten years following mobility. This wage-tenure profile is central to our dynamic analysis of wages and mobility below. Our analysis below examines the impact of dynamic match-specific productivity on the wage-tenure profile when mobility and wages are jointly determined.

2.5 New evidence on mobility and lifetime earnings

The logical outcome of our results above is the following. Mobility is associated with lower wages both before and after a move. Thus, one may ask whether lower wages are caused by a transitory effect or by persistent productivity shifts. In the latter case, one may expect to see a substantial negative relationship between lifetime earnings and mobility. Indeed, we find that individuals with lower lifetime earnings tend to be more mobile in the PSID. The remainder of the section provides the details leading to this conclusion.

2.5.1 Individual-level mobility

To measure an individual's propensity to move, we construct several mobility indexes. First, consider the most parsimonious index: the individual-specific mean of the mobility dummy (over the period covered by the Retrospective Files). Denoting this index by $\mathcal{M}_i^{\mathrm{a}}$, we define it as: $\mathcal{M}_i^{\mathrm{a}} = \frac{1}{T_i-1}\sum_{t=1}^{T_i-1} m_{it}$, where T_i is the number of years of observations for individual i and m_{it} is a dummy variable for changing industries between the periods $t-1$ and t.

Second, to control for the fact that mobility varies with age and education (see Figure 7), consider the following normalized index: $\mathcal{M}_i^{\mathrm{b}} = \frac{1}{T_i-1}\sum_{t=1}^{T_i-1} m_{it}/\tilde{m}_{it}$, where $\tilde{m}_{it}$ is the average mobility rate among individuals in the same age and education group as person i at time t. This measures average mobility for the *individual*, relative to otherwise similar workers. To ensure the robustness of these measures, we also compute quantile versions of these two indexes. Let $\mathcal{M}_i^{\mathrm{c}}$ and $\mathcal{M}_i^{\mathrm{d}}$ denote the quantile versions of $\mathcal{M}_i^{\mathrm{a}}$ and $\mathcal{M}_i^{\mathrm{b}}$, respectively.[6]

2.5.2 Lifetime earnings

Similarly, lifetime earnings are measured with several indexes. $\mathcal{E}_i^{\mathrm{a}}$ is the individual fixed effect estimated from a fixed-effect regression of log hourly wage on total sector experience and dummy variables for age, year, state, education, and sector. $\mathcal{E}_i^{\mathrm{b}}$ is the individual-specific mean residual from an OLS regression of log hourly wage on total sector experience and dummy variables for age, year, state, education, and sector. This lifetime earnings index is proposed

[6] Because of the finite number of age and education cells and the low mobility rate, more than one person can share a particular value of the level index $\mathcal{M}_i^{\mathrm{a}}$ or $\mathcal{M}_i^{\mathrm{b}}$. For example, there are 37 unique values of $\mathcal{M}^{\mathrm{a}}$. To avoid any possible bias caused by the discrete nature of the indexes, we construct the quantile indexes by imposing the same quantile on those who are tied to the associated level index.

by Moffitt and Gottschalk (2002) using PSID. $\mathcal{E}_i^{\mathrm{c}}$ and $\mathcal{E}_i^{\mathrm{d}}$ are the quantile versions of $\mathcal{E}_i^{\mathrm{a}}$ and $\mathcal{E}_i^{\mathrm{b}}$, respectively.

Table 1: Individual-Level Sectoral Mobility and Lifetime Earnings

	Lifetime earnings			
	$\mathcal{E}^{\mathrm{a}}$	$\mathcal{E}^{\mathrm{b}}$	$\mathcal{E}^{\mathrm{c}}$	$\mathcal{E}^{\mathrm{d}}$
Mobility across sectors				
$\mathcal{M}^{\mathrm{a}}$	-0.200	-0.141	-0.205	-0.124
$\mathcal{M}^{\mathrm{b}}$	-0.157	-0.152	-0.161	-0.145
$\mathcal{M}^{\mathrm{c}}$	-0.220	-0.168	-0.226	-0.155
$\mathcal{M}^{\mathrm{d}}$	-0.189	-0.172	-0.197	-0.169
Mobility across occupations				
$\mathcal{M}^{\mathrm{a}}$	-0.266	-0.187	-0.268	-0.221
$\mathcal{M}^{\mathrm{b}}$	-0.194	-0.191	-0.194	-0.210
$\mathcal{M}^{\mathrm{c}}$	-0.273	-0.191	-0.280	-0.228
$\mathcal{M}^{\mathrm{d}}$	-0.207	-0.208	-0.213	-0.229

Notes: *Individual-level mobility* ($\mathcal{M}$) refers to the number of sectoral (or occupational) moves a worker made during the sample period. *Lifetime earnings* ($\mathcal{E}$) refers to the overall wage level of a particular worker over the sample period. The table displays pairwise correlations of various measures of the two variables described in Section 2.5. The p-values associated with these correlations are all less than 0.01.

2.5.3 Correlation of lifetime earnings and mobility

Table 1 displays the correlations for each pair of lifetime earnings and mobility indices. All of the correlations are negative and significant, indicating that individuals with lower lifetime earnings are more mobile. On the surface, the aforementioned relationships between wages and mobility appear to contradict the theory that workers move to pursue better employment opportunities and wages. What a worker's wage *would have been* had they decided to stay in the original sector is unobservable, however. This represents the key difficulty in drawing conclusions from the data. To disentangle the potential forces driving the patterns established above, the next section presents a dynamic model with joint determination of wages and mobility.

3 The benchmark model

To uncover the forces underlying the relationship between wages and sectoral mobility, the benchmark model builds on McLaughlin and Bils (2001), who use Roy's (1951) framework to study mover-stayer wage differences. As mentioned earlier, McLaughlin and Bils (2001) consider a two-period model with permanent match quality. In contrast, we consider a dynamic Roy model with a *persistent* idiosyncratic productivity shock specific to the worker-sector match.

There are two other important elements in the model. First, in our model, as in Rogerson (2005) and Kambourov and Manovskii (2009), workers may acquire a sector-specific skill premium. Similarly to the islands model of Lucas and Prescott (1974), workers within a particular sector are subject to a common productivity shock referred to as the sectoral shock. The sectoral shock causes net mobility, while the worker-sector match shock creates excess mobility.

3.1 Environment

We present the model in terms of two sectors. The model, however, can be recast as an economy with $N > 2$ sectors. (Appendix B.14 provides further details.) The two sectors are denoted by 0 and 1. Each sector is inhabited by a large number of workers. A worker's wage in a particular sector is determined by three components: a sector-specific skill premium, a sectoral shock, and a shock specific to a worker-sector match.

3.1.1 The sector-specific skill premium

For sector-specific skill, we adopt the specification of Rogerson (2005) and Kambourov and Manovskii (2009). Individuals are either skilled or unskilled in their current sector, and a worker can only be skilled in one sector at a time.[7] A skilled worker is more productive than an otherwise identical unskilled worker (in the same sector). In each period, an unskilled worker becomes skilled in the current sector with probability p. Let π denote the skill premium this worker receives. Notice that the longer an agent remains in the current sector, the more likely she is to be skilled; therefore, tenure is required to become skilled. In addition, this skill premium is "general" in the sense that all skilled workers within a sector receive the same premium. This is in contrast to the idiosyncratic dynamic comparative advantage we introduce below. Each period, a worker exits the labor market with probability δ, while newly born workers enter the economy.

[7] See Lazear (2009) and Gathmann and Schonberg (2009) for alternative views of human capital where there is transferability of skills across sectors. Also, similarly to Kambourov and Manovskii (2009), we do not allow workers to exert effort to increase their specific skills.

3.1.2 The sectoral shock

There also exists a sectoral shock. It affects the productivity of all workers in one sector relative to another sector. Specifically, all workers in sector 1 are subject to the shock z_t. This shock has a stationary transition function $\Pr(z_{t+1} < z' \mid z_t = z) = G(z' \mid z)$ given by the following autoregressive process:

$$z_{t+1} = rz_t + u_t, \tag{1}$$

where $r \geq 0$ and u_t is a zero-mean random variable. Let s denote the unconditional standard deviation of z_t: $s = \mathrm{Std}(z_t)$.

3.1.3 The match shock

Each worker's productivity is also subject to an idiosyncratic shock. The magnitude of the shock depends on the worker's current sector of employment. Let (e_0, e_1) denote these labor income shocks. If $e_0 > e_1$, then the worker has a relative comparative advantage in sector 0, all else equal. The pair of shocks, (e_0, e_1), is drawn for each worker in each period. The labor income shocks e_0 and e_1 are correlated across time and sectors. They are given by the following autoregressive process:

$$\begin{cases} e_{0,t} = \rho e_{0,t-1} + u_{0,t} \\ e_{1,t} = \rho e_{1,t-1} + u_{1,t}, \end{cases} \tag{2}$$

where $\rho \geq 0$ and the innovations $(u_{0,t}, u_{1,t})$ are drawn from a zero-mean, bivariate distribution. We will talk about the structure of these shocks shortly in Section 3.4.

3.1.4 Wages

The current wage for workers in sector 0 is given by,

$$w_{0,t} = \omega + \pi h_{0,t} + e_{0,t} \tag{3}$$

and for workers in sector 1,

$$w_{1,t} = \omega + \pi h_{1,t} + e_{1,t} + z_t, \tag{4}$$

where $h_{j,t}$ is an indicator variable equal to 1 if the worker is skilled in sector $j \in \{0, 1\}$ at time t, or equal to zero otherwise. The constant term ω is introduced to normalize the average wage to 1. Below we decompose the shocks $(e_{0,t}, e_{1,t})$ in equations (3) and (4) to further simplify the solution of the dynamic model.

3.2 Comparisons with earlier models

As mentioned earlier, Jovanovic and Moffitt (1990) and McLaughlin and Bils (2001) are among the first to analyze the wage difference between movers and stayers. Relative to these existing models of the wage-mobility relationship, we introduce two key innovations: (i) directed mobility and (ii) evolving match quality. To help in the understanding of the importance of the two elements, we contrast our model with those of Jovanovic and Moffitt (1990) and McLaughlin and Bils (2001).

3.2.1 Undirected mobility and transitory productivity

In Jovanovic and Moffitt (1990), idiosyncratic match-specific productivity remains constant if a worker stays in the current sector. If a worker decides to leave the current sector, (i) she leaves without knowing her idiosyncratic match productivity at the destination, and (ii) her new productivity at the destination is drawn from an *exogenous* distribution. Thus, the new shock is uncorrelated with the current or previous shocks. This type of mobility is referred to as undirected mobility. In the existing literature, the main reason for using undirected mobility is computational. Under the assumption of undirected mobility, the state space of the dynamic problem and the simulation is substantially reduced (see, for example, Coen-Pirani, 2010 and Lkhagvasuren, 2012).[8]

3.2.2 Directed mobility and permanent productivity

McLaughlin and Bils (2001) study a model where the worker knows her wage at the destination before making the mobility decision. This feature of their model is inherent in Roy's (1951) model and present in much of the literature on sectoral selection (see, for example, Heckman and Taber, 2008). This key feature of Roy's model is referred to as directed mobility. Directed mobility is essential to understanding the wage gap between movers and stayers. For example, a mover may have lower wages relative to incumbents in a particular sector, but given the evolution of their idiosyncratic match-specific productivity, their wage in the new sector is higher than what it would have been had they remained in their old sector. Understanding such relationships requires a dynamic model with directed mobility.

McLaughlin and Bils (2001) also relax the assumption of Jovanovic and Moffitt (1990) that the idiosyncratic productivity shocks are drawn from an exogenous distribution. Indeed, McLaughlin and Bils (2001) allow the shocks to be correlated; however, they assume match productivity is permanent (*i.e.*, e_0 and e_1 are constant over time). Therefore, their model

[8] As explained below in Section 6.1, under directed mobility, keeping the record of sectoral history and wages of each worker and using this information for every iteration of the calibration procedure represents a major computational demand for our analysis.

considers only net mobility driven by the sector-wide shock (z), and ignores large excess flows.

3.2.3 Directed mobility with evolving match quality and skills

The main deviation of our model from the models in earlier work on the wage gap between movers and stayers is the combination of two elements: (i) directed mobility, and (ii) evolving match quality. With this combination, the wage distribution gap among movers versus stayers arises endogenously. This setup of the model allows us to focus on the impact of a persistent match shock without additional assumptions/restrictions on the income processes of stayers versus movers. This is important, especially when analyzing the wage-mobility relationship.

Finally, in the models of both Jovanovic and Moffitt (1990) and McLaughlin and Bils (2001), a worker lives for only two periods and there exists no skill premium. We extend these two-period, pre- and post-move wage comparison analyses by tracing the mover's wage over their lifetime while considering a skill premium and an evolving match shock.

3.3 Timing of the events

In our model, each period consists of four stages. In the first stage, individuals observe the sectoral shock z and the match shock e_i. In the second stage, after observing these shocks, individuals decide in which sector to work. A worker changing sectors starts as unskilled in the new sector. Production (or work) occurs during the third stage. Here each worker supplies one unit of labor and receives the wage. In the fourth stage, some of the unskilled workers become skilled. Simultaneously, some workers leave the labor market and new, unskilled workers enter the market.

3.4 Decomposing the match shock

While wages are subject to the idiosyncratic shocks e_0 and e_1, a certain component of the two shocks is not important for our main purposes, as these components do not affect mobility and the mover-stayer wage gap. To retrieve the component that is relevant for mobility and thus the wage-mobility relationship, below we decompose the idiosyncratic income shocks $(e_{0,t}, e_{1,t})$ into two orthogonal components.

First, we decompose the innovations $(u_{0,t}, u_{1,t})$ in equation (2) as follows:

$$\begin{cases} u_{0,t} = \zeta_t + \epsilon_t \\ u_{1,t} = \zeta_t - \epsilon_t, \end{cases} \tag{5}$$

where ϵ_t and ζ_t are independent, zero-mean shocks. Then, by inserting equation (5) into equation (2) and repeatedly substituting for the lagged values of e_0 and e_1, we have

$$\begin{cases} e_{0,t} = y_t + x_t = \rho(y_{t-1} + x_{t-1}) + \zeta_t + \epsilon_t = \rho e_{0,t-1} + u_{0,t} \\ e_{1,t} = y_t - x_t = \rho(y_{t-1} - x_{t-1}) + \zeta_t - \epsilon_t = \rho e_{1,t-1} + u_{1,t}. \end{cases} \tag{6}$$

According to the decomposition in equation (6), y_t represents the common component between $e_{0,t}$ and $e_{1,t}$, while x_t is the part of the match shock that drives a worker's mobility decision. Thus, x_t is important for mobility and the wage gap between movers and stayers (see Appendix B.1 for a more detailed discussion). Given the above, to simplify the analysis below, we characterize workers' mobility decisions using $(x_t, -x_t)$ shocks, rather than $(e_{0,t}, e_{1,t})$ shocks.

Notice that the persistence of the x shock is the same as the persistence of $e_{0,t}$ and $e_{1,t}$. Let $F(x' \mid x)$ denote the transition function given by the following AR(1) process:

$$x_{t+1} = \rho x_t + \epsilon_{t+1}, \tag{7}$$

where σ denotes the standard deviation of x_t, *i.e.*, $\sigma = \text{Std}(x_t)$. In our quantitative analysis below, persistence ρ and the dispersion σ of the x shock will be our main focus.

3.5 Re-defining the wages

Given the above decomposition, we redefine the wages of those working in sector 0 as

$$w_{0,t} = \omega + \pi h_{0,t} + x_t, \tag{8}$$

and the wages of those working in sector 1 as

$$w_{1,t} = \omega + \pi h_{1,t} - x_t + z_t. \tag{9}$$

We solve the model using the wages in equations (8) and (9), implying that the y shock in equation (6) is omitted in the model. This is because the y shock affects neither the workers' mobility decisions, nor the wage-tenure profiles. We do take the effect of y into account, however, when calculating certain simulated moments that require wage dispersion (see Appendix B.10).

With wages defined as in equations (8) and (9), individual productivity is *perfectly-negatively* correlated across sectors; the best-matched workers in sector j are the worst-matched workers of sector $1-j$.[9] For example, suppose an unskilled worker currently employed

[9] As stated earlier, the model can be recast as an economy with $N > 2$ sectors, by interpreting x as the worker's match shock in the current sector (*i.e.*, sector 0), and $-x$ as the highest of the $N-1$ match shocks

in sector 1 receives a shock $x_t > 0$ at the beginning of period t. This shock makes the worker more productive in sector 0 relative to sector 1 when $z_t = 0$. Therefore, the worker may prefer to move to sector 0.

3.6 Value functions

Let $U_j(h, x, z)$ denote the lifetime utility of a worker in sector $j \in \{0, 1\}$ with skill level $h \in \{0, 1\}$, where x and z represent the match and sector shocks, respectively. This represents the utility associated with the moment following the realization of the shocks, but preceding the mobility decision.

3.6.1 Skilled stayers

For a skilled worker in sector j, the lifetime utility of staying in j is given by

$$S_j(1, x, z) = w_j(1, x, z) + \beta(1 - \delta) \iint U_j(1, x', z') dF(x' \mid x) dG(z' \mid z), \tag{10}$$

where β is the time-discount factor.

3.6.2 Unskilled stayers

For an unskilled worker in sector j, the lifetime utility of staying in j is given by

$$\begin{aligned} S_j(0, x, z) = \quad & w_j(0, x, z) + \beta(1 - \delta) \left\{ p \iint U_j(1, x', z') dF(x' \mid x) dG(z' \mid z) \right. \\ & \left. + (1 - p) \iint U_j(0, x', z') dF(x' \mid x) dG(z' \mid z) \right\}. \end{aligned} \tag{11}$$

3.6.3 Movers

The lifetime utility for a worker moving from sector j to $1 - j$ is given by

$$M_j(x, z) = S_{1-j}(0, x, z). \tag{12}$$

3.6.4 The mobility decision

Given the value functions S_j and M_j, the lifetime utility of a worker with skill level h is given by

$$U_j(h, x, z) = \max\{S_j(h, x, z), M_j(x, z)\}. \tag{13}$$

from the remaining $N - 1$ sectors. Section 7.1 provides further details.

Let Ω_j denote the decision rule governing whether a person in sector j stays in her current sector:

$$\Omega_j(h,x,z) = \begin{cases} 1 & \text{if } S_j(h,x,z) \geq M_j(x,z), \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

Further details of the mobility decision are provided in Appendix B.2.

3.7 Measures

Let τ denote the number of periods a person has worked in their current sector since entering the labor market or since the last move. This sector tenure τ is measured at the end of each period, *i.e.*, $\tau \in \{1,2,3,\cdots\}$. At any t, a worker in sector j is fully characterized by her skill level h, match shock x, and sector tenure τ. Let $\mu_{j,t}(h,x,\tau)$ denote the number of workers in state (h,x,τ) in sector j at the end of period t. The next period's measures $(\mu_{0,t+1}(h,x,\tau), \mu_{1,t+1}(h,x,\tau))$ are determined by the current measures $(\mu_{0,t}(h,x,\tau), \mu_{1,t}(h,x,\tau))$ and the new sectoral shock z_{t+1}.

The total number of workers in the economy is given by

$$L_t = \sum_h \sum_\tau \int (\mu_{0,t}(h,x,\tau) + \mu_{1,t}(h,x,\tau))dx \tag{15}$$

for all t. The number of new workers born in sector j is proportional to the measure of the unskilled workers of the sector:

$$\mu_{j,t}(0,x,0) = \frac{\delta L_t \sum_{\tau \geq 1} \int \mu_{j,t}(0,x,\tau)dx}{\sum_{\tau \geq 1} \int (\mu_{0,t}(0,x,\tau) + \mu_{1,t}(0,x,\tau))dx} \tag{16}$$

for all x. The law of motion governing sectoral dynamics and the definition of the equilibrium are contained in Appendices B.3 and B.4.

4 What drives the wage-mobility relationship?

Before going to our quantitative analysis, it is useful to discuss how mobility and wages are interrelated in this dynamic extension of Roy's (1951) model and to understand what drives the wage gap between movers and stayers. In our model, there are two potential sources driving the wage gap between movers and stayers within a given sector: (i) the skill premium, π, and (ii) the dynamic comparative advantage component, given by the persistent idiosyncratic match shock, x.

4.1 Skill premium

The role of the skill premium, π, is generally well understood in the existing literature. Indeed, in our model, it is straightforward to see how the skill premium affects wages and mobility. First, a skilled worker has a higher wage than an unskilled worker and moves less frequently as a result. Second, because of directed mobility, skilled workers "require" higher wages at the destination relative to unskilled workers. These effects make the wages of stayers higher than those of movers. Furthermore, since those with higher tenure are more likely to be skilled, the skill premium generates a positive wage-tenure profile. These predictions are consistent with many data features discussed in Section 2.

4.2 A persistent match shock

The skill premium, however, does not represent the only explanation for the aforementioned wage-mobility patterns. It turns out that a persistent match shock can also generate these features of the wage-mobility relationship. To demonstrate this effect both numerically and analytically, we consider a simplified version of the model with *no skill premium* and *no sector-wide shock* where workers are infinitely-lived; *i.e.*, $\pi = 0$, $z_t = 0$ for all t, and $\delta = 0$. We refer to this version of the model as a Roy model with a match shock. The following section contains numerical and theoretical results on how a persistent match shock in this Roy model can generate a positive wage-tenure relationship and other data patterns presented in Section 2.

5 The role of a dynamic match shock

In this section, we highlight the role played by a match shock analytically, and then we illustrate the results via a simple numerical simulation.

5.1 A simple Roy model with a match shock

There are two sectors: 0 and 1. An individual's productivity is subject to an idiosyncratic match shock, x_t, which follows the AR(1) shock in equation (7). If a person works in sector 0, her current wage is given by

$$w_0(x_t) = \omega + x_t, \tag{17}$$

where ω is a constant. However, if the person works in sector 1, her current wage is given by

$$w_1(x_t) = \omega - x_t. \tag{18}$$

These wages imply that if the current match shock x_t is positive, a worker is better off in sector 0. However, if the match shock is negative, a worker is better off in sector 1. Thus, we have the following simple decision rule:

$$\begin{cases} \text{a worker in sector 0 moves at period } t \text{ if } x_t < 0, \text{ or stays if } x_t \geq 0; \\ \text{a worker in sector 1 moves at period } t \text{ if } x_t > 0, \text{ or stays if } x_t \leq 0. \end{cases}$$

5.2 Analytical results: a transitory match shock

For comparison, we first consider the case of a transitory match shock. When the match shock is transitory, half of the workers move across sectors each period. The average match quality is identical across tenure levels. Specifically, the mean value of the match shock at each level of sectoral tenure is given by $\mathbf{E}(\epsilon_t \mid \epsilon_t \geq 0)$. With a transitory shock, the wage-mobility relationship in this simple economy is summarized by the following claim.

Claim 1 (**A transitory shock**). *When the match shock is transitory (*i.e., $\rho = 0$*),*

1. *the mean wage of newcomers is equal to the mean wage of incumbent workers,*
2. *the mean wage of future movers is equal to the mean wage of future stayers,*
3. *the mean wage of recent movers remains constant with sectoral tenure, and*
4. *overall earnings and individual-level mobility are uncorrelated over a finite number of periods.*

Proof. See Appendix B.6. □

Thus, a transitory match shock has no effect on the mover-stayer wage gap or the wage-tenure profile. This further implies no correlation between lifetime earnings and mobility. Since mobility occurs via an i.i.d. shock process, lifetime earnings are also unaffected by mobility. When the match shock is persistent, however, the wage-mobility relationship in the model becomes similar to the wage-mobility relationship in data, as shown in the next subsection.

5.3 Analytical results: a persistent match shock

Here we analytically explore the effects of the persistence of the idiosyncratic match shock on wages and mobility.

5.3.1 Mobility and a persistent match shock

Consider workers in sector 0, that is, workers for whom $x \geq 0$. These workers leave their current sector if next period's shock, x' is negative, *i.e,* $x' = \rho x + \epsilon < 0$. Let ξ be a random shock with zero mean and finite variance. Then, the mobility rate is given by the conditional probability

$$m(\rho) = \mathrm{Prob}[\rho x + (1-\rho^2)^{1/2}\xi < 0 \mid x \geq 0].$$

Claim 2 (**The mobility rate**). *When the match shock follows a continuous, non-degenerate distribution, a more persistent match shock implies lower mobility.*

Proof. We need to show that $m(\rho)$ decreases with ρ. Let H and K denote the distribution functions of x and ξ, respectively. It can be written that

$$m(\rho) = \frac{\int_0^\infty K\big(-\frac{\rho x}{\sqrt{1-\rho^2}}\big)dH(x)}{\int_0^\infty dH(x)}.$$

Since K is a strictly increasing function, the kernel of this integral, $K\Big(-\frac{1}{\sqrt{1/\rho^2-1}}x\Big)$, decreases with ρ for $0 < \rho < 1$. Therefore, the mobility rate $m(\rho)$ decreases with ρ for $0 < \rho < 1$. $\square$

5.3.2 Wages and lifetime earnings under a persistent match shock

Now we explore how the persistence of the match shock affects the wage gap between movers and stayers, wage-tenure profiles, and lifetime earnings. (The proofs of the followings propositions are contained in Appendices B.7 to B.9.)

Proposition 1 (**Wages of newcomers**). *When the match shock is non-degenerate and persistent (*i.e., *$\sigma > 0$ and $\rho > 0$), the mean wage of the newcomers is lower than the mean wage of the incumbent workers.*

Proposition 2 (**Wages of future movers**). *When the match shock is non-degenerate and persistent (*i.e., *$\sigma > 0$ and $\rho > 0$), the mean wage of future movers is lower than the mean wage of future stayers.*

According to Propositions 1 and 2, when $\rho > 0$ (*i.e.,* not transitory), movers have lower average wages in both the origin and destination sectors. Next, we analyze how a persistent match shock affects the wage-tenure profile.

Proposition 3 (**The wage-tenure profile**). *When the match shock is nondegenerate and persistent (*i.e., *$\sigma > 0$ and $\rho > 0$), the average wage increases with sectoral tenure, over any finite range of sectoral tenure.*

The results in Proposition 3 imply that, when the match shock is persistent, the negative wage gap between newcomers and incumbent workers shrinks over time. The above three propositions also imply that when the match shock is nondegenerate and persistent (*i.e.,* $\sigma > 0$ and $\rho > 0$), total earnings and individual-level mobility are negatively related, over a finite number of periods.

5.4 Illustrative numerical examples

In this numerical example, we set the persistence of the match shock, ρ, to 0.9 and the unconditional standard deviation of the match shock, σ, to 0.5. We start with 10 million workers, and simulate the match shocks and mobility of these workers according to the algorithm outlined in Appendix B.5.

5.4.1 The evolution of the match shock

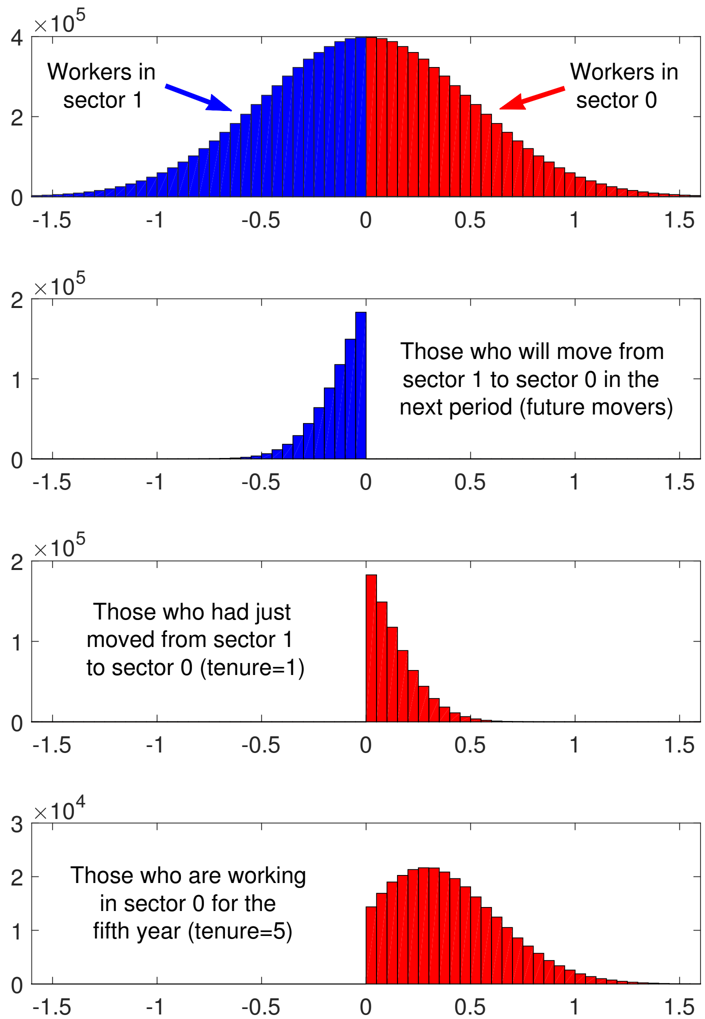
We summarize the main pattern of the distribution of the match shock in Figure 3. The distribution of all workers by the x shock is shown in the topmost (first) panel of Figure 3. Given the decision rule described above, the right half of this distribution corresponds to workers in sector 0 (with $x \geq 0$) and the left half to workers in sector 1 (with $x < 0$).

In the second panel from top, we display the distribution of those in sector 1 who will move to sector 0 in the next period. Comparing this distribution of future movers with the lower half of the distribution in the topmost panel, it can be seen that the average shock for those who will move from sector 1 to sector 0 in the next period is higher than the average shock of the workers in sector 1. Since for sector 1 workers, the wage is negatively related to the match shock (*i.e.*, $w_{1,t} = \omega - x_t$), the average wage of future movers is lower than the average wage of stayers. In other words, future movers represent the relatively less productive (and thus lower-wage) workers in their current sector.

In the third panel, we display the distribution of shocks for these workers after they have moved from sector 1 to sector 0 (*i.e.*, those who are working for the first year in sector 0 since their last move). Comparing this distribution with the upper half of the distribution in the topmost panel, the average value of the x shock among these newcomers is lower than the average shock of all workers in sector 0. Since the wages of sector 0 workers are positively related to the match shock (*i.e.*, $w_{0,t} = \omega + x_t$), the average wage of newcomers is lower than the average wage of incumbents.

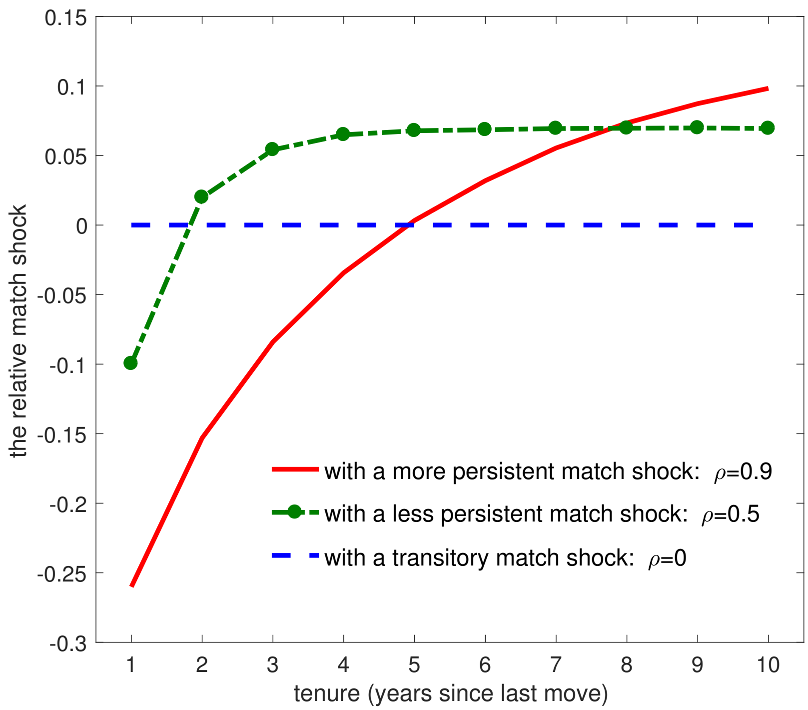
Finally, in the bottom panel, we show the distribution of match shocks for those working in sector 0 for the fifth consecutive year since their last move. Comparing this distribution with the distribution of the newcomers in the third panel, the average wage increases with sectoral tenure. Taking all of these observations together, this simplified model, with only a persistent idiosyncratic match shock, has generated the key wage-mobility relationships described in Section 2: (i) movers have a lower wage relative to stayers at both the origin and destination and (ii) the average wage increases with sectoral tenure.

Figure 3: Distribution of a Match Shock (the Model with Only a Match Shock)



Notes: This figure shows the distribution of the match shock in a simple economy with no skill accumulation, no sector-wide shock, and no labor market exit ($\pi = 0$, $z_t = 0$ for all t, and $\delta = 0$). In this parsimonious model, the decision rule is given by the x shock only, as explained in Section 5. Recall that as shown in equations (8) and (9), the wages in sector 0 are positively related to the match shock x, while the wages in sector 1 are negatively related to the match shock x. The figure is based on 10 million workers. It shows that, on average, movers have a lower wage than stayers at both the origin and destination. Also, the average wage grows with sectoral tenure. See Propositions 1 to 3 and Figure 4 for further details.

Figure 4: Match Quality by Tenure (the Model with Only a Match Shock)



Notes: This figure shows the wage-tenure profiles of a simple economy with no skill accumulation, no sector-wide shock, and no labor market exits ($\pi = 0$, $z_t = 0$ for all t, and $\delta = 0$). The three profiles are associated with three different values of ρ, persistence of the match shock x. The unconditional standard deviation of the match shock, σ, is set to 0.5. Each wage-tenure profile is based on the wages and mobility of 10 million workers. The vertical axis measures the magnitude of the match shock minus the mean value of the match shock in sector 0. The mean value of the match shock of sector 0 is given by $\mathbf{E}\big(x \mid x \geq 0\big)$ and equal to 0.399. Recall that wages in sector 0 are positively related to the match shock. The figure shows that persistence of the match shock raises both the mover-stayer wage gap, in absolute terms, and the overall slope of the wage-tenure profile. See Propositions 1 to 3 and Figure 3 for further details.

5.4.2 The wage-tenure profile

To further illustrate the role of a persistent match shock, consider Figure 4 where we present cases with different values of ρ. In Figure 4, we show how the average value of the match shock evolves with sectoral tenure for $\rho = 0$ (a transitory shock), $\rho = 0.5$, and $\rho = 0.9$. Figure 4 shows that when the match shock is transitory, there is no wage gap between movers and stayers, and the wage-tenure profile is constant. Moreover, the figure shows that the persistence of the match shock raises the wage gap between movers and stayers as well as the overall slope of the wage-tenure profile.

5.5 Discussion

We have shown that under directed mobility, a persistent worker-sector match shock can generate the main patterns of the wage-mobility relationship in PSID.[10] The above theoretical results suggest that in order to account for the substantial wage gap between movers and stayers in data, one can use either a highly persistent match shock in the absence of the skill premium, or the combination of a substantial skill premium and a transitory match shock. However, as shown below, these two approaches have very different quantitative implications on the labor mobility pattern. Therefore, in order to understand the relative role of the skill premium and the match shock, one needs to consider the actual mobility pattern measured in data. It is important to keep this in mind when going to the quantitative analysis in Section 6.

6 Quantitative analysis of the benchmark model

The model in Section 3 provides a flexible framework to analyze the relationship between wages and mobility. This section examines, quantitatively, how well the model captures the main facts in Section 2. Importantly, this quantitative analysis also examines what role the dynamic match shock (*i.e.*, comparative advantage) plays in the evolution of wages and mobility.

6.1 Computation

Finding the mobility decision rule in the benchmark model involves accounting for both the dynamic match and sectoral shocks in the presence of the sector-specific skill premium. Moreover, one must keep track of wages for each worker by their sectoral tenure. It should be

[10] While we consider the wage-mobility relationship in the U.S. using PSID, our results have an implication on cross-country differences. The analytical results in Section 5 imply that higher persistence of the match shock means lower mobility and a steeper wage-tenure profile. This is consistent with Deelen (2012) who finds in cross-country data including France, Germany, Netherlands, U.K., and U.S. that countries with lower labor mobility tend to have a steeper wage-tenure profile.

emphasized that finding workers' decision rules and simulating the wage and mobility of these workers are not that computationally demanding per se. However, keeping the record of sectoral history and past wages of *each worker* and using this information for every iteration of the calibration procedure (finding the minimum distance between the empirical and targeted moments) imposes the main computational demand.

6.1.1 Numerical solution

The model is solved by discretizing the state space along x and z. The sectoral technology shock, z, is approximated by a three-state Markov chain. A relatively fine grid for x is necessary to generate the observed level of mobility and the wage gap between movers and stayers. For this reason, the stochastic process for x is approximated by a 51-state Markov chain. The Markov chains are constructed using the finite-state process of Rouwenhorst (1995).[11] Then, using value function iteration, we find the decision rule in equation (14) for each sector j and for each discrete value of x, z, and h.

6.1.2 Simulation

To simulate the model, we draw the sequence of the three-state shock, z, for $T = 2000$ periods while keeping track of the distribution of heterogeneous agents over (j, h, x, τ). The first 500 periods are discarded, and T is set so that increasing it does not affect the moments. For the initial measures, $\mu_{j,0}(h, x, \tau)$, $j \in \{0, 1\}$, we consider the case that all workers are unskilled (*i.e.*, $h = 0$) and distributed equally between the two sectors. To measure life-cycle income and individual-level mobility, we consider the wages and mobility decisions of 50,000 individuals. The remaining moments are measured by normalizing the total number of workers to 1.

6.2 Calibration details

Several of the model parameters can be set directly from the data. The remaining parameters are calibrated to match certain moments in the data.

6.2.1 Parameters calibrated directly from data

The time period is one year. We set $\beta = 1/1.04$, consistent with an annual interest rate of 4%. The probability of leaving the labor market (or retiring), δ, is set to 0.025, implying an expected working lifetime of forty years. The probability of becoming skilled, p, is set by following Kambourov and Manovskii (2009). Specifically, we observe that the positive slope of the wage-tenure profile decreases at tenure levels of eight to twelve years. Accordingly, we set

[11] Galindev and Lkhagvasuren (2010) show that the method of Rouwenhorst (1995) outperforms the other commonly used discretization methods for highly persistent AR(1) shocks.

$p = 1/10$, implying an average duration of ten years to become skilled in a particular sector. Below we analyze how the main predictions of the model respond to the parameter p.

As mentioned earlier, the constant term of the wage function in equations (8) and (9) is introduced merely to normalize the average wage to 1. In the benchmark model, $\omega = 0.7422$. To measure the sector-specific productivity shock z_t, we use annual per-worker output from 1987 through 2012, tabulated by the Bureau of Labor Statistics (BLS). Analogous to Blanchard and Katz (1992), we measure z_t as the difference between per-worker output in the U.S. manufacturing sector relative to per-worker productivity of the Non-farm business sector, measured in log-differences. We take the standard deviation and annual autocorrelation of this relative productivity from the trend (HP filtered with smoothing parameter 100). This yields $s = 0.0068$ (standard deviation) and $r = 0.4236$ (persistence). Sensitivity of these numbers to the sample period is discussed in Appendix B.12.

6.2.2 Parameters calibrated by targeting moments

This leaves the parameters governing the match shock, ρ and σ, and the skill premium π. We calibrate these parameters by targeting three moments in data.

First, we target the annual gross mobility rate of 6.78% (see Figure 7). The second targeted moment is volatility of sectoral employment. For sectoral employment, we take the log of manufacturing employment minus the log of aggregate employment. As with per-worker output above, both employment series are tabulated by the BLS. We use annual employment data from 1987 to 2012 and calculate an unconditional standard deviation of sectoral employment (from its HP trend with the smoothing parameter 100) of 0.0059 (0.59%).

The third moment we target is repeat mobility. Repeat mobility is the probability that a worker moves, conditional on having moved in the previous period (see Farber, 1994). According to the PSID data, repeat mobility is 27%; *i.e.*, approximately one quarter of current movers move again in one period. Note that repeat mobility captures a certain feature of the distribution of workers by sectoral tenure: the number of workers with two years of tenure is 27% lower than the number of workers with one year of tenure.

The main intuition behind our calibration is the following. As mentioned earlier, overall mobility declines with the skill premium π. Also, a more persistent match shock implies a smaller probability of repeat mobility. Finally, a higher dispersion of the match shock implies lower net mobility, which in turn implies a lower volatility of sectoral employment. This is because a higher dispersion of the match shock implies fewer workers on the margin between moving and staying.[12] In fact, we show in Appendix B.13 that reducing σ (the standard deviation of the match shock) raises the density of these marginal workers, making sectoral

[12]In Figure B.2, it can be seen that with higher σ, fewer workers are indifferent between moving and staying.

employment more volatile. The above data targets imply $\sigma = 0.1112$, $\rho = 0.5440$, and $\pi = 0.3137$. Table 2 displays the benchmark parametrization.

Table 2: Benchmark Parameters

Parameters	*Values*	*Description*
Using data and normalization		
β	0.9615	time discount factor
δ	0.0250	probability of retirement
ω	0.7422	the constant term of the wage function
r	0.4236	persistence of the sector shock
s	0.0068	standard deviation of the sector shock
p	0.0100	probability of becoming skilled
Targeting moments		
π	0.3137	skill premium
ρ	0.5440	persistence of the match shock
σ	0.1112	standard deviation of the match shock

Notes: The table summarizes the key parameters of the benchmark model. The constant term of the wage function, ω, is chosen by normalizing the mean wage to 1.

6.3 Results

Table 3 presents the main results. First, the benchmark model performs well, capturing the main patterns of mobility. Specifically, it generates the observed level of gross, net, and repeat mobility, the main targets in our calibration.

To evaluate the model's performance, Table 3 displays how well the model matches several un-targeted moments. Specifically, we consider movers' wage at their origin and the destination, average wage growth among recent movers, and the correlation of lifetime earnings and mobility. It is worth noting that all of the un-targeted moments are wage related, while the targeted moments are exclusively mobility related. In this regard, the model's success in capturing the un-targeted moments is promising.

6.3.1 The mover-stayer wage gap

First, we consider the model's performance matching movers' wage at their origin. From Table 3, the model slightly overpredicts this wage gap. According to the data, one year before moving, movers have average wages 18.02% below the average wage in the origin industry. Our model predicts a wage gap of 19.60%. Next, the model also slightly overpredicts the

Table 3: Main Predictions

Moments	*Data*	*Bench-mark* $p=\frac{1}{10}$	*Faster skill accum.* $p=\frac{1}{8}$	*Slower skill accum.* $p=\frac{1}{12}$
Moments targeted in the benchmark model				
mobility	0.0678	0.0683	0.0580	0.0778
volatility of sectoral employment	0.0059	0.0059	0.0055	0.0064
repeat mobility	0.2729	0.2247	0.2123	0.2332
Key predictions				
movers' mean wage at the origin	-0.1803	-0.1960	-0.2045	-0.1879
movers' mean wage at the destination	-0.1786	-0.1794	-0.1877	-0.1715
wage growth among recent movers	0.0259	0.0263	0.0296	0.0237
corr. of lifetime earnings and mobility	-0.1523	-0.1685	-0.1649	-0.1681

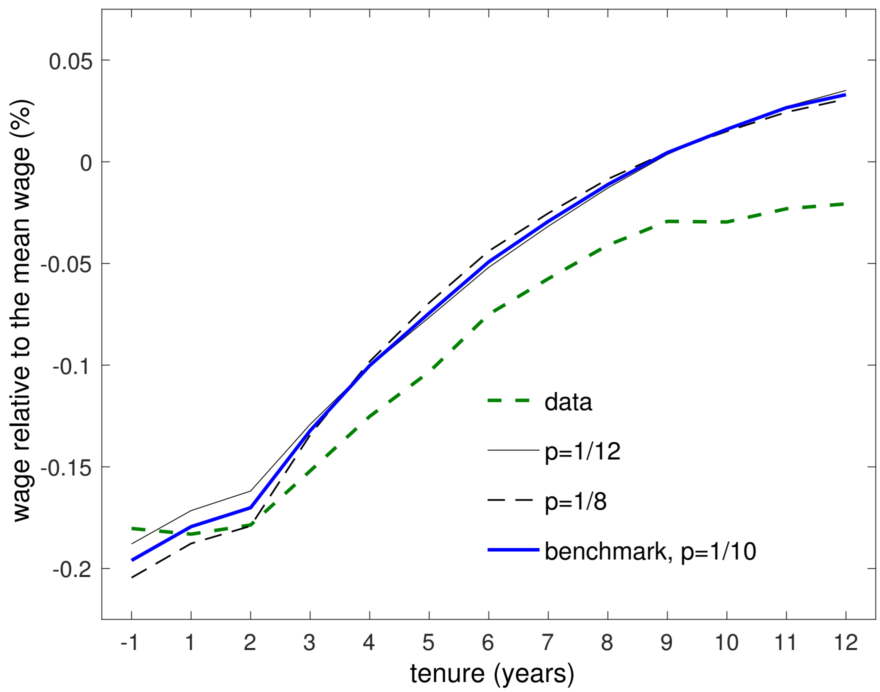
Notes: The columns denoted by *Data* and *Benchmark* summarize moments measured from the PSID data and the benchmark models respectively. In the benchmark model, it takes on average ten years for workers to become skilled in a sector, *i.e.*, $p = \frac{1}{10}$. In the last two columns of the table, we examine the impact of this parameter, where the model is simulated for different values of p, holding the other parameters at their benchmark values. When calculating the correlation of lifetime earnings and mobility in simulated data, we must allow for the impact of the wage variation not accounted by the x-shock. The details of this calculation are contained in Appendix B.10. In the table, *repeat mobility* refers to the probability that a worker will move, conditional on having moved in the previous period.

mover-stayer wage gap in the destination sector. In the data, movers have average wages 17.86% below the average in the destination sector, while the model predicts a wage gap of 17.94%.

6.3.2 The wage-tenure profile

In Figure 5, we compare the wage-tenure profile in the data to that generated by the model. Here, we plot the evolution of wages, starting one year before the move, and following wages for twelve years. The figure shows that the wage-tenure profile in the model lies above the empirical wage-tenure profile. Thus, our model slightly overpredicts wage growth among recent movers. Specifically, as Table 3 shows, in the PSID data, workers experience an annual wage growth of 2.59% during the first five years following mobility, while the model predicts amatannual wage growth of 2.63%. Nevertheless, the model is able to capture the main pattern of the wage-tenure profile.

Figure 5: The Wage-Tenure Profile



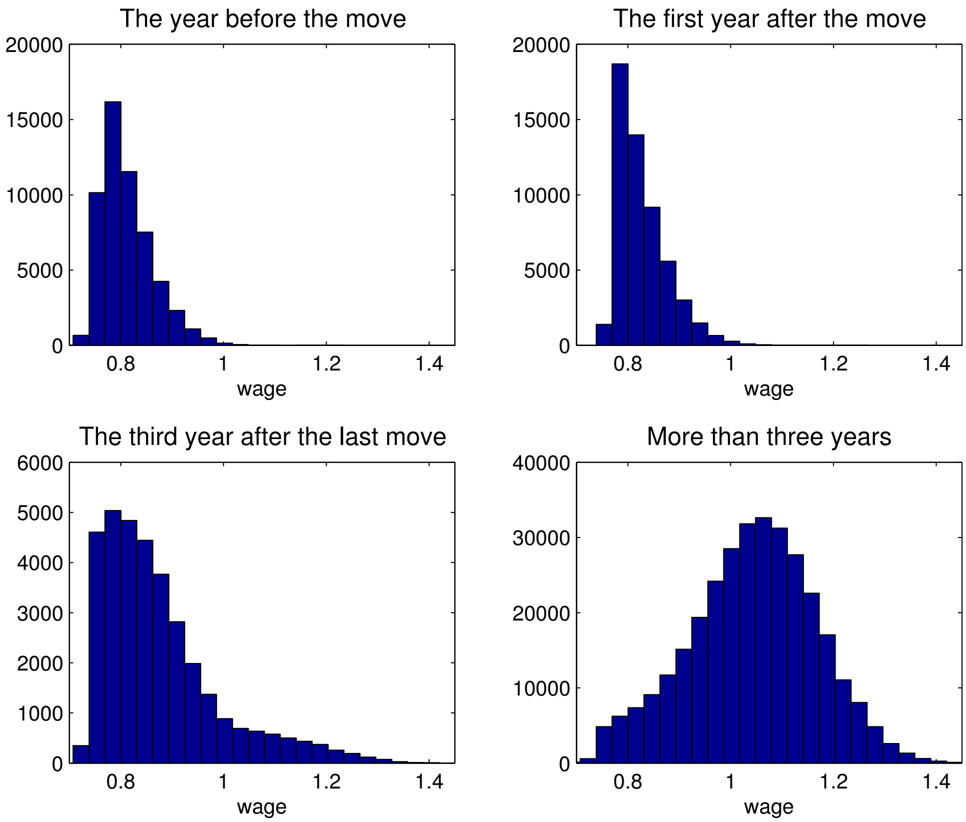
Notes: This figure plots how well the model captures the evolution of wages for recent movers and the effect of the parameter p on this wage-tenure profile. A tenure of 1 refers to the first year after mobility. A tenure of -1 refers to the year before mobility. The empirical wage profile labeled "data" is the specification where industry, year, age, and education effects are controlled for (*i.e.*, the thick, dashed curve shown in Figure 2). The profile labeled "benchmark" is generated by the benchmark model. The other two profiles are generated by the model for different values of p, the probability of becoming skilled. A lower value of p implies workers become skilled faster, increasing the slope of the wage-tenure profile early in the spell, while flattening it later in the spell. As seen in Table 3, faster skill accumulation also reduces mobility. (Also, see Propositions 1 to 3, and Table 3.)

6.3.3 Evolution of the wage distribution

Figure 1 shows evidence supporting different wage distributions for movers and stayers. To examine the model's predictions along this dimension, and to illustrate how wages evolve for movers in the model, we plot the wage distribution for different tenure levels in Figure 6. It shows that movers are drawn more from the lower tail of the wage distribution both before and after the move. As workers' tenure in the sector increases, their wages are drawn from a distribution with a higher mean.

This occurs for two reasons. First, as workers stay longer in a sector, they become more skilled, which can be seen from the evolution of the right tail of the wage distribution among recent movers shown in the lower-left panel of Figure 6. Second, by referring to Proposition 3

Figure 6: The Wage Distribution and Tenure in the Benchmark Model



Notes: The figure plots the wage distribution in the benchmark model. It shows that movers tend to be drawn more from the lower end of the wage distribution both before and after mobility, consistent with the empirical facts shown in Figure 1 and the analytical results contained in Propositions 1 and 2. The mean wage in the entire economy is 1. This figure also shows how wages evolve for recent movers. Among recent movers, those who stay longer in their current sector are those who draw, on average, better subsequent shocks (also see Proposition 3 and Figure 3) and become more skilled. The distributions in the figure are based on 800,000 observations.

and by looking at the left tail of the distributions in Figure 6, those who remain in a sector longer tend to have better match shocks, while movers are drawn more from the lower end of the wage distribution (also see Figure 3).

6.3.4 Lifetime earnings and mobility

The empirical analysis in Section 2 introduced a novel fact: lifetime income and mobility are negatively correlated. In Table 3, we show that the model captures the negative correlation between lifetime income and mobility measured in Table 1.[13] The comparison is made using the correlation between indexes $\mathcal{M}^{\mathrm{b}}$ and $\mathcal{E}^{\mathrm{b}}$ described in Section 2. According to the data, the correlation is -0.1523 compared to the model-generated value of -0.1685.

There are two main forces behind this negative correlation. First, workers with less mobility have longer tenure in a sector. In contrast, a worker changing industries several times over their lifetime are less likely to realize the wage gains from tenure. Second, as Proposition 3 and Figure 6 show, workers changing sectors more frequently on average have a lower quality match shock at both the origin and the destination. Since the shock is persistent, this poor match quality persists; as a result, these workers suffer lower lifetime income. Below in Section 8, we show that in the absence of skill accumulation, a persistent match shock alone can generate a substantial negative correlation between lifetime income and mobility.

6.3.5 Additional evidence: mobility by labor market experience

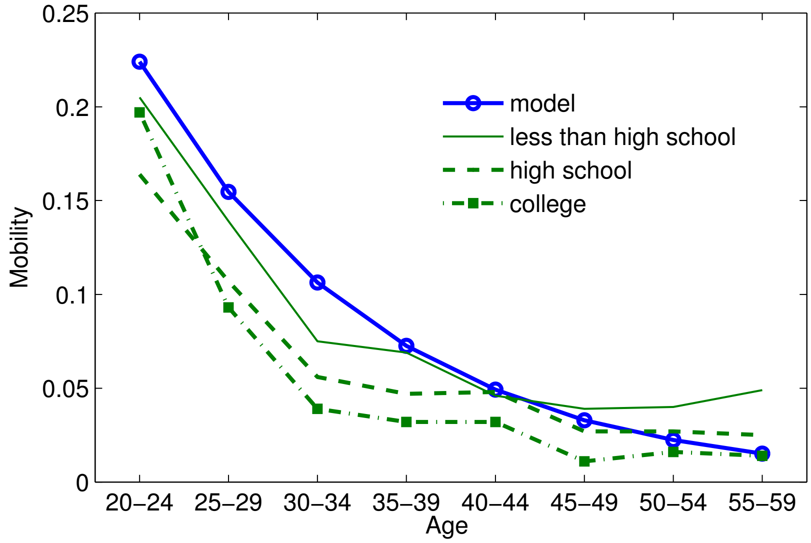
Although we do not target any age-mobility moment in the model, we can examine how mobility changes with overall labor market experience. Figure 7 shows that younger workers are more likely to move relative to older workers. It shows that the model performs well in producing the negative relationship between mobility and labor market experience. Thus, a skill premium and a persistent match shock, which are the key forces driving the evolution of the wage distribution shown in Figure 6, also account for the negative relationship between mobility and age. This is consistent with Farber (1994) who argues that evolving match quality might be important for the negative relationship between mobility and labor market experience.

7 Numerical experiments

Up to this point, we have established important links between wages, tenure, mobility, and lifetime income and demonstrated that the dynamic model accounts for these facts. This

[13] If we allow workers to exert effort that affects skills, the persistence of the two income shocks (x and z) could substantially raise the correlation between lifetime earnings and mobility.

Figure 7: Mobility and Labor Market Experience



Notes: This figure shows that the model captures the negative relationship between mobility and labor market experience. The curve with circles shows the mobility rate in the model, assuming that a worker enters the labor market at 20 years of age. The average labour market experience in the model is forty years. The simulated data contain 20,000 individuals. The other curves (green) show the mobility rate among different age and educational groups in PSID. In data, the annual mobility rate is 6.78%.

section performs several numerical experiments, quantifying the role of a persistent match shock.

7.1 Sectoral versus match shocks

We start our numerical experiments by examining the importance of the sectoral shock. In particular, we simulate the model by turning off the sectoral shock, *i.e.*, by setting $z = 0$. Table 4 summarizes the quantitative predictions and compares them to those of the benchmark model. In the absence of the sectoral shock, the individual-level relationship between mobility and wages remains almost identical to the baseline case. Of course, the model does not generate volatility in sectoral employment, but all other moments are virtually unchanged in this case. This suggests that excess mobility driven by a persistent match shock remains key to understanding the patterns of wages and mobility. While incorporating the sectoral shock helps us to identify and calibrate the dispersion of the match shock, σ, it does not drive our main results on the wage-mobility and wage-tenure relationships. This is in contrast to much of the literature that has focused on net mobility driven by sector-wide or aggregate shocks.

In Appendix B.13, we also consider a model where mobility is primarily driven by the sectoral level shock rather than the match shock. While such a model performs well along many dimensions, it generates implausibly high volatility in sectoral employment. This further underscores the important role played by the dynamic worker-sector match shock, not only for the individual level wage-mobility relationship, but also for sectoral level dynamics.

Table 4: Impact of the Sectoral Shock

Moments	*Benchmark model*	*No sectoral shock* (z=0)
mobility	0.0683	0.0683
volatility of sectoral employment	0.0059	0
repeat mobility	0.2247	0.2247
the mean wage of movers at the origin	-0.1960	-0.1963
the mean wage of movers at the destination	-0.1794	-0.1798
annual wage growth among recent movers	0.0263	0.0263
correl. of lifetime earnings and mobility	-0.1685	-0.1693

Notes: This table shows that in the absence of the sectoral shock, the individual-level relationship between mobility and wages remains almost identical to the baseline case. The column denoted by *Benchmark model* summarizes the predictions of the benchmark model. The column denoted by *No sectoral shock* corresponds to the predictions of the model where the sectoral shock is turned off (*i.e.*, set $z = 0$ in the benchmark model.)

7.2 The impact of a match shock on average income

In Table 5, we conduct two counterfactual experiments to quantify the impact of mobility and match shocks. The first experiment is to prohibit mobility, which provides a measure for the overall impact of mobility. Table 5 shows that when mobility is prohibited in the benchmark model, overall labor income declines by 0.7% (=100-99.3). That is, mobility raises overall labor income by 0.7%.

The second experiment asks what happens when the effect of the match shock on wages is zero (*i.e.,* $x = 0$), but mobility remains as it is in the benchmark model. The results of this experiment show that the match shock accounts for 2.5% (=100-97.5) of labour income. Using the numbers in Table 5, one can say that the net gain from mobility is 1.8% ($= 2.5\% - 0.7\%$) of labor income.

These numbers are much lower than the impact of mobility measured by Jovanovic and Moffitt (1990). Using a two-period model, they find that sectoral mobility raises income by 8.5% to 13%. The main reason behind this large difference between their numbers and ours is the nature of idiosyncratic match productivity. In Jovanovic and Moffitt (1990), match productivity in the initial sector remains constant. Therefore, if a worker is not allowed to move, the initial bad productivity affects future earnings permanently. In contrast, in our model, match productivity evolves over time and the impact of the initial match shock diminishes over time. This suggests that the evolution of the match shock might be an essential element for quantifying the impact of mobility.

Table 5: The Mean Wage under Different Restrictions

the benchmark model	100.00%
when mobility is prohibited in the benchmark model	99.30%
mobility as in the benchmark model, but no match shock (*i.e.,* $x = 0$)	97.54%

Notes: This table compares the mean wage in the benchmark model with that in other restricted versions of the model. Mobility raises overall labor income by 0.7% (=100-99.3). The match shock accounts for 2.46% (=100-97.54) of labour income. See Section 7.2 for a further discussion.

7.3 The cost of exogenous separation

The results in the above counterfactual experiments have an important implication on the value of a job. Topel (1991) estimates the value of a job by measuring the cost of an exogenous separation using a wage-tenure profile. In Topel (1991), the loss upon separation is based entirely on the skill-premium, and the value of a job is measured relative to the value of the average job with no experience. That is, the cost of an exogenous separation is the wage loss the worker experiences, which in Topel (1991) is measured exclusively by the skill premium. Topel (1991) finds that ten years of current tenure raises the wage of the typical male worker in the United States by over 25%.

Our analysis allows for a similar exercise, while providing further insights into what else is lost upon exogenous separation. According to our model, an exogenous separation causes a loss of the specific skills *and* the value of the "quality" of the match. Suppose that we displace a worker from sector 0 and let her work in sector 1. Using the wage equations (8) and (9), the worker's wage loss in this case is given by:

$$w_{0,t} - w_{1,t} = \pi h_{0,t} + 2x_t. \tag{19}$$

In this equation, the wage loss consists of two parts: the specific skill (π), emphasized by Topel (1991), and the loss of the match quality (x_t).

Table 6: Re-Calibration

	data	*the model with no skill premium* $\pi = 0$
Key parameters		
the persistence of the match shock, ρ		0.9352
dispersion of the match shock, σ		0.4488
Moments		
mobility	0.0678	0.0679
repeat mobility	0.2729	0.0884
volatility of sectoral employment	0.0059	0.0059
wage of movers at the origin	-0.1803	-0.2821
movers wage at the destination	-0.1786	-0.1868
annual wage growth among recent movers	0.0259	0.0281
correlation of lifetime earnings and mobility	-0.1523	-0.2007

Notes: The table shows the results of the re-calibration that omit sector-specific human capital (*i.e.*, $\pi = 0$), where the parameters of the match shock, σ and ρ, are re-calibrated by targeting gross and net mobility in the data while setting the skill premium parameter π to zero. The rest of the parameters, except for ω, are as in the benchmark model. The constant term of the wage function under the re-calibration is $\omega = 0.6422$.

How big is the second component? The counterfactual experiment displayed in Table 5 helps us quantify the magnitude of the second component. The last row of Table 5 shows that when we set the match shock to zero ($x = 0$) while keeping mobility and skill accumulation as in the benchmark model, the average wage drops by 2.46%. This is the mean value of x among those employed in sector 0. As shown in equation (19), we must multiply this number by two. Thus, our numerical experiment suggests that the wage loss resulting from the loss of the match quality is 5% of the mean wage, implying that the dynamic match quality represents an important component in the value of a job.

8 Re-calibration in the absence of skill premium

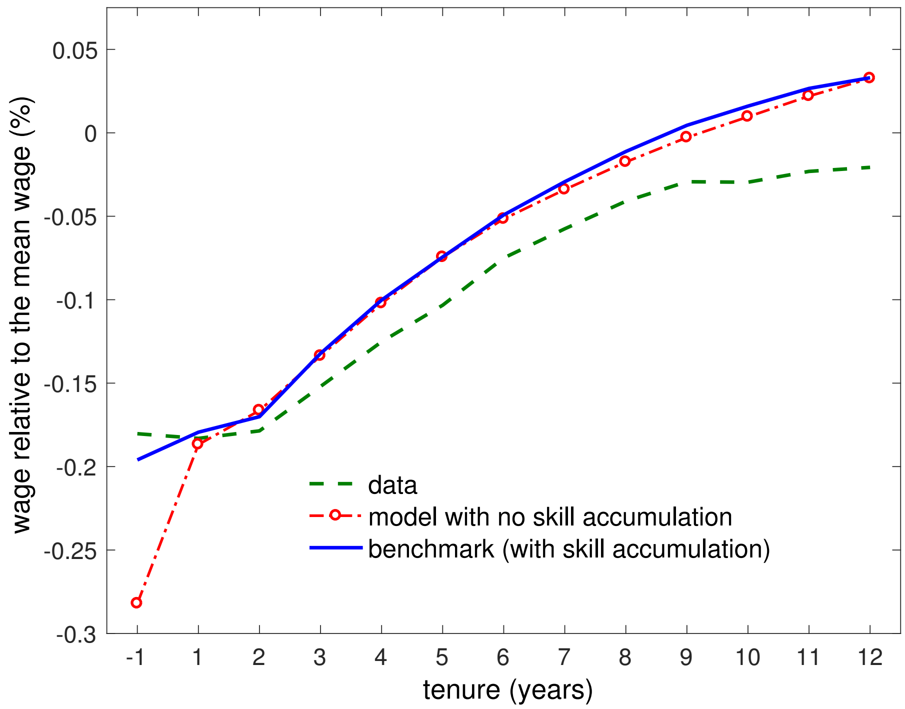
The dynamic worker-sector match shock represents the key innovation of this paper. In Section 5, we have shown analytically that a model featuring only this match shock, with no skill premium, can capture the main patterns of the wage-mobility relationship *qualitatively*. Here, we re-calibrate the model while ignoring the skill premium. The specific calibration strategy sets the skill premium to zero, *i.e.*, $\pi = 0$, and then selects the parameters of the match shock (ρ and σ), targeting gross mobility and employment volatility (or, equivalently, net mobility). We show that this re-calibrated version of the model can also capture the key facts of the wage-mobility relationship *quantitatively*. These facts include the positive wage-tenure profile, the negative wage gap between movers and stayers, and the negative correlation between lifetime income and individual-level mobility.

Table 6 presents the key moments and the parameter values associated with this re-calibration. Similarly to our benchmark calibration, this new calibration matches several key un-targeted moments from the data. As shown in Figure 8, it is remarkable that a model with only a dynamic match shock generates a substantial positive slope for the wage-tenure profile. This result suggests that that a significant, positive relationship between wages and sectoral tenure does not necessarily imply, or result exclusively from, sector-specific skill accumulation.

Figure 9 shows the evolution of the wage distribution in this re-calibrated model. Recall that the mean wage is normalized to 1. Thus, the figure shows that movers have lower-than-average wages before and immediately following a move and that the average wage among recent movers grows with tenure. As the analytical results in Section 5 demonstrate, when the match shock is persistent, those who find a suitable match are more likely to continue to have a favorable match shock, and thus stay in the current sector. This dynamic selection effect is able to generate the negative wage gap between movers and stayers, the positive wage-tenure relationship, and the negative correlation between mobility and lifetime earnings.

The last row of Table 6 shows that the negative correlation between mobility and lifetime earnings in the model are highly comparable with its empirical counterparts. Overall, the re-

Figure 8: The Wage-Tenure Profile in the Model With No Skill Premium



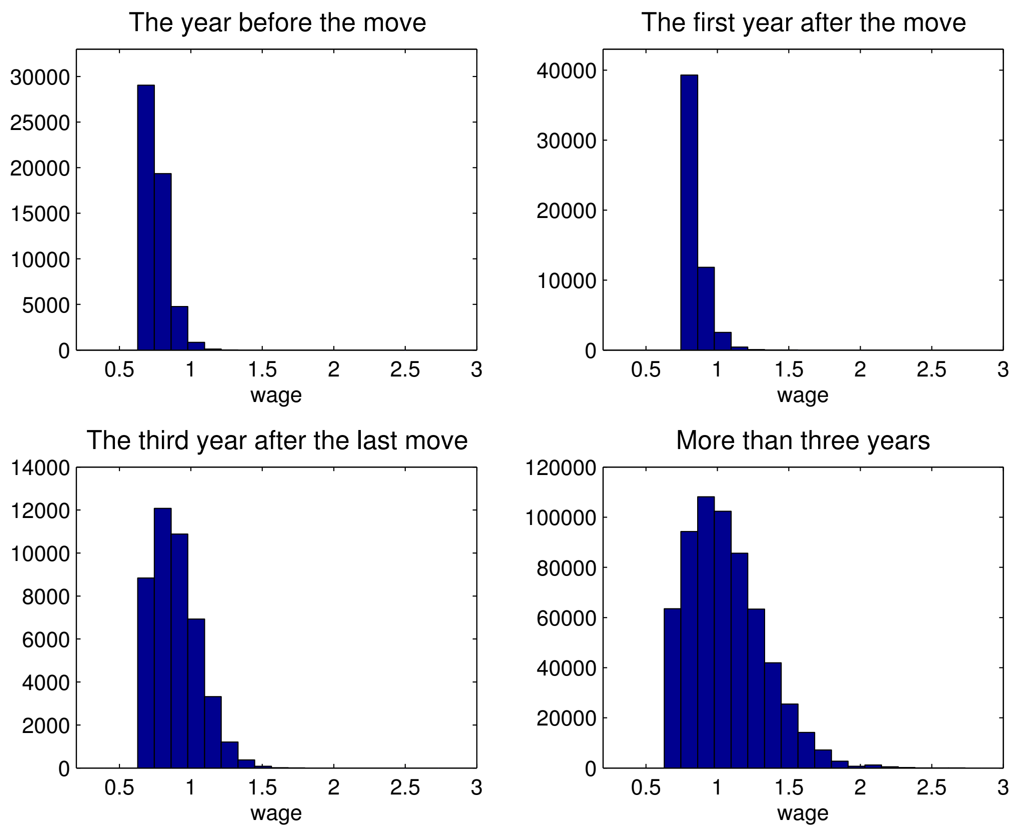
Notes: This figure plots the wage-tenure profile in a model with no skill premium and compares the profile with those in PSID data and the benchmark model. A tenure of 1 refers to the first year after mobility. A tenure of -1 refers to the year before mobility. The details of this re-calibrated model are contained in Section 8.

calibrated model does well, replicating the key features of the wage-mobility and wage-tenure relationships. These results further underscores the power of a persistent worker-sector match shock in accounting for the individual-level wage-mobility relationship.

Since there are fewer parameters in the absence of the skill premium, the re-calibrated model is expected to perform weaker relative to the benchmark model. Indeed, repeat mobility is substantially lower in the model with no skill premium. When the x shock is overly persistent, it becomes very unlikely for a worker to receive a shock moving her to a new sector in period t, and another shock making her leave the new sector in period $t+1$ (*i.e.*, repeat mobility). The pre-move wage gap represents another dimension along which the re-calibrated model performs relatively poorly, at least quantitatively. In Table 6, the pre-move wage gap is -0.28 as opposed to -0.18 in data.

Comparing the wage-tenure profile of the re-calibrated version of the model in Figure 8 with that of the benchmark model in Figure 5, it can be seen that there is strong trade-off between the skill premium and the persistence of the match shock. A substantially positive wage-tenure

Figure 9: The Wage Distribution in the Model With No Skill Premium



Notes: The figure plots the wage distribution generated by the model when it is re-calibrated in the absence of a sector-specific skill premium. The distributions in the figure are based on 800,000 observations. It shows that the movers tend to be drawn more from the lower end of the wage distribution both before and after mobility, which is consistent with the empirical facts shown in Figure 1 and the analytical results contained in Propositions 1 and 2. Among recent movers, those who stay longer in their current sector are those who draw, on average, better subsequent shocks, which is consistent with Proposition 3 and Figure 3.

profile can be generated either by using an highly persistent match shock in the absence of the skill premium, or by using the combination of a substantial skill premium and a less persistent match shock. However, these two approaches have very different implications on individual-level mobility patterns, including repeat mobility. All of this points to the importance of considering a quantitative, dynamic model while allowing for a persistent match shock.

9 Conclusion

We explore the role of a dynamic and persistent match shock on the wage-mobility relationship. Analytically, we characterize the effects of such a shock, and find they qualitatively match the corresponding facts in PSID data. According to this data, movers tend to have a lower wage than non-movers both prior to and after a move. The wage increases with sectoral tenure. Lifetime earnings are negatively related to sectoral mobility. A sector experiences simultaneous inflows and outflows of workers, while the corresponding net flows are very small relative to the gross flows. Labor mobility decreases with labor market experience. Recent movers are more likely to move again. These facts can be accounted for by a very simple, but persistent worker-sector match productivity process.

Both the analytical and numerical results in the paper highlight the important effects of a persistent match shock. Incorporating such a shock into a standard multi-sector model with directed mobility allows for endogenous determination of the key wage-mobility relationships. Quantitatively, the persistent match shock plays an important role in the mover-stayer wage gap, the wage-tenure profile, and the negative correlation between lifetime earnings and mobility, a novel fact we establish in this paper.

According to the model, the impact of net mobility driven by a sector-wide shock on wages is relatively small, while excess mobility driven by a worker-sector match productivity shock plays the key role for wage growth and lifetime earnings. Sectoral mobility raises overall labor income by 0.7%, and the sector-specific match shock accounts for 2.5% of average labor income.

There is strong trade-off between the skill premium and the persistence of the match shock. A positive relationship between wages and sectoral tenure does not necessarily imply sector-specific skill accumulation. Moreover, ignoring the persistence of a sector-specific match shock may introduce an upward bias to the sector-specific skill premium. A substantially positive wage-tenure profile and a large negative wage gap between movers and stayers can be generated either by using a highly persistent match shock in the absence of skill accumulation, or by using a combination of a substantial skill premium and a transitory match shock. However, these two approaches have very different implications for the individual-level mobility pattern. Therefore, in order to understand the role of mobility for wages, one may need to consider endogenous mobility, an endogenous distribution of workers by sectoral tenure, and the implied wage-tenure relationship. The model developed in the paper offers a parsimonious framework to accomplish this goal.

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ONLINE APPENDICES

A Data appendix

This appendix provides additional details of mobility and wages in PSID.

A.1 Sample description

For mobility, wages and lifetime earnings, we use data from the Panel Survey of Income Dynamics (PSID) of 1968-1997. For certain mobility measures, the empirical analysis further restricts attention to the Retrospective Occupation-Industry Supplemental Data Files, released in 1999. Kambourov and Manovskii (2009) find that the Retrospective Files for the period of 1968-1980 provide a more accurate measure of labor mobility across industries and occupations than the main PSID data. This enhanced precision is essential as we focus on the individual-level relationship between mobility and wages.[14]

A.2 Data limitations

Although our empirical findings concern mobility across both industries and occupations, in the quantitative analysis of the model, we focus on broad industries for the following data limitation reasons.

First, in order to capture the relative magnitude of gross versus net mobility across sectors, we need highly accurate time series of sectoral productivity and sectoral employment. Such data are available only for broadly-defined sectors. In particular, we use per-worker output of the Manufacturing sector relative to aggregate productivity and total employment of the Manufacturing sector relative to total non-farm business employment in the U.S.. These series are available from the Bureau of Labor Statistics (BLS) as explained in Section 6.

Second, the sample size represents another main reason for considering broad industries as sectors. McLaughlin and Bils (2001) argue that to measure the wage gap between inter-sectoral movers and stayers, one needs large sectors, as movers are a small fraction of the labor force. On the other hand, in PSID, the wages and industries are available for the heads of the households and the spouses. Then, if one considers much finer sectors (or more narrowly-defined sectors and job types) in PSID, the number of movers within each sector is too low to measure the wage gap between movers and stayers. Moreover, the small sample size makes it hard, if not impossible, to construct a reliable wage-tenure profile. For these measurement considerations, we focus on the above broad sectors.

A.3 Mobility

[14]While we focus on the individual-level differences within a sector, others have analyzed more aggregate features of sectoral dynamics. For examples, see Lee and Wolpin (2006), Kambourov and Manovskii (2009), Jaimovich and Siu (2012) and Autor and Dorn (2013).

A.3.1 Mobility across industries

Table A.1 summarizes sectoral mobility in the Retrospective Occupation-Industry Supplemental Data Files. The table shows most of industry mobility occurs between the Services and the Manufacturing sector. To a certain extent, it also suggests that net mobility is much smaller than excess mobility. Also, see Figure 7 for age-specific mobility and a related discussion.

Table A.1: Sectoral Mobility in the Retrospective Files, 1968-1980

	current sector			
previous sector	agriculture	manufacturing	service	public
agriculture	1,101	86	74	7
manufacturing	70	10,371	541	63
service	71	534	10,328	103
public	5	57	104	1,795

Notes: Overall mobility across these sectors is 6.78% per year.

A.3.2 Mobility across occupations

Table A.2 shows labor mobility across the following eleven occupations:

1. Professional, Technical, and Kindred Workers;
2. Managers and Administrators, Except Farm;
3. Sales Workers;
4. Clerical and Kindred Workers;
5. Craftsmen and Kindred Workers;
6. Operatives, Except Transport;
7. Transport Equipment Operatives;
8. Laborers, Except Farm;
9. Farmers and Farm Managers;
10. Farm Laborers and Farm Foremen; and
11. Service Workers and Private Household Workers.

This classification is provided by PSID.

Table A.2: Occupational Mobility in the Retrospective Files

	current occupation										
	1	2	3	4	5	6	7	8	9	10	11
previous occupation											
1. technical	3,090	110	22	15	29	13	10	16	8	1	12
2. manager	67	2,998	62	27	75	23	21	19	7	2	16
3. sales	13	79	808	15	27	13	15	11	3	1	9
4. clerical	41	44	13	1,143	48	46	25	26	2	2	24
5. craftsmen	51	95	33	45	5,611	183	83	111	15	11	61
6. operative	22	25	14	35	226	2,898	85	125	5	11	70
7. transport	13	35	11	19	86	70	1,456	68	6	14	47
8. laborer	21	28	7	35	151	122	86	1,670	3	22	66
9. farmer	4	2	7	4	18	7	5	10	579	6	3
10. farm laborer	2	2	4	1	19	15	13	19	8	356	8
11. service	18	28	6	28	53	54	36	55	2	3	1,615

Notes: Overall mobility across these occupations is 14.20% per year.

A.4 Wages and mobility

Wages are measured as real hourly wages, computed as annual labor income divided by annual hours and deflated by the *Consumer Price Index for All Urban Consumers* provided by the Bureau of Labor Statistics (BLS).

Tables A.3 and A.4 present the results of the OLS regressions of the log real hourly wage on a dummy for mobility (across sectors), along with various combinations of controls. Controlled effects include full sets of dummies for age, education, race, year, state, current and previous industries, and sector tenure. These results indicate that mobility is associated with significantly lower wages. Also, see Figures 1 and 2 for further details.

Tables A.3 and A.4 also show that the mover-stayer wage gap remains high even after controlling for unemployment spells from the previous and current years. The unemployment spells are constructed using *HEAD'S ANNUAL HOURS OF UNEMPLOYMENT* from the PSID Family-level data for years 1968-1974 and *ANNUAL HOURS OF UNEMPLOYMENT* from the PSID Individual-level data for years 1975-1980.

Tables A.5 and A.6 extend the same regression analysis to occupational mobility and show that the mover-stayer wage gap is also high across occupations.

Table A.7 shows the evolution of the log wages by occupational tenure. Specifically, it reports the mean log wage of workers with a given tenure level minus the log wage of all workers, while controlling for age, education, year and occupation.

Table A.3: Wage Regressions on Sectoral Mobility: Main Sample

Variables	*Specifications*			
mobility	-0.391 (0.019)	-0.273 (0.030)	-0.153 (0.026)	-0.151 (0.029)
current and past sectors, and tenure dummies		✓	✓	✓
age, education, race, year and state dummies			✓	✓
(hours unemployed in the past year)$\times 10^{-3}$				-0.055 (0.020)
(hours unemployed in the current year)$\times 10^{-3}$				-0.075 (0.018)
R-squared	0.029	0.159	0.401	0.411

Notes: The table shows the results of the log wage regressions with different specifications, where the variables with the checkmarks are controlled for. The first column shows that an average mover makes 39.1% less than an average non-mover. Standard errors are in parenthesis. See Figures 1 and 2 for further details.

Table A.4: Wage Regressions on Sectoral Mobility: Workers with No Unemployment Spell

Variables	*Specifications*		
mobility	-0.377 (0.031)	-0.268 (0.055)	-0.195 (0.044)
current and past sectors, and tenure dummies		✓	✓
age, education, race, year and state dummies			✓
R-squared	0.018	0.162	0.414

Notes: The table shows the results of the log wage regressions among those with no unemployment spell. The variables that are controlled for are denoted by a checkmark. For example, the first column shows that an average mover makes 37.7% less than an average non-mover. Tenure is captured by dummies for yearly tenure levels. This subsample consists of 17029 year-person observations. See Figures 1 and 2 for further details.

Table A.5: Wage Regression on Occupational Mobility: Main Sample

Variables	*Specifications*			
mobility	-0.303 (0.014)	-0.162 (0.021)	-0.107 (0.018)	-0.101 (0.019)
current and past occupations, and tenure dummies		✓	✓	✓
age, education, race, year and state dummies			✓	✓
(hours unemployed in the past year)$\times 10^{-3}$				-0.055 (0.020)
(hours unemployed in the current year)$\times 10^{-3}$				-0.075 (0.018)
R-squared	0.033	0.259	0.401	0.412

Notes: The table shows the results of the log wage regressions with different specifications, where the variables with the checkmarks are controlled for. Occupational mobility dummy is constructed using Retrospective Occupation-Industry Supplemental Data Files of 1968-1980. The other variables are constructed similarly to those in Table A.3. See Tables A.3 and A.4 for further details.

Table A.6: Wage Regressions on Occupational Mobility: Workers with No Unemployment Spells

Variables	*Specifications*		
mobility	-0.264	-0.154	-0.101
	(0.019)	(0.031)	(0.027)
current and past occupations, and tenure dummies		✓	✓
age, education, race, year and state dummies			✓
R-squared	0.020	0.269	0.410

Notes: The table shows the results of the log wage regressions among those who switch sectors with no unemployment spell. Occupational mobility dummy is constructed using Retrospective Occupation-Industry Supplemental Data Files of 1968-1980. The other variables are constructed similarly to those in Tables A.3 and A.4.

Table A.7: Wages by Occupational Tenure

yearly tenure	*control variables*				*N*
	occupation year	occupation year age	occupation year education	occupation year age education	
-1	-0.185	-0.124	-0.187	-0.099	3669
1	-0.194	-0.139	-0.193	-0.110	3669
2	-0.110	-0.088	-0.110	-0.072	2022
3	-0.056	-0.054	-0.057	-0.048	1350
4	-0.013	-0.028	-0.013	-0.023	1007
5	0.010	-0.028	0.013	-0.021	752
6	0.030	-0.010	0.038	0.009	566
7	0.056	-0.006	0.065	0.003	432
8	0.098	0.030	0.115	0.027	331
9	0.102	0.030	0.116	0.025	216
10	0.096	0.024	0.134	0.026	149
11	0.083	0.022	0.110	0.034	97
12	0.149	0.052	0.210	0.084	37

Notes: The table shows the log wage by occupational tenure relative to the mean log wage of workers of all tenure levels. The values at -1 refer to the year before mobility. N denotes the number of observations. The pattern of mobility across the occupations is shown in Table A.2.

B Further details of the model

This section contains further details of the model and the proofs of the analytical results in the paper. It also demonstrates the role of the key elements in the model and provides some robustness checks for the baseline parameterization.

B.1 The distribution of the match shock

Figure B.2 illustrates the distribution of the match shock across workers. A worker's pair of the match shocks (e_0, e_1) is described by a point in the graph. The iso-probability contours reflect the bivariate distribution of the match shocks. To simplify the discussion, suppose for now that there is no sectoral shock, *i.e.*, $z = 0$.

B.1.1 Indifference line

Consider an unskilled worker. If the two shocks are the same, the worker will be indifferent between the two sectors. In other words, workers "lying" along the line $e_0 = e_1$, are indifferent between the two sectors. This line is referred to as the Indifference line. If the worker's pair of shocks are given by a point above the Indifference line, the person prefers sector 1. Conversely, if the pair of the shocks lies below the line, the person prefers sector 0.

B.1.2 Which component of the shock is relevant for mobility?

Suppose that the worker is in sector 0 at time t, and let her pair of match shocks be given by point A (at time t). Since the point lies below the Indifference line, at time t the person has a better match shock in sector 0 than in sector 1: $e_0 > e_1$. Now, suppose that at the beginning of $t + 1$, the worker is hit by new shocks given by point B. In this case, the person does not move, as the match quality remains below the Indifference line. If instead the new shock is given by point C, the person may decide to move. Therefore, any component of the shock that shifts match productivity along the Indifference line is not relevant for mobility. It is important to keep this in mind when we decompose the match shocks in Appendix B.1.

B.1.3 The sectoral shock and skill premium

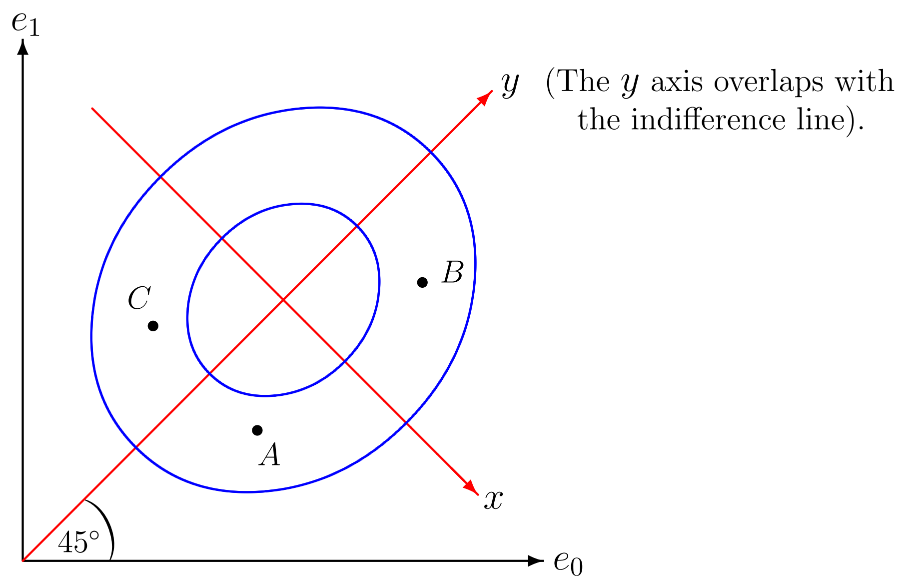
Figure B.2 illustrates the case where there is no skill premium ($\pi = 0$) and no sector-wide shock ($z = 0$). Consistent with equation (14), the Indifference line shifts with the sectoral shock, z. For example, if sector 1 is hit by a positive z shock, the Indifference line shifts downward. Moreover, the Indifference line is different between skilled versus unskilled workers. For example, for a skilled worker of sector 0, the Indifference line should be above the one depicted in Figure B.2. See McLaughlin and Bils (2001) for a further discussion on how the Indifference line in the standard Roy (1951) model responds to the z shock, when the match productivity is permanent.

B.2 The mobility decision and the match shock

Figures B.3 and B.4 help understand how x affects the mobility decision. In Figure B.3 we plot the wage as a function of the match shock, x, for both unskilled and skilled workers.[15]

[15] In both Figures B.3 and B.4 we simplify by setting $z = 0$. This abstraction is done to simplify the intuition, which carries over for a non-zero z.

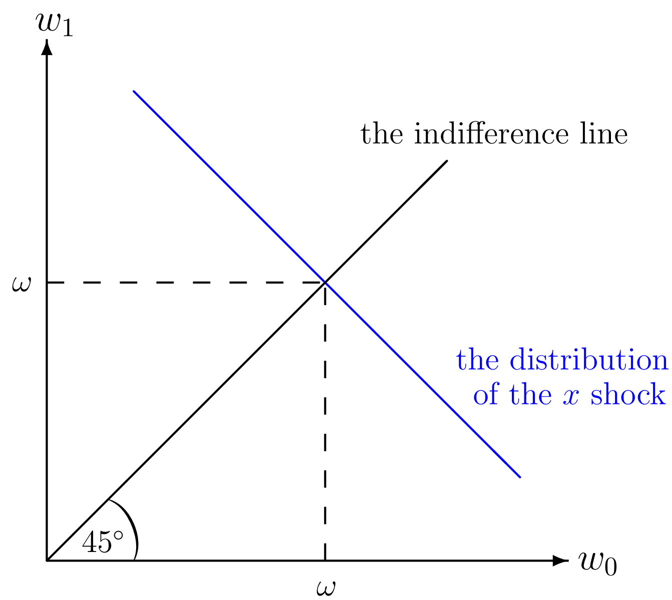
Figure B.1: Decomposing the Match Shocks



Notes: The figure illustrates the distribution of the match shocks among *unskilled workers* when there is no sectoral shock (*i.e.*, $z = 0$). A pair of the match shocks (e_0, e_1) of a person is described by a point in the graph. For example, at point A, e_0 is higher than e_1, while at point C, e_1 is higher than e_0. The iso-probability contours reflect the bivariate distribution of the match shocks.

The line at which workers are indifferent between the sectors is referred to as the *Indifference line*. When $z = 0$, this line is given by the 45° line. Given the decomposition in equation (6), y shock is along the Indifference line, while the x shock is orthogonal to the Indifference line. The x shock of a particular person is positive if the worker is below the Indifference line (for a worker at A or B), while it is negative if the person is above the Indifference line (for a worker at C). See Appendix B.1 for a further discussion.

Figure B.2: Comparative Advantage



Notes: The figure illustrates the range of the match shock among *unskilled workers* when there is no sectoral shock ($i.e.$, $z = 0$). As the match shock evolves, workers move between the sectors. There is comparative advantage in that those who are better matched in one sector is poorly matched with the other sector. See Appendix B.1 for a further discussion.

For positive values of x, the worker is more productive in sector 0 relative to sector 1, which in turn affects the mobility decision. This is evident in Figure B.4 where we plot the value of remaining in sector 0, $S_0(\cdot)$, and of moving to sector 1, $M_0(\cdot)$.

At any point in time (or for each value of z), for each skill level h, there exists a critical value of the match shock, $\hat{x}_0(h, z)$, where a worker prefers to move from sector 0 to sector 1 if the match shock of the person is below the critical value, *i.e.*, $x < \hat{x}_0(h, z)$. Among workers in sector 0, this critical value is higher for an unskilled worker than for a skilled worker: $\hat{x}_0(1, z) < \hat{x}_0(0, z)$ (see Figure B.4). Therefore, a skilled worker is less likely to change sectors, all else equal. Indeed, the skill premium, π, acts as an implicit moving cost. In our quantitative exercises below, we further highlight the role of π in replicating observed mobility patterns.

B.3 Sectoral dynamics

Let $\nu_{j,t}(h, x)$ denote the measure of workers after the realization of the match shocks in period t:

$$\nu_{j,t}(h, x, \tau) = \int \mu_{j,t-1}(h, \tilde{x}, \tau) \frac{\partial F(x \mid \tilde{x})}{\partial \tilde{x}} d\tilde{x}. \tag{B.1}$$

Also, let $m_{1-j,t}$ denote the total number of workers moving from sector j to $1-j$ in period t. Then, one can write

$$m_{1-j,t}(x) = (1-\delta) \sum_h \sum_\tau (1 - \Omega_j(h, x, z_t)) \nu_{j,t}(h, x, \tau), \tag{B.2}$$

where z_t is the sectoral shock at time t. At the end of the current period, these movers will have worked for one period at their destination; therefore, the measure of workers in sector j with $(h, x, \tau) = (0, x, 1)$ is equivalent to the measure of workers that move to sector j in period t. That is,

$$\mu_{j,t}(0, x, 1) = m_{j,t}(x). \tag{B.3}$$

For stayers, sectoral dynamics are captured by the following two equations. The measure of unskilled workers in sector j at the end of period t with tenure $\tau + 1$ is given by

$$\mu_{j,t}(0, x, \tau + 1) = (1-\delta)(1-p)\Omega_j(0, x, z_t)\nu_{j,t}(0, x, \tau), \tag{B.4}$$

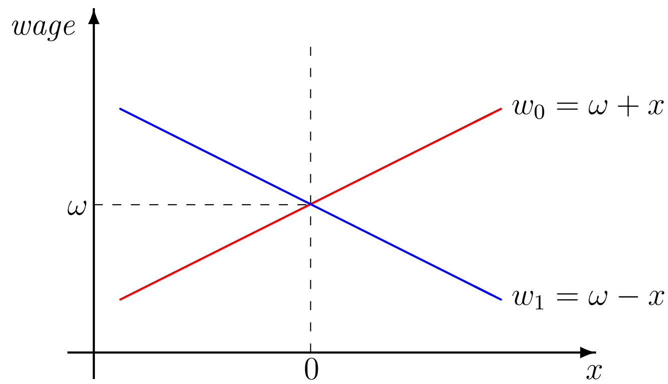
which is the probability of surviving and remaining unskilled (the $(1-\delta)(1-p)$) term) multiplied by the measure of unskilled workers with tenure τ (entering period t) who receive the match shock x during period t (the $\nu_{j,t}(0, x, \tau)$ term) and who decide to stay in sector j (the $\Omega_j(0, x, z_t)$ term). Similarly, for the measure of skilled workers,

$$\mu_{j,t}(1, x, \tau + 1) = (1-\delta) \left(\Omega_j(1, x, z_t)\nu_{j,t}(1, x, \tau) + p\Omega_j(0, x, z_t)\nu_{j,t}(0, x, \tau) \right), \tag{B.5}$$

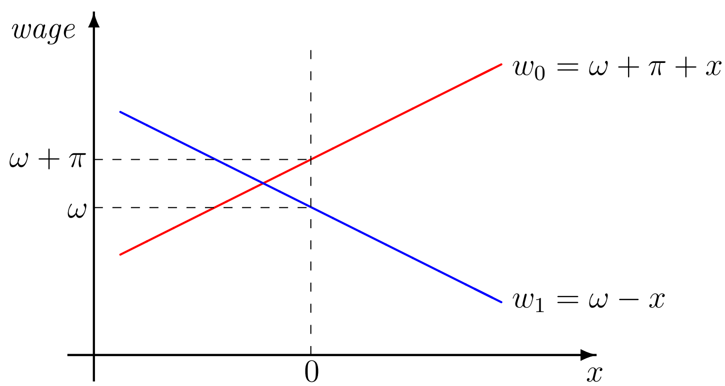
where $\tau \in \{1, 2, \cdots\}$.

Figure B.3: The Match-Specific Wage

Panel A. An Unskilled Worker of Sector 0



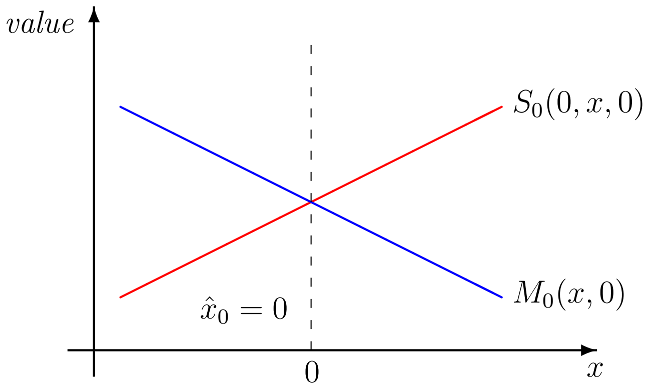
Panel B. A Skilled Worker of Sector 0



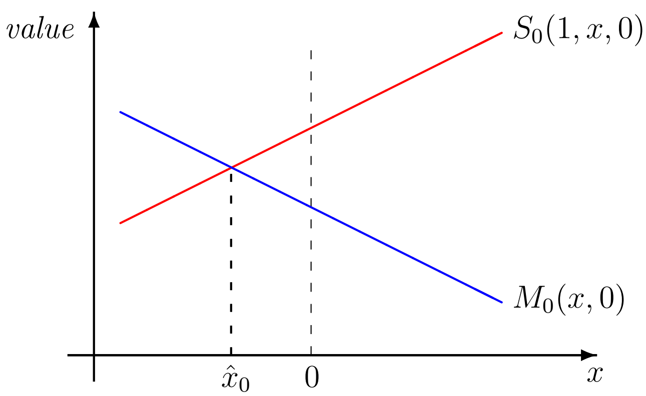
Notes: The figure shows the wages when the sector-wide shock z is zero (*i.e.*, $z = 0$). w_0 is the wage of a worker staying in sector 0 when his or her match-specific productivity for the sector is x. w_1 is the wage of a worker moving to sector 1, when his or her match productivity for sector 0 is x. (Also, see Figure B.4.)

Figure B.4: The Mobility Decision

Panel A. An Unskilled Worker of Sector 0



Panel B. A Skilled Worker of Sector 0



Notes: The figure shows who moves and who stays behind when the sector-wide shock z is zero (*i.e.*, $z = 0$). S_0 is the value to a worker of staying in sector 0 when his or her match-specific productivity for the sector is x. M_0 is the value of leaving sector 0. Among those in sector zero, those hit by a match-specific shock below $\hat{x}_0$ will move across sectors. For unskilled workers, when $z = 0$, $\hat{x}_0 = 0$. Among skilled workers, when $z = 0$, $\hat{x}_0 < 0$. (Also, see Figure B.3.) Using the figures and recalling the persistence of the match shock, it can be seen that, on average, a) movers are those with worse match shocks and b) skilled workers move less.

B.4 Definition of the equilibrium

An equilibrium consists of a set of value functions $\{U_j, S_j, M_j\}$, a decision rule Ω_j, a sequence of the sectoral technology shock $\{z_t\}_{t=1}^T$ for an integer $T > 0$, and the sequence of numbers of workers $\{\mu_{j,t}(0, x, \tau), \mu_{j,t}(1, x, \tau)\}_{t=0}^T$ for any j, τ and x such that

1. stayer: given U_j, the value function $S_j(h, x, z)$ solves equations (10) and (11);
2. mover: given S_j and M_j for each j, the decision rule $\Omega_j(h, x, z)$ and the value function $U_j(x)$ solve equation (13); and
3. consistency of the law of motion: for each (x, j, τ), $\{\mu_{j,t}(0, x, \tau), \mu_{j,t}(1, x, \tau)\}_{t=1}^T$ satisfy equations (15) and (16), and (B.1) through (B.5), subject to the sequence of the sectoral technology shock $\{z_t\}_{t=1}^T$ and the initial measure $\{\mu_{0,0}(h, x, \tau), \mu_{1,0}(h, x, \tau)\}$.

B.5 Simulating the simple economy in Section 5

1. For each worker, the match shock for period 1 is drawn from the normal distribution $\mathcal{N}(0, \sigma^2)$ independently. If the shock is positive or equal to zero, assign the worker to sector 0, otherwise to sector 1.

2. For each person, generate the subsequent match shocks according to the AR(1) process in equation (7), where ϵ_t is drawn from $\mathcal{N}(0, (1-\rho^2)\sigma^2)$ independently.

3. For a given worker, if the sign of the match shock changes between t and $t+1$, move the worker across sectors. Otherwise, keep the worker in the same sector. (If $x_{t+1} = 0$, consider it as the same sign as x_t.)

B.6 Proof of Claim 2

1. For those who just moved from sector 1 to sector 0, the mean wage in sector 0 is given by $\mathbf{E}(\epsilon_t \mid \epsilon_{t-1} \leq 0 \text{ and } \epsilon_t > 0)$. For those who stayed in sector zero during $t-1$ and t, their mean wage at t is given by $\mathbf{E}(\epsilon_t \mid \epsilon_{t-1} \geq 0 \text{ and } \epsilon_t \geq 0)$. Since, ϵ_t is independently distributed for all t, these two values are equal to the mean wage of sector 0, $\mathbf{E}(\epsilon_t \mid \epsilon_t \geq 0)$. Then, using equation (8), the mean wage is the same between the newcomers and incumbents.

2. For those who will move from sector 0 to sector 1 in the next period, the mean value of their current sectoral shocks is given by $\mathbf{E}(\epsilon_t \mid \epsilon_{t+1} < 0 \text{ and } \epsilon_t \geq 0)$. Similarly, the mean value of the current sectoral shocks of those who will stay in sector 0 is given by $\mathbf{E}(\epsilon_t \mid \epsilon_{t+1} \geq 0 \text{ and } \epsilon_t \geq 0)$. Since, ϵ_t is independently distributed for all t, these two values are equal to the mean wage of sector 0, $\mathbf{E}(\epsilon_t \mid \epsilon_t \geq 0)$. So, the average wage of the future movers is equal to the average wage of the future stayers.

3. Using the results of, it can be seen that for those with τ years of sectoral tenure, the mean wage is independent of tau and given by $\mathbf{E}(\epsilon_t \mid \epsilon_t \geq 0)$. So, the average wage remains constant with sectoral tenure.

4. Regardless of the mobility status and sectoral tenure, the mean value of the match shocks at each level of sectoral tenure is given by $\mathbf{E}\big(\epsilon_t \mid \epsilon_t \geq 0\big)$. Thus, the number of moves a person makes is uncorrelated with overall earnings, over a finite number of periods.

B.7 Proof of Proposition 1

Consider the workers who worked in sector 0 during periods t and $t+1$. For these workers, it is the case that $x_t \geq 0$ and $x_{t+1} = \rho x_t + \epsilon_{t+1} \geq 0$. Then, the mean shock of these incumbent workers of sector 0 in period $t+1$ is given by the following conditional mean:

$$\overline{x}_0^{\mathrm{Old}} = \mathbf{E}\big(\rho x_t + \epsilon_{t+1} \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t \geq 0\big).$$

Because $\rho > 0$, $\sigma > 0$ and ϵ_t-s are *i.i.d*,

$$\begin{aligned} \overline{x}_0^{\mathrm{Old}} &= \mathbf{E}\big(\rho x_t + \epsilon_{t+1} \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t \geq 0\big) \\ &> \mathbf{E}\big(\rho x_t + \epsilon_{t+1} \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t = 0\big) \\ &= \mathbf{E}\big(\epsilon_{t+1} \mid \epsilon_{t+1} \geq 0\big). \end{aligned} \tag{B.6}$$

On the other hand, for those who worked in sector 1 during period t and in sector 0 during period $t+1$, it is the case that $x_t \leq 0$ and $x_{t+1} = \rho x_t + \epsilon_{t+1} > 0$. Then, the mean shock of these newcomers of sector 0 in period $t+1$ is given by the following conditional mean:

$$\overline{x}_0^{\mathrm{New}} = \mathbf{E}\big(\rho x_t + \epsilon_{t+1} \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t < 0\big).$$

As before, because $\rho > 0$, $\sigma > 0$ and ϵ_t-s are *i.i.d*,

$$\begin{aligned} \overline{x}_0^{\mathrm{New}} &= \mathbf{E}\big(\rho x_t + \epsilon_{t+1} \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t < 0\big) \\ &< \mathbf{E}\big(\rho x_t + \epsilon_{t+1} \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t = 0\big) \\ &= \mathbf{E}\big(\epsilon_{t+1} \mid \epsilon_{t+1} \geq 0\big). \end{aligned} \tag{B.7}$$

Combining inequalities (B.6) and (B.7), we get

$$\overline{x}_0^{\mathrm{New}} < \overline{x}_0^{\mathrm{Old}}. \tag{B.8}$$

The latter means that, on average, the incumbent workers have a better shock than the newcomers. Further, using equation (8), on average, the incumbent workers have a higher wage than the newcomers.

B.8 Proof of Proposition 2

For those who will move from sector 0 to sector 1 at the beginning of period $t+1$ (future movers), the mean value of their current sectoral shocks (at period t) is given by

$$\overline{x}_0^{\mathrm{FM}} = \mathbf{E}\big(x_t \mid \rho x_t + \epsilon_{t+1} < 0 \text{ and } x_t \geq 0\big).$$

Similarly, the mean value of the current sectoral shocks of those who will stay in sector 0 during periods t and $t+1$ (future stayers) is given by

$$\overline{x}_0^{\mathrm{FS}} = \mathbf{E}\big(x_t \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t \geq 0\big).$$

Since $\rho > 0$ and $\sigma > 0$, for the future movers,

$$\overline{x}_0^{\mathrm{FM}} = \mathbf{E}\big(x_t \mid \rho x_t + \epsilon_{t+1} < 0 \text{ and } x_t \geq 0\big) < \mathbf{E}\big(x_t \mid x_t \geq 0\big) \tag{B.9}$$

and for the future stayers,

$$\overline{x}_0^{\text{FS}} = \mathbf{E}\big(x_t \mid \rho x_t + \epsilon_{t+1} \geq 0 \text{ and } x_t \geq 0\big) > \mathbf{E}\big(x_t \mid x_t \geq 0\big). \tag{B.10}$$

So, we have

$$\overline{x}_0^{\text{FM}} < \mathbf{E}\big(x_t \mid x_t \geq 0\big) < \overline{x}_0^{\text{FS}}. \tag{B.11}$$

Then, using equation (8), the average wage of future stayers is higher than that of future movers.

B.9 Proof of Proposition 3

Using equation (7), for any integer $k \in \{0, 1, \cdots, \tau - 1\}$, one can write

$$x_{t-k} = \rho^{\tau-k} x_{t-\tau} + e_{t,\tau,k},$$

where

$$e_{t,\tau,k} = \sum_{\ell=k}^{\tau-1} \rho^{\ell-k} \epsilon_{t-\ell}.$$

On the other hand, given any large finite number T, for those with tenure of $\tau < T$ years in sector 0 at the end of period t, it must be the case that $x_t \geq 0$, $x_{t-1} \geq 0$, $\cdots$, $x_{t-\tau+1} \geq 0$, $x_{t-\tau} > 0$ and $x_{t-\tau-1} \leq 0$. The mean value of their shocks at time t is given by

$$\overline{x}_0(\tau) = \mathbf{E}\big(\rho^\tau x_{t-\tau} + e_{t,\tau,0} \mid \rho^{\tau-k} x_{t-\tau} + e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau - 1, x_{t-\tau} > 0, x_{t-\tau-1} \leq 0\big).$$

Since $\rho > 0$ and the conditional mean increases with $x_{t-\tau}$, we can write that

$$\overline{x}_0(\tau) > \mathbf{E}\big(e_{t,\tau,0} \mid \rho^{\tau-k} x_{t-\tau} + e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau - 1, x_{t-\tau-1} \leq 0\big).$$

Because ϵ_t-s are $i.i.d$, it follows that

$$\begin{aligned} \overline{x}_0(\tau) &> \mathbf{E}\big(e_{t,\tau,0} \mid e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau - 1, x_{t-\tau-1} \leq 0\big) \\ &= \mathbf{E}\big(e_{t,\tau,0} \mid e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau - 1\big). \end{aligned} \tag{B.12}$$

Similarly, the mean shock of those who have worked for exactly $\tau - 1$ periods in sector 0 is given by

$$\overline{x}_0(\tau - 1) = \mathbf{E}\big(\rho^\tau x_{t-\tau} + e_{t,\tau,0} \mid \rho^{\tau-k} x_{t-\tau} + e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau - 2, x_{t-\tau+1} > 0, x_{t-\tau} \leq 0\big).$$

Then, since $\rho > 0$ and the conditional mean increases with $x_{t-\tau}$,

$$\overline{x}_0(\tau - 1) < \mathbf{E}\big(e_{t,\tau,0} \mid \rho^{\tau-k} x_{t-\tau} + e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau - 2, x_{t-\tau+1} > 0, x_{t-\tau} = 0\big).$$

Furthermore, because ϵ_t-s are $i.i.d$,

$$\begin{aligned} \overline{x}_0(\tau - 1) &< \mathbf{E}\big(e_{t,\tau,0} \mid e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau - 2, \epsilon_{t-\tau+1} > 0\big) \\ &= \mathbf{E}\big(e_{t,\tau,0} \mid e_{t,\tau,k} \geq 0, 0 \leq k \leq \tau - 1\big). \end{aligned} \tag{B.13}$$

Using inequalities (B.12) and (B.13),

$$\overline{x}_0(\tau-1) < \overline{x}_0(\tau) \tag{B.14}$$

for $\tau < T$. Then, using equation (8), over any finite range of sectoral tenure, the average wage increases with sectoral tenure.

B.10 Calculating the correlation of lifetime earnings and mobility

As equation (6) shows, the labor income shocks $e_{0,t}$ and $e_{1,t}$ have a common component denoted by y_t (also, see Figure B.2 in Appendix B.1). As mentioned before, this common component is omitted in the model as it does not affect mobility, the mover-stayer wage gap, and the wage-tenure profile. However, when the y_t component is omitted in the model, the variance of lifetime earnings generated by the model is lower than what it should be if the model were simulated using the labor income shocks $e_{0,t}$ and $e_{1,t}$ (as opposed to x and $-x$).

Therefore, in order to compare the correlation between lifetime earnings and mobility generated by the model with that in data, one needs to take into account the earnings dispersion driven by the y_t component. For this reason, when calculating the correlation using the simulated data, we allow for the impact of the y shock by considering the overall dispersion of lifetime earnings in PSID data. The details are described below.

Let $\mathcal{E}_i^x$ and $\mathcal{M}_i$ denote the lifetime earnings and overall mobility of person i in the model. Let $\mathcal{E}_i^y$ be the component of the lifetime earnings based on the y shock and $\mathcal{E}_i$ be the total lifetime earnings of person i: $\mathcal{E}_i = \mathcal{E}_i^x + \mathcal{E}_i^y$. Then, the correlation of the total lifetime earnings and mobility can be written as follows:

$$\mathrm{corr}(\mathcal{E}_i, \mathcal{M}_i) \quad = \quad \frac{E(\mathcal{E}_i^x + \mathcal{E}_i^y, \mathcal{M}_i) - E(\mathcal{E}_i^x + \mathcal{E}_i^y)E(\mathcal{M}_i)}{\mathrm{std}(\mathcal{E}_i^x + \mathcal{E}_i^y)\mathrm{std}(\mathcal{M}_i)} \tag{B.15}$$

Given the independence of $\mathcal{E}_i^y$ and $\mathcal{M}_i$,

$$\begin{aligned} \mathrm{corr}(\mathcal{E}_i^x + \mathcal{E}_i^y, \mathcal{M}_i) &= \frac{E(\mathcal{E}_i^x, m_i) - E(\mathcal{E}_i^x)E(m_i)}{\mathrm{std}(\mathcal{E}_i^x)\mathrm{std}(\mathcal{M}_i)} \frac{\mathrm{std}(\mathcal{E}_i^x)}{\mathrm{std}(\mathcal{E}_i^x + \mathcal{E}_i^y)} && \text{(B.16)} \\ &= \mathrm{corr}(\mathcal{E}_i^x, \mathcal{M}_i) \frac{\mathrm{std}(\mathcal{E}_i^x)}{\mathrm{std}(\mathcal{E}_i^x + \mathcal{E}_i^y)}. && \text{(B.17)} \end{aligned}$$

In the model, $\mathrm{corr}(\mathcal{E}_i^x, \mathcal{M}_i) = -0.655$ and $\mathrm{std}(\mathcal{E}_i^x) = 0.086$. Using the PSID data, $\mathrm{std}(\mathcal{E}_i^x + \mathcal{E}_i^y) = 0.368$. Inserting these numbers into the above equation, we get $\mathrm{corr}(\mathcal{E}_i, \mathcal{M}_i) = -0.152$.

B.11 Employment-to-employment transition

From a data-to-modeling perspective, our analysis requires matching moments for employment to employment changes in *wages*. Thus, productivity represents the key variable in the model. While we do not explicitly include unemployment in our model, one could view unemployment as a particularly bad productivity shock via either $e_{j,t}$ or z_t. Although this does not capture the full impact of unemployment, given our focus and the data limitations (for example, both the wage and sectoral mobility in the PSID can be measured only at an annual frequency), this is the most appropriate modeling strategy.[16]

On the empirical side, in the PSID, sectors and unemployment spells are recorded at an annual basis. Using the PSID variable *HEADS ANNUAL HOURS OF UNEMPLOYMENT*, we find that for two thirds of sectoral movers, there were no unemployment spells in the two calendar years over which their sectoral switches were recorded. Clearly, among the remaining one third of movers, there can be workers whose unemployment spell and mobility do not coincide. Therefore, employment-to-employment sectoral mobility is much higher than that measured using annual frequency data.

More important, in Tables A.3 to A.6, we measure the mover-stayer wage gap across both sectors and occupations while controlling unemployment spells. The main patterns of the wage-mobility relationship remains robust.

B.12 Robustness: a more volatile sectoral shock

In the model, the sectoral shock is governed by the following process:

$$z_{t+1} = rz_t + u_t,$$

where $\mathrm{Var}(z_t) = s^2$. In the quantitative analysis of Section 6, the persistence and standard deviation of this sectoral shock, r and s, are calibrated using the relative productivity series of the manufacturing sector of 1987 to 2012. One could argue that the length of this productivity series may not be sufficient enough to precisely measure the volatility of the sectoral shock.

To address this issue, we re-scale the two parameters using much longer aggregate productivity data of 1947 to 2012. For this purpose, let $\rho^{\mathrm{L}}_{\mathrm{agg}}$ and $\sigma^{\mathrm{L}}_{\mathrm{agg}}$ be the persistence and standard deviation of aggregate US productivity in the longer sample (1947 to 2012). Also, let $\rho^{\mathrm{S}}_{\mathrm{agg}}$ and $\sigma^{\mathrm{S}}_{\mathrm{agg}}$ be the persistence and standard deviation of aggregate US productivity in the shorter sample (1987 to 2012). Then, one can consider the following re-scaling:

$$\begin{cases} r^{\mathrm{L}} = r^{\mathrm{S}} \times \dfrac{\rho^{\mathrm{L}}_{\mathrm{agg}}}{\rho^{\mathrm{S}}_{\mathrm{agg}}}, \\ s^{\mathrm{L}} = s^{\mathrm{S}} \times \dfrac{\sigma^{\mathrm{L}}_{\mathrm{agg}}}{\sigma^{\mathrm{S}}_{\mathrm{agg}}}, \end{cases} \tag{B.18}$$

where r^{S} and s^{S} are the persistence and standard deviation of relative productivity of the manufacturing sector from 1987 to 2012. These equations imply a slightly more volatile, but less persistent, shock where $r^{\mathrm{L}} = 0.3683$ and $s^{\mathrm{L}} = 0.0090$.

Table B.1 summarizes the key predictions of the model with these two parameters of the sector shock. The results indicate that considering a slightly more volatile sectoral shock does not have much impact on the wage-mobility relationship. However, most of its impact is on sectoral employment. This is simply because a more volatile sectoral shock raises net mobility and thus raises sectoral employment.

[16]See Moscarini (2001), Lkhagvasuren (2012), and Carrillo-Tudela and Visschers (2014) for related multi-sector models of unemployment.

Table B.1: Predictions under Re-scaled Sectoral Shock

	Data	*Benchmark model*	*A more volatile sectoral shock*
Key parameters			
persistence of the sectoral shock		0.4236	0.3683
st.dev. of the sectoral shock		0.0068	0.0090
Moments			
mobility	0.0678	0.0683	0.0683
volatility of sectoral employment	0.0059	0.0059	0.0078
repeat mobility	0.2729	0.2247	0.2247
movers' mean wage at their origin	-0.1803	-0.1960	-0.1955
movers' mean wage at destination	-0.1786	-0.1794	-0.1789
annual wage growth of recent movers	0.0259	0.0263	0.0262
corr. of lifetime earnings and mobility	-0.1523	-0.1685	-0.1680

Notes: The table summarizes predictions of the benchmark model with that of the model with a more volatile sectoral shock.

B.13 High net mobility

Next, we consider a counterfactual case where mobility is driven primarily by the sectoral shock. Specifically we set the dispersion of the match shock σ to one fifth of its benchmark value. This means that the role of the sectoral shock is higher when compared with the benchmark model. Table B.2 shows the results of this alternative parametrization. The key issue is that it generates implausibly high volatility in sectoral employment. Specifically, employment is five times more volatile than that in data (0.0294 versus 0.0059). This further underscores the important role played by the dynamic worker-sector match shock. Given observed volatility in sectoral productivity and employment, sectoral shocks (*i.e.*, net mobility) alone are insufficient to explain the overall observed pattern of mobility.

B.14 An implication on the number of sectors

It is shown in Section 7.1 that the sectoral-level shock (z) has no significant effect on the wage-mobility relationship. This has an important implication that one can account for the key features of wage and mobility data without introducing sector-wide shocks. Then, under directed mobility, the model can be recast as an economy with $N > 2$ sectors. Specifically, one can interpret x as the worker's match shock in the current sector, and $-x$ as the best of the $N-1$ match shocks from the remaining $N-1$ sectors and directly extend the main quantitative results to an economy with N sectors.

Table B.2: The Model with a Low Dispersion of the Match Shock

	data	*the model with a low dispersion of the match shock:* $\sigma = 0.0222$
mobility	0.0678	0.0689
repeat mobility	0.2729	0.2249
volatility of sectoral employment	0.0059	0.0294
wage of movers at the origin	-0.1803	-0.2361
movers wage at the destination	-0.1786	-0.2328
annual wage growth among recent movers	0.0259	0.0293
correlation of lifetime earnings and mobility	-0.1523	-0.2127

Notes: The tables shows the key parameters and predictions of the model when the dispersion of the match shock is set to one fifth of its benchmark value (*i.e.*, $\sigma = 0.0222$), and compares the predictions with data. The rest of the key parameters are as in the benchmark model.

C Absolute advantage with a persistent match shock

In the main model of the paper, mobility is driven by dynamic comparative advantage. This is commonly referred to as a Roy model with horizontal sorting. Papageorgiou (2014) finds evidence of substantial horizontal worker sorting across broad occupational categories based on the comparative advantage. Recently, Groes et al. (2015) argue that sectoral mobility may involve both horizontal and vertical sorting. The latter refers to mobility driven by absolute advantage. In this appendix, we show that a positive wage-tenure profile can also arise in an economy with vertical sorting when the match shock is perststent.

C.1 A simple model with vertical sorting

Consider the following model of absolute advantage. Let the idiosyncratic shock x follow the AR(1) process given by equation (7). Let the current wage for a worker in sector 0 be

$$w_0(x_t) = \omega + \gamma x_t \tag{C.1}$$

and for a worker in sector 1 be

$$w_1(x_t) = \omega + x_t \tag{C.2}$$

where ω is a normalization constant, and $\gamma > 1$. In this specification, we let sector 0 to be the high wage sector and thus let $\gamma > 1$.[17]

C.2 High versus low wage sectors

Using the above wages, one can consider the diagram shown in Figure C.1. As can be seen from the figure, those who are below the indifference line will choose sector 1, while those who are above the indifference line will choose sector 0. Therefore, depending on the match shock x_t, the decision rule is as follows:

$$\begin{cases} \text{a worker in sector 0 moves at period } t \text{ if } x_t < 0, \text{ or stays if } x_t \geq 0; \\ \text{a worker in sector 1 moves at period } t \text{ if } x_t > 0, \text{ or stays if } x_t \leq 0. \end{cases}$$

The match-specific wages are shown in Figure C.2. The wage in sector 0 will be equal to or more than ω, while the wages in sector 1 will be less than or equal to ω. In other words, sector 0 is the high wage sector, while sector 1 is the low wage sector. As the match shock of the workers evolves persistently, on average, sector 0 will lose its worse workers to sector 1, while sector 1 will lose its best workers to sector 0.

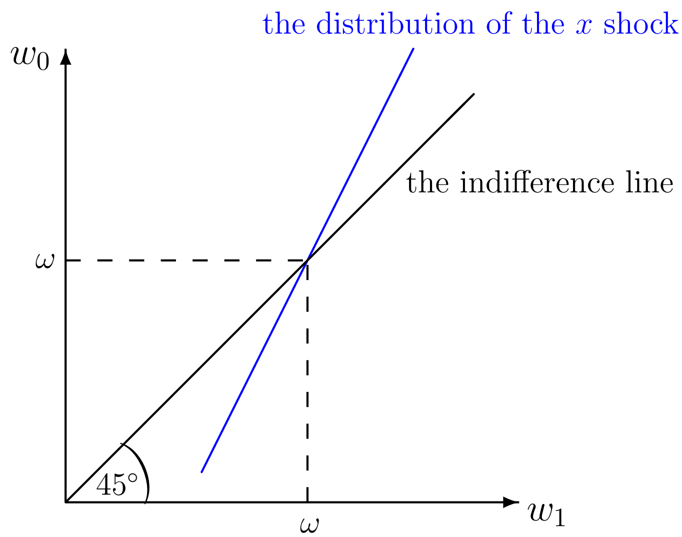
C.3 Wage-tenure profile under vertical sorting

C.3.1 A numerical example

We simulate the above simple model of dynamic absolute advantage for the following parameter values: $\rho = 0.9$, $\sigma = 0.2$ and $\gamma = 3$. The results are summarized in Figure C.3. The

[17] One can also have vertical sorting by assuming that $0 \leq \gamma < 1$. However, by switching the sectors and re-scaling the dispersion of the zero-mean x shock, one can arrive at an equivalent specification.

Figure C.1: Absolute Advantage and Vertical Sorting



Notes: The figure illustrates the case with vertical sorting. The range of the match shock is given by the blue line. The average wage is higher in sector 0 than in sector 1. Under a persistent match shock, low performers of the high wage sector and high performers of the low wage sector will move more frequently. Also, see Figure C.2.

figure shows that in the high-wage sector, the wage-tenure profile is positive, while the profile is negatively-sloped in the low-wage sector. However, the overall wage-tenure profile in the economy is increasing. This is because the slope of the wage-tenure profile is larger in the high-wage sector than in the low-wage sector, in *absolute terms*. Given the positive slope of the economy-wide wage-tenure profile, on average, movers earn less than stayers.

C.3.2 Analytical results

The positive relationship between the wages and sectoral tenure in the entire economy can also be established analytically.

The mover-stayer wage gap. Propositions 1 and 2 show that when the match shock is persistent, the average match shock among stayers is higher than that among movers, in *absolute value*. (See inequalities B.8 and B.11 in Appendix B.)

Now, let the average match shock among newcomers and incumbents be $\overline{x}_{New}$ and $\overline{x}_{Old}$ respectively. We have $|\overline{x}_{New}| < |\overline{x}_{Old}|$. Then, given the wage equations (C.1) and (C.2), the average wage among all movers, $(\gamma - 1)|\overline{x}_{New}|$, is lower than the average wage among incumbents, $(\gamma - 1)\overline{x}_{Old}$. Analogously, it can be shown that the average wage among all future movers is lower than the average wage of all future stayers.

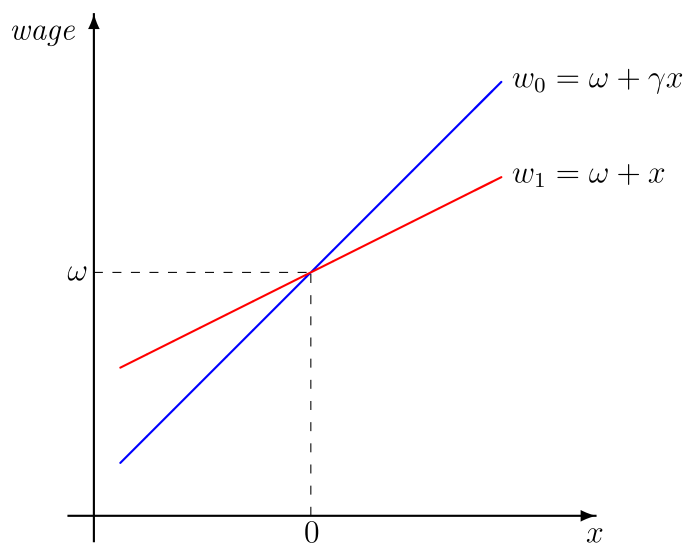
The wage-tenure profile. Using Proposition 3 in Section 5, it can be seen that at each tenure level τ, the average match shock between the sectors are the same, in absolute terms:

$|\overline{x}_0(\tau)| = |\overline{x}_1(\tau)|$ for each τ. Proposition 3 also states that these absolute values of the average match shock, increases with tenure, τ. (See inequality B.14 in Appendix B.) Therefore, for recent movers, the average wage $(\gamma - 1)|\overline{x}_0(\tau)|$ increases with tenure τ.

C.4 Discussion

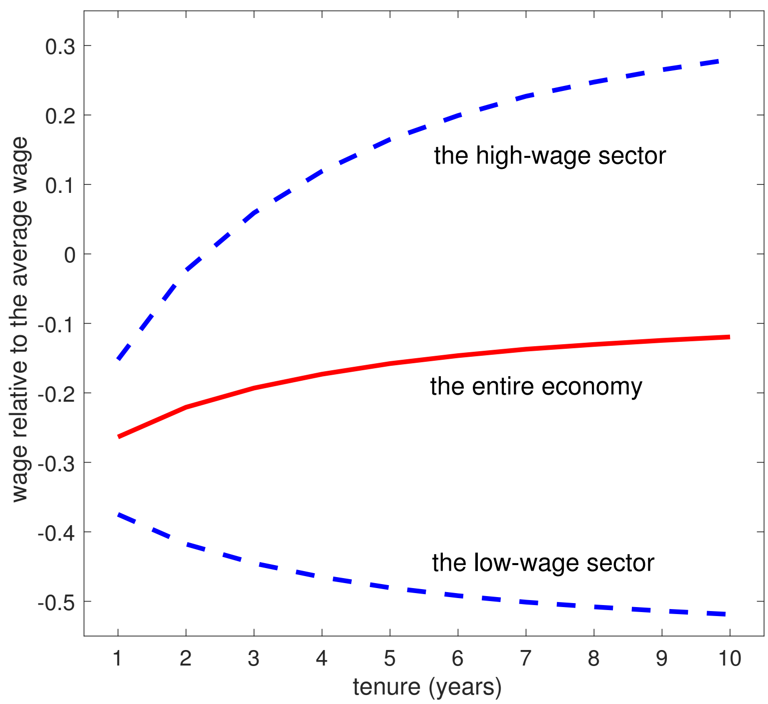
In the simple model considered in this appendix, we do not explore the impact of the relative size of sectors. Moreover, the issue of how high wage sectors emerge and how low wage sectors disappear is omitted. Clearly, in the economy where mobility is driven by income prospects, the employment size and the average wage are intimately related at the sectoral level. Since the relative order of different sectors changes over time, vertical and horizontal sorting might be both important. Nevertheless, the simple model with only a match shock shows that the persistence of the match shock accounts for the wage-tenure profiles and lifetime earnings.

Figure C.2: The Match-Specific Wage under Absolute Advantage



Notes: The figure shows the wages under vertical sorting where $\gamma > 1$. w_j is the wage of a person working in sector j when his or her match-specific productivity shock is x. If the match shock is positive, the person works in sector 0. If the shock is negative, the person works in sector 1. Also, see Figure C.1.

Figure C.3: Dynamic Absolute Advantage and the Wage-Tenure Profile



Notes: This figure plots the wage-tenure profile in a model with dynamic absolute advantage and vertical sorting.