Eligibility, Experience Rating, and Unemployment Insurance Take-up*

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September 2, 2016

Abstract

In this paper we investigate the causes and consequences of “unclaimed” unemployment insurance (UI) benefits. A search model is developed where the costs to collecting UI benefits are endogenous. Experience rated taxes give firms an incentive to challenge a worker’s UI claim. This imposes a utility cost on the individual applying for benefits, causing some eligible unemployed not to collect. Exploiting data on improper denials of UI benefits across states in the U.S. system, an empirical analysis shows a statistically significant negative relationship between the improper denials and the UI take-up rate, providing empirical support for our model. The endogenous collection costs imply the take-up rate responds slower to changes in UI benefits relative to a model with fixed collection costs; as a result, the unemployment rate also responds slower to UI benefit changes. Moreover, the model captures the negative correlation between the unemployment rate and take-up rate across U.S. states that we observe in our empirical analysis. Finally, the endogenous UI collection costs impose an externality that amounts to a welfare loss of as much as 3%.

JEL Classification Numbers: E61, J32, J64, J65.
Keywords: Unemployment Insurance, Take-up Rate, Experience Rating, Matching Frictions, Search.

*We are grateful to Guido Menzio, B. Ravikumar, Ofer Setty, as well as participants at several conferences and workshops for comments that greatly improved the paper. We would also like to thank financial support from the Chaire “Sécurisation des Parcours Professionnels at Sciences Po and GENES.”
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1 Introduction

The U.S. unemployment insurance (UI) system is characterized by sizable “unclaimed” benefits. These are UI benefits that eligible unemployed do not collect, and they amount to around 25% of total UI expenditures, on average. Since UI benefits provide around 50% of a worker’s previous wage, the presence of unclaimed benefits suggests there exist non-trivial costs to participating in the system. Unfortunately, little is understood about such costs or how they may interact with other parameters of the UI system. Characterizing these links is essential to understanding the effects of UI benefits on equilibrium outcomes. This paper provides a micro-foundation for the UI collection costs and explores their implications.

We develop a search model with matching frictions (in the class of Pissarides (2000)) where worker-firm interactions introduce an endogenous UI collection cost. There exist both UI eligible and ineligible workers in the model, with eligibility achieved by accumulating sufficient work history. Eligibility status is known by the firm, but unknown to the UI agency. Given this, there exists the possibility of UI fraud, as an ineligible worker can apply for UI benefits and potentially receive them. Similarly to the current U.S. system, firms have the possibility of challenging a worker’s eligibility. Firms optimally choose how often to challenge eligibility, and when they do it imposes a utility cost on the worker. This cost represents the cost of collecting UI benefits and leads to unclaimed benefits. The endogeneity of collection costs arises from firm behavior which is related to the financing of UI benefits.

Specifically we model the experience rating feature of UI finance in the U.S. system where firms pay a tax rate based on their history, or experience, sending workers to insured unemployment. Thus, firms face a marginal tax cost of separating from a worker who decides to collect UI benefits. Given this, firms have an incentive to challenge UI claims of separated workers. This occurs for both UI eligible and ineligible workers. Thus, verifying eligibility provides a deterrent to ineligible workers committing UI fraud.

The verification technology is imperfect, however, and only reveals the true eligibility status with some probability. As a result, some UI fraud occurs in equilibrium. More importantly, in some instances eligible workers who apply are denied UI benefits. These cases are referred to as improper denials. Workers are heterogenous in the utility cost of a verification, giving rise to unclaimed UI benefits; some workers find the expected benefits of collecting UI insufficient to cover the expected
utility cost of eligibility verifications.

Our focus on the costs of improper denials is supported by our empirical analysis. To determine what factors are correlated with the UI take-up rate, we exploit the variation across U.S. states in unclaimed benefits. The take-up rate is estimated with the methodology of Auray, Fuller, and Lkhagvasuren (2016) (who build on the methodology of Blank and Card (1991)), using the March supplement of the CPS and detailed state-level eligibility criteria. Focusing on the period from 2002 – 2011, we find an average take-up rate of 74.6%.

Improper denials are negatively related to the UI take-up rate, after controlling for state-fixed effects, the unemployment rate, replacement rate, and UI fraud rate in each state. That is, a state with a relatively high rate of improper denials has a relatively low take-up rate, all else equal. We also adjust for the fact that when improper denials increase, the take-up rate necessarily decreases as the denials imply fewer eligible unemployed are able collect. Moreover, the analysis finds a surprising negative relationship between the unemployment rate in a state and its take-up rate.

We use the data to calibrate the search model and perform several policy experiments. The model provides a natural link between the level of experience rating and different labor market outcomes. As the level of experience rating increases, the take-up rate decreases, as it is optimal for firms to increase their use of the eligibility verifications. Moreover, despite increased use of verifications, the increased costs associated with higher experience rating lead firms to decrease vacancy creation, increasing the unemployment rate. In this experiment, the unemployment rate and take-up rate are negatively related via their link with experience rating. This result provides one avenue to potentially explain the negative relationship between the unemployment rate and take-up rate uncovered in the empirical analysis.

The link with experience rating relates this paper to the existing literature examining the implications of experience rated UI taxes. This literature has focused almost exclusively on how experience rating affects separations. For example, the work of Feldstein (1976), Topel (1983), Albrecht and Vroman (1999), Wang and Williamson (2002), and Cahuc and Malherbet (2004) all examine the effects of experience rating, finding positive welfare effects and that experience rating decreases the unemployment rate. While we abstract from the separation dimension, our results indicate that incorporating the take-up decision has important implications for the effects of experience rating on equilibrium outcomes. Indeed, firm responses to changes in experience rating
are irrelevant if workers do not collect UI benefits.

Allowing for endogenous costs of collecting UI benefits matters when considering the effects of different UI policies. Specifically we consider changes in the level of UI benefits under our endogenous UI collection cost model, and a model with costs fixed at the baseline level. The take-up rate responds slower to UI benefits when the costs of collecting are endogenous. When UI benefits increase, so does the tax paid by firms (for a fixed level of experience rating); as a result, firms increase the probability of an eligibility verification. This increases the expected costs of collecting UI benefits, reducing the net effect of the UI benefit increase. The slower increase in the take-up rate also implies a slower response of the unemployment rate to changes in UI benefits, relative to the model with fixed UI collection costs.

In this paper, we provide a micro-founded mechanism to generate unclaimed UI benefits and explore the general equilibrium implications of UI policies in this setting. The existing literature examining the equilibrium effects of UI benefits generally ignores the issue of UI take-up, or assumes the take-up rate to be exogenous. Exceptions include Blasco and Fontaine (2016), who examine take-up of UI benefits in the French system focusing on the effects of unemployment durations on UI take-up. Kroft (2008) represents an example with endogenous UI collection costs. Kroft (2008) incorporates endogenous costs based on a “social” effect; more unemployed collecting UI reduces the costs to collecting, further increasing the take-up rate. The focus of Kroft (2008) is on determining the optimal UI replacement rate using the method of Baily (1978).

In our model, a welfare analysis indicates that the informational frictions associated with eligibility imply an externality that reduces welfare in equilibrium. The externality arises as firms do not consider the utility cost to worker’s when deciding how often to challenge a UI claim. We compare equilibrium welfare to that of a social planner’s solution, and to a full commitment equilibrium. The social planner prefers to never utilize the eligibility verification technology. Depending on non-collector flow income, this is accomplished either by setting the probability of verification so high as to discourage all applications or by never verifying and allowing all unemployed to collect. The full-commitment equilibrium assumes that ineligible unemployed never apply for UI benefits, and firms commit to never verifying eligibility. Welfare gains range from 1.79% to 3.03%, depending on non-collector flow income.

The remainder of the paper proceeds as follows. Section 2 presents the data and our empirical
analysis of the UI take-up rate across U.S. states. Sections 3 and 4 develop the model and equilibrium. Section 5 analytically derives some key properties of equilibrium and Section 6 parameterizes the model based on the data in Section 2. Section 7 then performs a number of counter-factual experiments and Section 8 conducts welfare experiments. Section 9 concludes.

2 Data

This section presents the key facts regarding unemployment benefit receipt across U.S. states. Each state has control over its UI benefit system. Although there are certain federal-level rules and guidelines, the operation of UI benefit systems is autonomous across states. Indeed, there exists variation in the level of UI benefits offered, taxes levied, the specific eligibility requirements, and perhaps more significantly, in the administration of these requirements.

Two states with equivalent eligibility requirements may enforce them quite differently. We seek to exploit these differences across states to provide possible clues regarding what drives the variation in take-up rates. The results from this analysis motivate the micro-foundations for UI collection costs we develop in Section 3.

2.1 Experience Rating

The U.S. UI system is unique relative to most developed countries’ systems on one dimension: its financing. In the U.S., benefits are financed via a payroll tax levied on employers. Moreover, the specific tax rate a firm faces depends on their “experience” sending workers to insured unemployment. A firm that has previously sent a relatively large fraction of its payroll to insured unemployment will in general pay a higher tax rate than a firm with less “experience.” This feature represents an important component of the model described in Section 3. Given its use in modeling, in this section we describe the main aspects of experience rating and present the available data.

Each state has a particular formula for calculating a firm’s tax rate. The actual extent of experience rating depends on how a firm’s tax rate responds to changes in its experience with insured unemployment. That is, how much will the firm’s taxes increase if they send a worker to insured unemployment, and how does the increase in taxes relate to the total amount collected by the worker. This increase in future payroll taxes represents the marginal cost to the firm (in terms of UI taxes) of separating from a worker who collects UI benefits.
The U.S. system is “partially” experience rated. That is, on average the marginal cost of separating from a worker is less than one; firms do not fully pay for the benefit expenditures of their former employees. Partial experience rating stems primarily from minimum and maximum tax rates. A firm at the minimum tax rate generally has a marginal cost higher than one, while a firm at the maximum tax rate has a marginal cost below one. The minimum and maximum tax rate vary significantly across U.S. states, as do the wages subject to the tax (referred to as the “taxable wage base”).

To capture this, our model in Section 3 requires an estimate of the marginal tax cost of separating from a worker. That is, when a firm separates from a worker who decides to collect UI benefits, how does this increase the taxes of the firm? To provide an estimate of this cost, we use data from the Department of Labor who tabulates an index referred to as the “Experience Rating Index,” or “ERI.” The specific calculation is:

\[ \text{ERI} = \left( \frac{\text{BEN}-(\text{IEC}+\text{IAC}+\text{NNC})}{\text{BEN}} \right) \times 100 \]

\( \text{BEN} \) refers to total benefits charged in a given state. \( \text{IEC} \) represents “ineffective charges.” To compute these, employers are aggregated into 30 groups based on their experience factor. Within each group, the difference between benefits charged to the employers (i.e. benefits collected by former employees) and the benefits contributed by those employers. Summing over the 30 groups produces the \( \text{IEC} \). It is a measure of how much of benefit expenditures are not completely financed by firm taxes. \( \text{IAC} \) represents benefits charged to employers who have gone out of business (and thus from whom no taxes may be collected). Finally, \( \text{NNC} \) represents benefits collected that were not charged to any particular employer.

Thus, the ERI is a measure of how “responsible” employers in a given state are for the benefits charged by their former employees, providing one estimate of the average marginal cost of separating from a worker (in terms of UI taxes). As expected, given the specificity of the tax rate calculations, there exists noticeable variation across states in the ERI. Figure 1 displays the average ERI in each state over the period from 1989 – 2004.\(^1\) The standard deviation of these average ERIs is 8.01 with

\(^1\)From 2005 – 2007, the ERI is not tabulated by the U.S. Department of Labor. The tabulations are available from 2008 – 2012, however, the calculation is now different. Since 2008, the preferred metric is now “The Average Increase in an Employer’s Per employee Tax for Incurring Benefit Charges Equivalent to 1% of its Taxable Payroll.” The idea of this index is to calculate the average additional cost an employer will incur if it sends an employee to
an average of 60.02, for a coefficient of variation of 0.13. Topel (1983) also calculates an estimate of the marginal cost using the details of each state’s tax scheme, generally finding a higher marginal cost (around 80%), although for an earlier time period.

Figure 1: Experience Rating Index Across U.S. States

Notes: The figure displays the average value of the ERI in each U.S. State over the period from 1989 – 2004. The vertical axis is in percent.

2.2 Take-up Rate Estimates

“Unclaimed benefits,” or unemployed individuals who do not collect UI benefits they are eligible for, is an important issue in the U.S. UI system. In this paper we argue that the “take-up” rate of UI benefits represents a key variable to understanding the effect of UI benefits on labor market outcomes. Despite its relevance for policy considerations, there does not exist any readily available tabulations of the UI take-up rate. In this section we describe our estimates of the take-up rate and explore some of its features.

insured unemployment. From the Significant Measures of State UI Tax Systems, published by the BLS (Bureau of Labor Statistics), it is calculated as: “The difference between the maximum per employee cost at the tax base and the minimum per employee cost, divided by the difference between the experience rating percent (either Reserve Ratio or Benefit Ratio) corresponding to the maximum statutory tax rate and the experience rating percent corresponding to the minimum statutory tax rate.”
The “take-up” rate is the fraction of unemployed eligible for UI benefits who collect them. Eligibility for UI benefits in the U.S. is determined by three factors: monetary criteria, separation criteria, and duration criteria. The specifics of each criteria vary considerably across U.S. states, but the general notion of each is described below.

Monetary criteria specify that an individual must have accumulated sufficient work experience prior to becoming unemployed. In most U.S. states, the monetary criteria require certain threshold for earnings. A worker must have earned more than a multiple of their Weekly Benefit Amount (WBA), which is the amount of UI benefits they receive each week. For example, a state may specify that in the previous year, the worker must have earned at least $40 \times \text{WBA}$. Since the WBA is typically 50% of previous weekly earnings, this criteria is approximately equivalent to requiring at least 20 weeks worked in the previous year. Other states simply have a number of weeks of previous employment required, and some have a hybrid requirement.

The separation criteria attempt to prevent workers who do not fall under the category “unemployed through no fault of their own.” Thus, workers who voluntarily quit their jobs, or are fired for cause (such as tardiness or poor performance) are not eligible to collect UI benefits. Finally, the duration criterion arises from the fact that benefits have a limited potential duration. In most states an individual may collect UI benefits for at most 26 weeks.\(^2\) Once an individual has exhausted their benefits, they are no longer eligible until they have a new employment spell satisfying the monetary criteria.

Calculating the take-up rate thus involves determining eligibility. To do so, we use the method of Auray, Fuller, and Lkhagvasuren (2016) (who build on the method of Blank and Card (1991)). Specifically, we use data from the March CPS supplement. This contains earnings information for the previous year, in addition to current labor market status. The take-up rate is calculated by first determining who among the unemployed would be eligible for UI benefits. Eligibility is determined based on the individual’s state of residence and that particular state’s eligibility criteria.\(^3\)

Thus, for each year in each state, we calculate the fraction of unemployed eligible for benefits.

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\(^2\)In periods of high unemployment, benefits may be extended an additional 13 or 20 weeks depending on the state’s unemployment rate. During the period from 2009 – 2013, benefits had a potential duration of 99 weeks in states with high unemployment rates.

\(^3\)See Auray, Fuller, and Lkhagvasuren (2016) for the details on how each eligibility criteria is handled specifically in the CPS data.
Figure 2: State averages from 2002 – 2011. Each graph plots the average value of the variable in each state. The upper left figure plots the Take-up Rate and the upper right figure the Improper Denial Rate, both as fractions. The lower left figure plots the Replacement Rate and the lower right figure the Unemployment Rate.
Then, the number of insured unemployed (i.e. those collecting UI benefits) is determined from the fraction of insured unemployed. This fraction is available from the BLS for each state, and represents the number of insured unemployed divided by total unemployed. We refer to this fraction as the “IUR.” Thus, dividing the IUR by the fraction of unemployed eligible gives the take-up rate.

![Figure 3: Take-up Rates Over Time](image)

**Notes**: The figure displays the average take-up rate for the U.S. over the period from 1989 – 2012. Take-up rates are calculated following the method in Auray, Fuller, and Lkhagvasuren (2016). The dashed line plots the Insured to Total Unemployed Ratio. Both lines have the same numerator, but differ in the denominator.

Figure 2(a) plots the take-up rate across U.S. states during the 2002 – 2011 period. Each bar corresponds to the average take-up rate in a particular state over the 2002 – 2011 period. The U.S. average was 74.6% during this period, with a standard deviation across states of 0.17 for a coefficient of variation of 0.23. The standard deviation is calculated across states for the average take-up rate over the period (i.e. the standard deviation evident in Figure 2(a)). Figure 3 plots the average take-up rate (across all states in each year) from 1989 – 2011 (solid line) along with the average IUR (dashed line).

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4The fraction of insured unemployed is taken from the BLS tabulations, and refers to the fraction of unemployed persons collecting *regular program* UI benefits. “Regular program” benefits are those benefits available for the typical 26 weeks; therefore, this does not include any individuals collecting extended benefits.
Figure 4: UI Filing Methods

This graph displays how individuals have filed their initial UI claims over time. The solid line is the fraction of initial UI claims filed in person and the dashed line is the fraction filed online or by phone.

2.3 UI Collection Costs

UI benefits provide unemployed individuals with insurance against lost income. Since 25% of those eligible do not collect these benefits, however, it is clear there exist some costs to collecting the benefits. The exact nature of these costs has not been well documented in the existing literature examining UI take-up rates.\(^5\) Indeed, an exploration of the micro-foundations for such a cost represents one contribution of this paper.

The eligibility requirements detailed above require individuals to apply for UI benefits if they become unemployed. This application provides the UI agency the opportunity to verify the individual’s eligibility. Thus, the time associated with filing this application represents one possible cost that may prevent some eligible unemployed from deciding to collect the benefits. To gauge the extent of this potential cost we examine how individuals actually file their initial UI benefit claim. Data on the initial filing method is available from the Benefit Accuracy Measurement (BAM) program run by the U.S. Department of Labor. The BAM program audits a random sample of UI

\(^5\)Anderson and Meyer (1997) examine the effect of changes in the tax treatment of UI benefits on the take-up rate in the 1980s. They provide some survey results regarding why workers do not file, but no clear reason emerges.
claims to determine their accuracy. This data provides many variables of interest for an analysis of the U.S. UI program. For example, data regarding claimants weekly benefits and prior earnings are used below to calculate replacement rates across states.

In Figure 4, we present the data on the initial filing method, from 1988 – 2011. There exist five possible initial filing methods. These include, in person, mail (including e-mail), telephone, employer filed claim, and internet claim. Figure 4 plots the fraction of agents who file in person, compared to the fraction filing by phone and/or internet.\(^6\) The graph indicates that there has been a large shift in how unemployment benefit applications are filed in the U.S.; since 2000, in person claims and phone and internet claims have switched as the dominant method.

This change has almost certainly had an effect on the explicit application costs of applying for UI. First, since an in person application is typically no longer required, at a minimum, the time associated with filing a claim has been dramatically reduced. Second, applying via phone or internet makes the process more anonymous, which reduces any negative stigma associated with applying for benefits. The negative stigma argument proposes that some workers do not collect UI as they may view collecting “government benefits” negatively, or are concerned others will view their receipt of them negatively. This is perhaps surprising since UI benefits in the U.S. are generally not a government benefit. Indeed, UI benefits are insurance. The worker pays premiums via their employer’s experience rated tax, similar to any other insurance scheme (health insurance for example). While the government manages the system, it is more appropriately viewed as insurance rather than government benefits, or “welfare.”

If the costs associated with the actual application were a significant portion of the total “cost” of applying for UI benefits, then as the process has become less costly the take-up rate should increase over this time period. In Figure 3, we plot the take-up rate (average over all U.S. states) over the same 1989 – 2011 time period. Outside of some cyclical fluctuations, the take-up rate has remained relatively constant since 1989. Thus, there does not appear to be strong empirical support for a simple time/application cost. Indeed, Ebenstein and Stange (2010) find no evidence to support time/application costs as a reason for low UI take-up. Below we propose one potential source of the costs of applying for UI benefits.

\(^6\)The other filing methods account for a small fraction of the total. Moreover, prior to 2002, there were no internet claims observed, so this represents a recent phenomenon.
2.4 Improper Denials

Above we have argued that the direct application costs of filing a UI claim have decreased significantly in the past 10−15 years. This has occurred with no noticeable change in the UI take-up rate. We argue instead that the costs of verifying eligibility in the application process fall primarily on the firm. Combined with experience rated taxes, this gives firms incentives to occasionally challenge the eligibility of a UI claimant. The resulting process does impose a cost on the claimant, which is the cost we model below in Section 3.

When a worker files a claim for unemployment benefits, the UI authority in that U.S. state contacts the worker’s previous employer(s) to verify the relevant information. For example, they verify the worker’s wages to determine eligibility and calculate the proper benefit amount. They also have to verify that the nature of the separation is proper, since certain separations render the worker ineligible for benefits. If indeed the individual is eligible, then benefits are provided; however, if it is determined that the individual is not eligible, then no benefits are provided. When disagreements between the worker and the firm arise, the case may move to the legal system to resolve the dispute. Thus, even before paying taxes, the administrative costs related to a worker filing a claim for benefits may be substantial.

In some instances, an individual is improperly denied UI benefits; that is, they were eligible but were incorrectly determined as ineligible. Improper denials obviously pose a cost to those applying for UI benefits. The process of verifying eligibility described above is costly, and the prospect of going through the process and being denied benefits lowers the net expected gain from collecting UI benefits.

Data on improper denials is available from the BAM program. This information is available beginning in 2002 for each state. The Improper Denial Rate is the fraction of all denied applications that are determined to be improper. Figure 2(b) plots the average Improper Denial Rate in each state over the 2002−2011 time period. Notice, several states do not have information on improper denials. During this period, the average is 0.103; around 10% of denied applications are improper. The standard deviation is 0.042, for a coefficient of variation of 0.41.
2.5 UI Fraud

Since eligibility status is not perfectly observable to the UI agency, it also remains possible for an ineligible unemployed worker to apply for and receive UI benefits. This is referred to as UI “fraud.” For the analysis here only two types of UI fraud are relevant. The first is misreported base period earnings, weeks, days, or hours worked. In other words, a worker did not accumulate sufficient work history to be eligible, but committed fraud by collecting benefits anyway. Separation issues represent the second form of fraud we include. This includes workers who quit or were fired for cause but still collected benefits. These two types of UI fraud are the relevant forms with regards to the eligibility verification process we discuss above and model below in Section 3.

We characterize UI fraud using the aforementioned BAM data. Overall, UI fraud accounts for around 3.0% of total UI benefit expenditures. From 2002 – 2011, benefits collected under the two types of fraud discussed above amounted to around 0.5% of total UI benefit expenditures.\(^7\)

2.6 Benefit Generosity Across U.S. States

The replacement rate represents the most common measure of UI generosity. It is calculated as the weekly benefit amount (WBA) divided by weekly earnings in the previous (i.e. pre-unemployment) job. Most generally, the U.S. system offers a fixed replacement rate of 50%; an individual can expect to receive half of their previous weekly earnings. As with other elements of state UI systems, the specific rules for calculating an individual’s WBA vary across states. There are several important factors determining the observed or actual replacement rate an individual receives.

First is the specific formula the state uses. Most have a formula that corresponds to roughly a 50% replacement rate. The WBA, however, is adjusted based on factors such as number of dependents. Second, there also exists a maximum benefit amount that varies across states. Since the WBA cannot exceed this amount, individuals with relatively high previous earnings will have a lower replacement rate. Figure 2(c) plots the average replacement rate from 2002 – 2011 in each state.

\(^7\)Fuller, Ravikumar, and Zhang (2015) provide further discussion regarding other types of UI fraud. They show that “concealed earnings” fraud is the dominant form of UI fraud. This occurs when an unemployed worker finds a job but continues to simultaneously collect UI benefits. Fuller, Ravikumar, and Zhang (2015) focus on determining the optimal UI scheme with monitoring when concealed earnings fraud is present, and also present some general facts regarding UI fraud in the U.S. system.
state.

Over the 2002 – 2011 period the mean is 0.471, with a the standard deviation across states of 0.045, for a coefficient of variation of 0.096.

### 2.7 Analysis of State-level Variation in Take-up Rates

This section provides a relatively parsimonious statistical analysis of the relationship between the aforementioned variables. Specifically we examine how these variables impact the Take-up Rate in a particular state. Towards this end, we examine a linear regression model with state fixed-effects. The dependent variable is the Take-up Rate (\(TUR\)) in the state, with independent variables the Unemployment Rate (\(UR\)), the Replacement Rate (\(RR\)), the Fraud Rate (Fraud), and the Improper Denial Rate (\(ID\)).

There is a natural relationship between the Take-up Rate and Improper Denial Rate that may affect the results. Specifically, a state with a higher rate of improper denials will (all else equal) have a lower take-up rate. The take-up rate is calculated as the total number collecting UI divided by the total number of unemployed eligible to collect UI. If an eligible individual is improperly denied, they would in theory appear in the denominator but not in the numerator, decreasing the take-up rate.
To account for this, we make the following adjustment. We first compute the total number (in the population of denials) of those improperly denied. This is accomplished using the population numbers provided by the U.S. Department of Labor in the BAM reports on Denials. Next, we add these to the total number collecting UI benefits. This is the adjusted numerator, and the adjusted take-up rate is thus divided by the total number of eligible unemployed. We perform the state fixed-effects regression using the adjusted take-up rate as the dependent variable.\(^8\)

Tables 1 and 2 present the results from two different specifications. First consider Table 1, which includes UI fraud. Here we find a significant positive relationship between the replacement rate in a state and its take-up rate; higher UI benefits tend to imply higher take-up rates, all else equal. The unemployment rate is significantly negatively related to the take-up rate in a state. This relationship is surprising, as one typically expects a higher unemployment rate to be associated with a higher take-up rate. Below in Section 7.1 we explore some possible explanations for this relationship by linking it with experience rating.\(^9\)

The coefficient on “ID,” or improper denials, represents a key parameter in our discussion of the costs of collecting UI benefits. From Table 1 this coefficient is significantly negative, so that higher improper denials rates are correlated with lower UI take-up rates, after adjusting the take-up rate accordingly. States that more strictly enforce and challenge eligibility have higher improper denials. This implies higher expected initial cost of applying for benefits, reducing the net gain of collecting UI benefits, and thus reducing the take-up rate. Indeed we adopt this interpretation.

Since the process of verifying eligibility goes through the worker’s previous firm, the firm decides whether or not to challenge the information presented by the worker on their initial application. Given experience rated taxes, a firm prefers that a separated worker not collect UI benefits. Thus, the firm has an incentive to challenge the eligibility of a worker’s UI claim. Indeed, firms in states with higher improper denials see a higher probability of successfully denying a worker’s claim. Thus, they may respond by more frequently challenging claims, increasing the costs of applying for workers, reducing the take-up rate. This mechanism is detailed in Section 3.

\(^8\)As a further robustness check, we present the results with the unadjusted take-up rate for both specifications in Appendix A.

\(^9\)Unfortunately there does not exist sufficient data on experience rating to include it in the empirical analysis. As previously discussed, there is a gap in the availability of the ERI from 2005 – 2007. Moreover, from 2008 on the methodology to compute the ERI is significantly different that what is used prior to 2005. Lacking sufficient observations we were unable to include the ERI in our regression.
Finally, the coefficient on “Fraud” implies that states UI fraud rates are positively correlated with UI take-up rates. One possible explanation for this is those states with higher fraud rates may be more lenient when enforcing eligibility requirements. This implies a lower cost of applying for UI benefits, increasing the take-up rate. Of course, a similar problem exists here as with improper denials. A UI collector committing fraud appears in the numerator but not in the denominator (as they are ineligible). Unfortunately population estimates are not available for the fraud rates so we are unable to make the same adjustment as the case of improper denials. As a robustness check, Table 2 displays results from a specification that does not include “Fraud.”

3 Model

Given the empirical results of Section 2, the process surrounding improper denials appears to be an important determinant of the UI take-up rate. In this section, we develop a search-matching model with endogenous UI costs based on this process.

Time is continuous and lasts forever. There exists a unit mass of risk-neutral workers and a unit mass of risk-neutral firms. Both discount the future at rate $r$. All firms are ex-ante homogenous. Workers may be either employed or unemployed. Each worker has the same productivity level, denoted by $y$. Firms are composed of one job, either filled or vacant.

Employed workers enjoy flow utility from the wage $w$. They are separated from the job exogenously at Poisson rate $\lambda$. We also incorporate UI eligibility as follows. Upon forming a match, all workers begin employment as UI ineligible; this is regardless of whether or not they previously collected UI or not. At Poisson rate $\sigma$, the worker becomes UI eligible. This represents a stylized version of the monetary criteria described in Section 2.2. Since separations remain exogenous and UI benefits last forever, the other two eligibility criteria discussed in Section 2.2 are not relevant.

If separated, a worker decides whether or not to collect UI benefits. A worker collecting UI benefits receives flow utility $b$, while a non-collector receives flow utility $d$, with $b > d$. Both UI eligible and ineligible workers may decide to apply for and collect UI benefits. This feature captures the UI fraud (where UI ineligible workers collect benefits) described above in Section 2.5. Eligibility is monitored, however, which may deter some UI ineligibles from collecting. Below we describe the eligibility monitoring process in more detail.

Firms are free to enter and pay a flow cost to open a vacancy, denoted by $\gamma$. In the event
of a separation (at Poisson rate $\lambda$), the firm faces some workers who decide to collect UI benefits and some that do not. If the worker does collect benefits, the firm pays a flow cost of $\tau$. This “tax” captures the experience rating feature in the U.S. economy discussed in Section 2.1.\textsuperscript{10} If the worker does not collect, the firm does not pay this flow cost. As stated above, we assume that upon forming a match, all workers are UI ineligible. Moreover, we assume that eligibility is perfectly observable by the firm, so that the firm knows if a worker moves to the UI eligible state.

In addition, if a worker decides to apply for UI benefits (recall the process described in Section 2.4), the firm can decide whether or not to challenge the worker’s UI eligibility. The firm chooses the probability of such challenges optimally. Let $p_i, i \in \{B, N\}$ denote this probability. Notice, the firm chooses monitoring probabilities for eligible and ineligible workers separately. In the event of an eligibility review, the firm is successful with probability $s_i, i \in \{B, N\}$. Thus, with probability $s_N$, an ineligible worker attempting to collect UI benefits is found to be ineligible and denied benefits. For a UI eligible worker, a successful eligibility challenge by the firm occurs with probability $s_B$ and results in an improper denial.

If the firm decides to challenge eligibility, they pay a flow cost $c(p)$ that depends on $p$. We assume that $c(p) \in \mathbb{C}^2$, and is strictly increasing and strictly convex: $c'(p) > 0$ and $c''(p) > 0$. If eligibility is denied, the firm is credited back the tax, $\tau$.

Having an eligibility verification is also costly for the worker. In the event of a verification, the worker pays a flow cost $\chi$. For each worker, this cost is permanent, but varies across the population. Let $F(\chi)$ denote the population distribution of these costs. Firms know the distribution of $\chi$, and learn the worker’s value upon forming a match (this is described further when discussing wage determination).

### 3.1 Matching Technology

There exists a matching function that describes the relationship between the number of unemployed workers, vacancies, and the resulting number of matches formed. Let $u$ denote the number of unemployed workers, and $v$ denote the number of vacancies. The number of matches formed is given by $m = m(u, v)$. The matching function, $m$, is continuous, strictly increasing, strictly con-

\textsuperscript{10}Notice, this tax does not have to be self-financing. That is, $\tau$ simply represents the marginal tax cost to the firm of a worker leaving and collecting UI benefits.
cave (with respect to each of its arguments), and exhibits constant returns to scale. Furthermore, 
\( m(0, \cdot) = m(\cdot, 0) = 0 \) and \( m(\infty, \cdot) = m(\cdot, \infty) = \infty \). Following Pissarides (2000) terminology, define \( \theta \equiv v/u \), referred to as labor market “tightness.”

Each vacancy is filled according to a Poisson process. Define \( q(\theta) \equiv \frac{m(u,v)}{v} = m\left(\frac{1}{\theta}, 1\right) \). Given this, a vacancy is filled with Poisson arrival rate \( \frac{m(u,v)}{v} = q(\theta) \). Similarly, each unemployed worker finds a job according to a Poisson process with arrival rate \( \frac{m(u,v)}{u} = \theta q(\theta) \). Since matching is random, when the firm meets an unemployed worker, that worker is randomly drawn from the population according to the distribution \( F(\chi) \).

### 3.2 Value Functions

This section describes the value functions for both workers and firms. We begin with workers and then describe value functions for firms.

#### 3.2.1 Workers

The value functions for workers are given by the following. For the unemployed UI benefit collector and non-collector, respectively,

\[
r_U(\chi) = b + \theta q(\theta) [E_N(\chi) - U(\chi)]
\]

\[
r_N(\chi) = d + \theta q(\theta) [E_N(\chi) - N(\chi)]
\]

In Equation (1), unemployed UI collectors receive flow utility \( b \) and transition to employment at rate \( \theta q(\theta) \). Equation (2) has a similar interpretation for an unemployed non-collector. Notice, since all workers start new employment ineligible for UI benefits, the relevant employed value function for both collectors and non-collectors is \( E_N(\chi) \).

For employed workers, there exist two possible employment states: UI eligible \( (i = B) \) and UI ineligible \( (i = N) \). Denote the value functions as \( E_B(\chi) \) and \( E_N(\chi) \) for UI eligible and ineligible, respectively. For the UI eligible:\(^{11}\)

\[
r_{EB}(\chi) = w(\chi) + \lambda \max \left\{ p_B \left( -\chi + s_B \left[ N - E_B \right] + (1 - s_B) \left[ U - E_B \right] \right) \right. \\
+ (1 - p_B) \left[ U - E_B \right], N - E_B \right\}
\]

\(^{11}\)Note: the dependence of \( E_B, U, \) and \( N \) on \( \chi \) is suppressed in several places for brevity, but it is implied throughout.
For the UI ineligible:

\[
 rN(\chi) = w(\chi) + \lambda \max\left\{ p_N (\chi - s N [N - E_N] + (1 - s N) [U - E_N]) + (1 - p_N) [U - E_N], N - E_N \right\} + \sigma [E_B(\chi) - E_N(\chi)]
\]

In Equation (3), the employed UI eligible worker receives flow utility from the wage, \( w(\chi) \), and at rate \( \lambda \) the job is dissolved. If this occurs, the worker must decide whether or not to apply for UI benefits. If the worker applies, one of two events occur: (i) they are subject to an eligibility verification (occurs with probability \( p_B \)) or (ii) they are not (occurs with probability \( 1 - p_B \)). If the eligibility verification does occur, the worker pays the flow utility cost \( \chi \). Then, whether or not the worker enters unemployment state \( U \) and collects UI, or state \( N \) (does not collect) depends on the outcome of the eligibility verification. Since monitoring is imperfect, with probability \( s_B \) the worker is improperly denied UI benefits and enters state \( N \). If the worker decides not to apply for UI benefits, the change in expected lifetime utility is given by \( N - E_B \).

Equation (4) has a similar interpretation to Equation (3), with different a eligibility verification probability, \( p_N \), and success rate, \( s_N \). The last term in Equation (4) reflects the transition to UI eligible, occurring at Poisson rate \( \sigma \).

### 3.2.2 Firms

Next, consider the value functions for a firm. Let \( V \) denote the value of a vacancy, given by:

\[
 rV = -\gamma + q(\theta) \int_{\chi}^{\infty} [J_N(\chi) - V] dF(\chi)
\]

In Equation (5), the firm pays the flow cost of opening a vacancy, \( \gamma \), and matches with a worker at Poisson rate \( q(\theta) \). The worker is drawn randomly from the population according to the distribution \( F(\chi) \), and starts as UI ineligible (\( i = N \)).

Let \( J_i(\chi), i = N, B \) denote the value of a filled vacancy for a worker of “type” \( i \) and \( \chi \). For a filled job with a currently UI ineligible worker (\( i = N \)):

\[
rJ_N(\chi) = \max_{p_N} y - w(\chi) + \lambda \{ \Omega_N(\chi) [-\tau + p_N s_N \tau - c(p_N)] + V - J_N(\chi) \} + \sigma [J_B(\chi) - J_N(\chi)]
\]

In this case, the firm earns flow profits equal to \( y - w(\chi) \). At arrival rate \( \lambda \) the job is dissolved; in this event, some workers decide to apply for UI benefits, while others do not. Here \( \Omega_i(\chi), i \in \{B, N\} \)
is an indicator variable for the worker’s choice. It is equal to 1 if the worker applies for UI and 0 if not. Thus, if the worker applies for benefits, the firm must pay a flow tax, \( \tau \). The firm optimally chooses \( p_N \), the probability of initiating an eligibility verification. With probability \( s_N \), eligibility is denied. If eligibility is denied, the worker does not collect and the firm is credited back the tax \( \tau \). The firm also pays the flow cost of verifications, \( c(p_N) \). Finally, at Poisson rate \( \sigma \) the worker gains UI eligibility and moves to state \( i = B \).

For a UI eligible worker (\( i = B \)), the value of a filled vacancy is,

\[
J_B(\chi) = \max_{y} y - w(\chi) + \lambda \left\{ \Omega_B(\chi) \left[ -\tau + p_B s_B \tau - c(p_B) \right] + V - J_B(\chi) \right\}
\]

which has a similar interpretation to Equation (6).

The firm chooses \( p_i \) optimally to maximize the value of a filled vacancy. Verifying eligibility more frequently reduces UI tax costs, but the firm also incurs a higher flow cost of verification, \( c(p) \). Thus, \( p \) is chosen to maximize the expected value of challenging eligibility:

\[
p_i^* = \arg \max_{p_i} p_i s_i \tau - c(p_i)
\]

Solving Equation (8) yields the following F.O.C:

\[
s_i \tau = c'(p_i^*)
\]

### 3.3 Wage Determination

This section discusses the determination of wages in equilibrium, which occurs via Nash Bargaining. With the different levels of eligibility and UI collection status, the wage setting process is relatively complicated. To simplify, we assume that upon meeting a firm, the disagreement value of a worker is \( N(\chi) \), the value for a non-collector. This is assumed to be true regardless of whether or not the unemployed worker is currently collecting benefits or not. This assumption actually reflects current UI laws in the U.S. system; if a worker rejects a suitable job offer, they are no longer eligible to collect UI benefits. Although no offers are rejected in equilibrium, walking away from the bargaining table renders the worker UI ineligible, implying this represents the relevant threat option.

One may argue, however, that while current law prevents a worker from rejecting the firm’s offer and still collecting UI, it may be possible for the worker to commit fraud. That is, the worker
could conceal the job offer rejection from the authorities and continue collecting UI benefits. This may seem particularly relevant, since we allow ineligible workers to potentially collect UI after a separation. While potentially feasible, data on UI fraud imply a low incidence of such behavior. Specifically, according to the BAM data discussed in Section 2.5, fraud from rejecting suitable job offers represents a negligible fraction of total UI fraud.\textsuperscript{12} Thus, we maintain the assumption that all workers have disagreement value \( N \), and that the option to commit fraud via job rejections is unavailable. Overall, this assumption does not affect any of the main results of the paper, but simply provides valuable tractability.

Given these assumptions we now describe the Nash Bargaining solution. Letting \( \beta \) denote the bargaining parameter, the Nash Bargaining problem is given by:

\[
\max_{\chi} \left[ E_N(\chi) - N \right] - N \]

\[
\beta \left[ J_N(\chi) - V \right]^{1-\beta}
\]

The F.O.C. for this Nash problem is given by

\[
(1 - \beta) \left[ E_N(\chi) - N \right] = \beta \left[ J_N(\chi) - V \right]
\]

Recall, we assume that workers all bargain with the common disagreement value \( N \). Notice, if a worker gains UI eligibility (happens with arrival rate \( \sigma \)), their surplus changes from \( E_N(\chi) - N(\chi) \) to \( E_B(\chi) - N(\chi) \). If the worker re-bargains, the wage changes once UI eligibility is obtained. We assume that there is no such re-bargaining, so the wage is constant for the duration of the match. Thus, there is just one wage function, which we denote by \( w(\chi) \). Similar to the assumption of a common threat value, this assumption provides tractability but does not affect the main results of the paper.

Under these assumptions, we can now characterize the wage function, \( w(\chi) \). Specifically, this is a piecewise linear function of \( \chi \), depending on how the worker’s value of \( \chi \) relates to the critical values \( \chi_N^* \) and \( \chi_B^* \). This equation is given by:

\[
w(\chi) = \begin{cases} 
    w_1(\chi) & : \chi \leq \chi_N^* \\
    w_2(\chi) & : \chi_N^* < \chi \leq \chi_B^* \\
    \frac{\beta \sigma + \lambda + \theta q(\theta)}{\rho + \lambda + \theta q(\theta)} y & : \chi > \chi_B^*
\end{cases}
\]

\textsuperscript{12}See Table 1 in Fuller, Ravikumar, and Zhang (2015) for more details.
The algebraic derivations of these equations and specific forms of $w_1(\chi)$ and $w_2(\chi)$ are presented in Appendix B.1.

4 Equilibrium

Determining equilibrium involves finding the following objects: \{\theta, w(\chi), \Omega_i(\chi), p^*_i\}_{i \in \{B,N\}}. That is, given the model and value functions described above, determining equilibrium requires finding market tightness, \theta, wage functions $w(\chi)$, a UI take-up decision rule, $\Omega_i(\chi), i \in \{B,N\}$, and optimal eligibility challenge decision by firms, $p^*_i, i \in \{B,N\}$. In addition, equilibrium determines the stock of workers in each employment state: \{n^E_B, n^E_N, n^U_B, n^U_N\}, where $n^E_i, i \in \{B,N\}$ denotes the number of workers employed by UI eligibility, and $n^U_i, i = B,N$ the number of unemployed UI collectors ($i = B$) and non-collectors ($i = N$), respectively.

4.1 Equilibrium Decision Rules

The first step in characterizing equilibrium is to examine worker decisions regarding UI benefit applications. We show that these decision are characterized by cutoff values for $\chi$, denoted by $\chi^*_i, i \in \{B,N\}$. For values of $\chi$ below the cut-off the worker prefers to apply for UI benefits if separated. To characterize these cut-offs, recall that when separated the worker maximizes between two options:

$$\max \{p_i [-\chi + s_i(N-U)] + [U - E_i(\chi)], N - E_i(\chi)\} \quad (13)$$

Given these value functions, the decision to collect benefits or not is characterized by the following Proposition.

**Proposition 1** Given $p_i > 0$ and $s_i > 0$, there exists a unique value of $\chi^*_i$ such that the worker prefers to apply for UI benefits for all $\chi \leq \chi^*_i$, for $i \in \{B,N\}$. These values are given by

$$\chi^*_i = \frac{(1 - p_i s_i)(b - d)}{p_i r + \theta q(\theta)} \quad (14)$$

Equation (14) represents an important variable in our analysis. A higher $\chi^*_i$ implies a higher take-up rate, all else equal, as a larger fraction of workers ($\Phi(\chi^*_i)$) apply for UI benefits. Thus, we use Equation (14) to characterize the relationship between the take-up rate, several UI policy variables, and other equilibrium outcomes such as the unemployment rate. This provides a qualitative link between the model and the empirical results presented in Section 2.7.
Towards this end, it is useful to characterize the optimal choice of $p_i$, which is summarized in the following Proposition:

**Proposition 2** There exists a unique $p_i^* > 0$ solving Equation (9). For $c(p) = cp'$, $\nu > 0$ with $c$ a constant, this unique value is given by:

$$p_i^* = \left[ \frac{s_i r}{\nu c} \right]^{\frac{1}{\nu - 1}}$$  \hspace{1cm} (15)

### 4.2 Equilibrium Characterization

Equilibrium is characterized by the following set of equations. Given the parameters, Equations (14) and (15) determine the values of $\chi_i^*$ and $p_i^*, i = B, N$. Equation (12) then determines the wage function $w(\chi)$. Given these, market tightness is determined by the free-entry condition, or $V = 0$. From Equation (5),

$$\frac{\gamma}{q(\theta)} = \int_0^{\chi_N^*} J_N(\chi|\chi \leq \chi_N^*)dF(\chi) + \int_{\chi_N^*}^{\chi_B^*} J_N(\chi|\chi_N^* \leq \chi \leq \chi_B^*)dF(\chi)$$

$$+ \int_{\chi_B^*}^{\infty} J_N(\chi|\chi_B^* \leq \chi \leq \infty)dF(\chi)$$  \hspace{1cm} (16)

With $\{\theta, w, \chi^*, p^*\}$ determined, the four stocks, $\{n_{EB}^E, n_{EN}^E, n_{UB}^U, n_{UN}^U\}$, are determined by the following four equations.

$$\lambda n_{EB}^E = \sigma n_{EN}^E$$  \hspace{1cm} (17)

$$(\sigma + \lambda)n_{EN}^E = \theta q(\theta) [n_{UB}^U + n_{UN}^U]$$  \hspace{1cm} (18)

$$\lambda \left[ F(\chi_B^*)(1 - p_B s_B)n_{EB}^E + F(\chi_N^*)(1 - p_N s_N)n_{EN}^E \right] = \theta q(\theta)n_{UB}^U$$  \hspace{1cm} (19)

$$n_{EB}^E + n_{EN}^E + n_{UB}^U + n_{UN}^U = 1$$  \hspace{1cm} (20)

Equation (17) states that the flows into and out of UI-eligible employment must be equal. Equation (18) equates the flows into and out of UI-eligible employment, and Equation (19) equates the flows into and out of insured unemployment. Equation (20) normalizes the measure of workers.
to 1. Solving these equations for the four stocks yields:

\[
\begin{align*}
    n_E^B &= \frac{\sigma \theta q(\theta)}{[\lambda + \sigma][\lambda + \theta q(\theta)]} \\
    n_E^N &= \frac{\lambda \theta q(\theta)}{[\lambda + \sigma][\lambda + \theta q(\theta)]} \\
    n_U^B &= \frac{\lambda \left[ \lambda F(\chi^*_N)(1 - p_Ns_N) + \sigma F(\chi^*_B)(1 - p_BS_B) \right]}{[\lambda + \sigma][\lambda + \theta q(\theta)]} \\
    n_U^N &= \frac{\lambda \left[ \lambda + \sigma - \lambda F(\chi^*_N)(1 - p_Ns_N) - \sigma F(\chi^*_B)(1 - p_BS_B) \right]}{[\lambda + \sigma][\lambda + \theta q(\theta)]}
\end{align*}
\]

The unemployment rate is given by \(n_B^U + n_N^U\). Denoting the unemployment rate by \(u\), Equations \((23)\) and \((24)\) imply:

\[
u = \frac{\lambda}{\lambda + \theta q(\theta)}
\]

which is the same expression one obtains in the standard Pissarides framework. The take-up rate represents another key moment. It is given as the ratio of the number collecting UI benefits, \(n_B^U\), to the number of unemployed eligible for benefits. The number of unemployed eligible for benefits is simply the total number unemployed multiplied by the fraction of employment that is UI-eligible. Denoting the take-up rate by \(TUR\) we have,

\[
TUR = \frac{n_B^U}{u \frac{n_B^E}{u} n_B^N} = \frac{\lambda F(\chi^*_N)(1 - p_Ns_N) + \sigma F(\chi^*_B)(1 - p_BS_B)}{\sigma}
\]

It is important to note that this is the model equivalent to the take-up rate calculated in Section 2.2. Specifically, this is not corrected for those improperly denied or those committing UI fraud (ineligibles that collect). This is appropriate since the model is calibrated to data on the un-adjusted take-up rate.

Next, consider the improper denial and UI fraud rates. As discussed in Section 2.4, the improper denial rate is calculated as the fraction of denied claims that are improperly denied. The corresponding moment in the model is given by:

\[
\text{Improper Denial Rate} = \frac{n_B^E F(\chi^*_B)p_BS_B}{n_B^E F(\chi^*_B)p_BS_B + n_N^E F(\chi^*_N)p_Ns_N}
\]

The numerator is the number of eligible employed, \(n_B^E\), multiplied by the probability of filing an application and having it denied, \(F(\chi^*_B)p_BS_B\); this represents the number of improper denials. The denominator is the total number of UI applications denied, both proper and improper. Similarly,
the UI fraud rate is defined as:

\[ \text{Fraud Rate} = \frac{n_N E F(\chi^*_N)(1 - p^*_N s_N)}{n_B^U} \]  

(28)

which is the total number of ineligible workers who have an application accepted, divided by the total number of workers collecting UI benefits.

5 Properties of Equilibrium

In this section we describe several features of equilibrium. Our focus here is to discuss how the model’s qualitative predictions relate to the relationships summarized in Table 1. Towards this end, recall that the relationships in Table 1 are *partial equilibrium* in nature. For example, Table 1 describes how changes in the replacement rate affect the take-up rate, holding the other variables constant. To analyze the model’s qualitative predictions on these dimensions, we characterize the relationship between the take-up rate and several key variables in partial equilibrium.

Specifically we consider a change in one model parameter, holding the values of \( \theta \) and \( w(\chi) \) fixed. The following Lemmas help illuminate the key mechanisms.

**Lemma 1** The optimal choice of \( p^*_i \) is increasing in \( \tau \) and increasing in \( s_i \).

This can be seen by examining the optimal choice of \( p^*_i \) determined by Equation (9), given the properties of \( c(p) \); since \( s_i \) is fixed, the positive relationship between \( \tau \) and \( p^*_i \) follows directly. Similarly, the firm responds to an increase in the probability of a “successful” eligibility verification by increasing \( p^*_i \).

The cut-off values for \( \chi \) represent the key variables affecting the take-up rate. When any \( \chi^*_i, i = B, N \) increases, this increases the UI take-up rate, defined in Equation (26). From Equation (14) the effect of different parameters on \( \chi^*_i \) are relatively straightforward. The following Lemma summarizes these changes in \( \chi^*_i \).

**Lemma 2** The cut-off levels, \( \chi^*_i, i = B, N \) are increasing in \( b \) \( (\frac{\partial \chi^*_i}{\partial b} > 0) \) and decreasing in \( \tau \) \( (\frac{\partial \chi^*_i}{\partial \tau} < 0) \), \( s_i \) \( (\frac{\partial \chi^*_i}{\partial s_i} < 0) \), and \( \theta \) \( (\frac{\partial \chi^*_i}{\partial \theta} < 0) \).

Given the partial equilibrium responses of \( p^*_i \) and \( \chi^*_i \), we can now compare the model’s qualitative predictions to Table 1. Formally,
Proposition 3 The take-up rate has the following relationships:

(i). it is increasing in UI benefits: \( \frac{\partial TUR}{\partial b} > 0 \).

(ii). it is decreasing in the improper denial rate and increasing in the UI fraud rate: \( \frac{\partial TUR}{\partial s_i} < 0 \).

(iii). it is increasing in the unemployment rate (i.e. decreasing in \( \theta \)): \( \frac{\partial TUR}{\partial \theta} < 0 \).

(iv). it is decreasing in experience rating: \( \frac{\partial TUR}{\partial \tau} < 0 \).

Qualitatively, the model matches the relationships in Table 1 with the exception of the unemployment rate. Recall, from Table 1 the UI replacement rate has a positive impact on the take-up rate (holding the other variables fixed). An increase in \( b \) increases \( \chi_i^* \), increasing the take-up rate (again, holding \( \theta \) and \( w(\chi) \) constant).

Changes in the improper denial and fraud rate are accomplished by changing \( s_i, i = B, N \). An increase in the improper denial rate requires an increase in \( s_B \). This makes it more likely a UI eligible worker is denied benefits if they apply. Again, from Lemma 2, this decreases \( \chi_i^*, i = B, N \), decreasing the take-up rate. To increase the UI fraud rate, \( s_N \) must decrease, making it less likely a worker attempting to commit fraud is denied benefits. The decrease in \( s_N \) increases \( \chi_i^*, i = B, N \) increasing the take-up rate.

The effects of a partial equilibrium change in the unemployment rate can be analyzed by considering an exogenous decrease in \( \theta \). This decreases \( \theta q(\theta) \), increasing the unemployment rate (see Equation (25)). From Lemma 2, a decrease in \( \theta \) increases \( \chi_i^* \), increasing the take-up rate. Qualitatively, this prediction is in contrast to the relationship displayed in Table 1, which shows a negative relationship between the unemployment rate in a state and the UI take-up rate.

In our model, the intuition for the opposite result is clear: a longer unemployment duration increases the expected benefit of collecting UI benefits, and all else equal the expected costs of collecting are constant. The second dimension, the expected costs of collecting offers one potential avenue to further explore this relationship. Specifically, below in Section 7.1 we quantitatively explore the general equilibrium relationship between the level of experience rating and several equilibrium outcomes. We show that the results offer a possible explanation of the negative relationship between the unemployment and take-up rates across U.S. states.
This section has verified that the model qualitatively captures the relationships discussed in Section 2.7. Next we calibrate the model to the data and explore these relationships quantitatively.

6 Calibration

The model is calibrated to U.S. data for the time period from 2002 – 2011. Given the model in Section 3, the following parameters must be specified: \{r, \beta, \eta, \lambda, F(\chi), \tau, \gamma, b, d, s_B, s_N, c(p)\}. Several of the parameters are determined directly from the data. The time period is taken to be one month and the discount factor is set to capture a 4% per-annum interest rate; i.e. \( r = \frac{0.04}{12} \). Similarly, following Fredriksson and Holmund (2001), the bargaining parameter, \( \beta \), and matching function elasticity, \( \eta \), are set to \( \beta = \eta = 0.5 \).

This leaves \( \lambda, F(\chi), \tau, \gamma, b, d, s_B, s_N \) and \( c(p) \) to be determined. These parameters are calibrated targeting the appropriate moments in the data. The arrival rate of job separations is set to hit a target unemployment rate of 6.5%, which implies \( \lambda = 0.014 \), or an average employment duration of around 6 years. Related, the value of \( \gamma \) is set to match the average unemployment duration during the 2002 – 2011 time period. The average duration was 20 weeks (or 5.0 months), which implies \( \gamma = 5.47 \).\(^\text{13}\)

The distribution of UI application costs, \( F(\chi) \), is parameterized as follows. First, we assume that it follows an exponential distribution with rate parameter \( \mu_\chi \). That is, \( f(\chi) = \frac{1}{\mu_\chi} \exp(-\frac{1}{\mu_\chi} \chi) \) and \( F(\chi) = 1 - \exp(-\frac{1}{\mu_\chi} \chi) \). The value of \( \mu_\chi \) is set to match the average take-up rate from 2002 – 2011, which was 0.75; this implies a value of \( \mu_\chi = 109 \).

Before describing the calibration of \( s_i \), the cost function \( c(p_i) \) is parameterized. We assume \( c(p_i) = cp_i^2 \), where \( c \) is a constant. Thus, \( p_i^* \) follows from Equation (15) with \( \nu = 2 \). We normalize the value of \( c = 1 \). This particular normalization of \( c \) does not affect the main results.

The parameters \( s_B \) and \( s_N \) are set to match the improper denial and UI fraud rates, respectively. To match the 0.10 improper denial rate in the data requires \( s_B = 0.0053 \); this implies that when

\(^{13}\)Note, the calibrated value of \( \lambda \) is slightly lower than Shimer (2005) who finds a job separation rate of 0.033 on a quarterly basis. That the unemployment rate (see Equation (25)) is only a function of \( \lambda \) and \( \theta \) represents the primary issue. Since \( \theta \) pins down \( \theta q(\theta) \) and is matched to the average unemployment duration, only \( \lambda \) can further manipulate the unemployment rate. If, for example, workers could exert variable search effort, one could fix \( \lambda \) according to Shimer (2005), set \( \gamma \) to hit the unemployment duration, and fix another parameter (perhaps \( b \)) to hit the unemployment rate.

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Table 3: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tr>
<td>$r$</td>
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<td>Discount rate</td>
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<tr>
<td>$\beta$</td>
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<td>Bargaining parameter</td>
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<tr>
<td>$\eta$</td>
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<td>Elasticity of matching function</td>
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<tr>
<td>$\lambda$</td>
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<td>Job separation rate</td>
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<tr>
<td>$\tau$</td>
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<td>Experience rating parameter</td>
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<tr>
<td>$\mu_\chi$</td>
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<td>Rate parameter for $F(\chi)$</td>
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<tr>
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<td>Vacancy creation costs parameter</td>
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<tr>
<td>$b$</td>
<td>0.887</td>
<td>Unemployed collector flow utility</td>
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<tr>
<td>$d$</td>
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<td>Non-collector flow utility</td>
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<td>$s_B$</td>
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<td>Probability of successful audit, eligible</td>
</tr>
<tr>
<td>$s_N$</td>
<td>0.607</td>
<td>Probability of successful audit, ineligible</td>
</tr>
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<td>$c$</td>
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<td>Cost of audit parameter</td>
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</tbody>
</table>

firms choose $p_B$ optimally (e.g. from Equation (15)), $p_B = 0.007$. Similarly, $s_N$ is set to match the UI fraud rates from Section 2.5. With $s_N = 0.607$, the optimal choice of $p_N$ is $p_N^* = 0.8015$, giving a fraud rate of 0.005.

To calibrate the values of $b$ and $d$ we use the estimates in Gruber (1997) who finds that among UI collectors the ratio of unemployed consumption to employed consumption is 0.93; that is, consumption decreases by approximately 7% upon unemployment. For those who do not collect UI benefits, consumption decreases by an additional 22%, implying a ratio of unemployed to employed consumption of 0.71. Given this, we set $b$ to target $\frac{b}{\bar{w}} = 0.93$ and $\frac{d}{\bar{w}} = 0.71$, where $\bar{w} = \int w(\chi)dF(\chi)$ represents the average wage. With $b = 0.887$ and $d = 0.678$, the average wage in the economy is $\bar{w} = 0.952$ implying the appropriate consumption ratios. Note, this estimate of $b$ is in line with that in Hall and Milgrom (2008).

Finally, the value of $\tau$ is set to match the available data on experience rating. As modeled, after a separation, firms pay a UI tax, $\tau$ only if the worker applies for UI and is accepted. Therefore, $\tau$ represents the marginal cost to firms of a worker collecting UI benefits. Given the partial experience rating nature of the U.S. system described in Section 2.1, $\tau$ represents the portion of that worker’s
Table 4: Calibration Results

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate</td>
<td>6.5%</td>
<td>6.5%</td>
</tr>
<tr>
<td>Unemployment duration</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Take-up rate</td>
<td>0.74</td>
<td>0.75</td>
</tr>
<tr>
<td>Improper Denial Rate</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Fraud Rate</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>Consumption Ratio: UI Collectors</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>Consumption Ratio: Non-collectors</td>
<td>0.71</td>
<td>0.71</td>
</tr>
</tbody>
</table>

The first column lists the moment, the second column the model’s predictions, and the third column the value of the moment in the data.

UI benefits the firm is responsible for. To calibrate this, we use the ERI tabulations discussed in Section 2.1. Specifically, the average value of the ERI over the period from 1989 – 2004 is 60.02.\textsuperscript{14} Given this, we set \( \tau \) to be 60% of the UI benefits a separated employee is expected to collect; i.e. \( \tau = 0.6 \times \frac{b}{\theta q(\theta)} \). Table 3 lists the parameters and their values. Table 4 presents the results from the calibration.

To further evaluate how well the model captures the relevant data, consider Table 5. Here we examine several elasticities in the model and compare them to their counterparts in the data. Qualitatively, we demonstrate the model matches these moments in Section 5, while in Table 5 we examine the elasticities quantitatively.

Recall that the regression in Table 1 gives the elasticity of the take-up rate with respect to several variables. For example, the coefficient on \( RR \) of 0.46 in Table 1 implies an elasticity of the take-up rate with respect to the replacement rate of 0.37. Note, this assumes that we keep the other variables (improper denial rate, unemployment rate, and fraud rate) \textit{constant}. As a result,

\textsuperscript{14}Ideally, we would use the period from 2002 – 2011 to calculate the ERI. Recall, however, the ERI is only tabulated until 2004, and again after 2008, but under different methodology.
the appropriate elasticities implied from the model should also keep the other variables constant.

To generate the model elasticities we do the following. We increase the replacement rate by increasing \( b \), but we hold \( w, \theta, \) and \( p^*_i \) fixed at the baseline levels, simply recalculating \( \chi^*_i \) and \( n_{ij} = E, U, i = B, N \). This maintains the baseline unemployment rate, improper denial rate, and fraud rate, allowing a true comparison of model and data elasticities. The results in Table 5 show that our model tends to over-predict the elasticities. This is true of the improper denial rate and the replacement rate, while the model under-predicts the elasticity of the take-up rate with respect to the fraud rate. With respect to the unemployment rate, the model predicts a small but positive elasticity while the data indicates a negative elasticity. In Section 7.1 we explore one possibility that makes the model and data consistent on this dimension.

Generally speaking, a stationary model over-predicting elasticities is a positive sign. Especially given the comparison elasticities in Table 1 are based on an across state analysis. The stationary model does not have any of the shocks and other sources of randomness present in the data underlying Table 1 (state fixed effects are one such example). Without the additional sources of randomness, the model implies perfect correlations, increasing the predicted elasticities.

### Table 5: Elasticities

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improper Denial Rate</td>
<td>−0.33</td>
<td>−0.04</td>
</tr>
<tr>
<td>Replacement Rate</td>
<td>1.61</td>
<td>0.37</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>( 3.77 \times 10^{-4} )</td>
<td>−0.06</td>
</tr>
<tr>
<td>Fraud Rate</td>
<td>( 3.50 \times 10^{-4} )</td>
<td>0.01</td>
</tr>
</tbody>
</table>

7 Comparative Statics

This section presents the results of several comparative static policy experiments. While the elasticities computed in Table 5 were partial equilibrium exercises, in this section we examine the general equilibrium consequences of changes to the UI system. In particular we focus on changes in
the level of experience rating and UI benefits. The results offer insight into the negative relationship between the unemployment rate and take-up rate (across U.S. states) noted in Table 1, and they also highlight the importance of incorporating endogenous UI costs.

7.1 Experience Rating

The first experiment is to increase the level of experience rating by increasing \( \tau \). Recall, experience rating is a key mechanism in the model; it creates the incentives for firms to challenge the UI claims of both eligible and ineligible workers. As the level of experience rating varies, so will firm actions to reduce their UI bill via proper and improper denials, which in turn impacts several moments in the model.

Figure 5 displays the results. Note, in each graph, \( \tau \) is given as a fraction of the expected UI benefits \( \frac{b}{\theta q(\theta)} \). In the model economy, an increase in experience rating decreases the take-up rate (Figure 5(a)), has a non-monotonic relationship the unemployment rate (Figure 5(b)), increases the improper denial rate (Figure 5(c)), and has a non-monotonic relationship with the fraud rate (Figure 5(d)).

First consider the decrease in the take-up rate. As \( \tau \) increases, recall Lemma 1, which shows that \( p^*_i \) is increasing in \( \tau \). As the costs to the firm of a separated worker collecting UI increase, the firm increases the probability of challenging a claim. As \( p^*_i, i = B, N \) increases, the take-up rate decreases as workers are more reluctant to file a claim (from Lemma 2, \( \chi^*_i, i = B, N \) are decreasing in \( p_i \)).

Next consider the effect of experience rating on the unemployment rate in Figure 5(b). Initially the unemployment rate increases as \( \tau \) increases, but then it becomes decreasing in \( \tau \) for values above approximately 0.80. The non-monotonicity of the unemployment rate in response to an increase in \( \tau \) occurs from two competing effects on firm vacancy creation decisions. To begin, notice from Equation (25), since \( \lambda \) is fixed, the unemployment rate increases (decreases) when \( \theta q(\theta) \) decreases (increases), which happens when \( \theta \) decreases (increases).

As \( \tau \) increases, the two competing effects are as follows. First, from Equations (6) and (7) the value of a filled vacancy \( J_N(\chi) \) decreases for all \( \chi \leq \chi^*_B \). From Equation (16) this implies that \( q(\theta) \) must increase, so that \( \theta \) must decrease. Thus, one effect of increasing experience rating is to reduce the profits firms receive from a filled vacancy, reducing vacancy creation; this increases the
Figure 5: Each graph plots the response of a particular moment in response to a change in the level of experience rating, $\tau$. The upper left figure plots the response of the Take-up Rate and the upper right figure the response of the unemployment rate. The lower left figure plots the Improper Denial Rate and the lower right figure the Fraud Rate. On the horizontal axis, $\tau$ is expressed as a fraction of the expected UI benefits $\left(\frac{b}{\theta q(\theta)}\right)$; i.e. $\tau$ is expressed as the fraction of expected UI benefits the firm is responsible for.
Figure 6: Each graph plots the response of a particular variable in response to a change in the level of experience rating, $\tau$. The upper left figure plots the response of $p^*_N$ and the upper right figure the response of $p^*_B$. The lower left figure plots $\chi^*_B$ and the lower right figure $\chi^*_N$. On the horizontal axis, $\tau$ is expressed as a fraction of the expected UI benefits ($\frac{b}{\log(\theta)}$); i.e. $\tau$ is expressed as the fraction of expected UI benefits the firm is responsible for.
unemployment rate. The second effect arises from the effect of $\tau$ on the take-up rate.

Recall from above that as $\tau$ increases, the take-up rate decreases. This occurs as both $\chi^*_{N}$ and $\chi^*_B$ decrease; as a result, more weight is pushed to the portion of $J_N(\chi)$ where $\tau$ has no effect on firm profits as workers do not collect UI benefits (see Equation (16)). Thus, the second effect of increasing $\tau$ is to reduce the number of workers collecting UI, which decreases a firm’s expected UI tax bill in equilibrium. For higher values of $\tau$ this second effect begins to dominate and the unemployment rate decreases as $\tau$ increases.\(^{15}\)

Figures 5(c) to 5(d) show the responses of the improper denial and UI fraud rates, respectively. These movements follow from the changes in $p_i$ and $\chi^*_i, i = B, N$ to changes in $\tau$, and thus help illustrate the changes in the take-up and unemployment rates. The responses of $p_i$ and $\chi^*_i, i = B, N$ to changes in $\tau$ are displayed in Figures 6(a) to 6(d).

It is interesting to compare the results here to the existing literature examining the effects of experience rating on labor market outcomes. With respect to the effect of experience rating on the unemployment rate, the literature is inconclusive. Most find that experience rating reduces unemployment. The work of Feldstein (1976) and Topel (1983) focuses on the separation element, showing that a move from partial to full-experience rating reduces separations and the unemployment rate. Others, such as Burdett and Wright (1989) and Marceau (1993) find that experience rating may increase the unemployment rate when vacancies or the size of the firm are endogenous. The work of Albrecht and Vroman (1999), Wang and Williamson (2002), and Cahuc and Malherbet (2004), among others, also allow for endogenous vacancy creation and all find experience rating decreases unemployment.

The increasing unemployment rate in response to an increase in $\tau$ may offer insight into the negative relationship between a state’s unemployment rate and take-up rate (see Table 1). Specifically, notice that for a range of $\tau$, an economy with a higher level of experience rating has a lower take-up rate and a higher unemployment rate. That is, when allowing the level of experience rating to differ, the unemployment rate and take-up rate indeed may have a negative relationship. Ideally we could incorporate the data on the ERI into our empirical analysis in Section 2.7; however, since the ERI is only tabulated from 1989–2004, and again from 2008–2011 (but under different methodology and calculations), we lack sufficient data. The results in this section suggest that

\(^{15}\)The kink in Figure 5(b) occurs where $\bar{p}_N = 1$; that is, $\bar{p}_N$ is increasing as $\tau$ increases until it reaches its maximum. At this point, the fraction of ineligible workers no longer changes with $\tau$, accelerating the second effect.
exploring this aspect as more data becomes available represents a promising direction.

Our results show that an endogenous take-up rate represents an important consideration for the impact of experience rating on labor market outcomes. Since the tax only matters if workers collect UI benefits, the interaction between the tax and take-up rate determines how experience rating impacts the labor market. Moreover, incorporating endogenous UI collection costs further magnifies these effects, as the changes in firm behavior in response to changes in experience rating alter the costs of collecting UI benefits.

7.2 Unemployed Flow Utility

We now consider the effects of changes in UI benefits on equilibrium outcomes. In this experiment we change the level of $b$ while maintaining the baseline level of experience rating. That is, we change $\tau$ with $b$ to maintain $\frac{\tau}{b \ast \text{DUR}} = 0.6$, where DUR refers to the average duration of unemployment. Figure 7 presents the results.

As the unemployed flow utility increases from the baseline level of $b = 0.887$, both the take-up rate and unemployment rate increase. Figures 7(a) and 7(b) display these responses. The intuition for these results is straightforward. An increase in the flow utility for unemployed collecting UI (holding the non-collecting utility fixed) increases the benefits to collecting relative to the costs. The increase in $b$ also increases the unemployment rate and unemployment duration for the same reasons it increases in Pissarides (2000).

In Figures 7(c) and 7(d) we see that the improper denial rate and UI fraud rate are both non-monotonic in UI benefits. The improper denial rate initially decreases with UI benefits before eventually increasing, while the UI fraud rate initially increases before eventually decreasing. There are several effects that determine this response. The response of the endogenous costs of collecting UI, driven by the changes in $p_i$ in response to the changes in $\tau$, represents a key component. To further illuminate this mechanism, consider Figures 8(c) and 8(d).

In these graphs we compare the baseline model responses to an economy with fixed UI collection costs. That is, we increase $b$ but hold $\tau$ and $p_i$ at the baseline levels. In this case, the improper denial rate is monotonically deceasing in UI benefits while the UI fraud rate is monotonically increasing. Indeed, the different responses of $p_i^\ast$ are at the heart of the non-monotonicity of the improper denial and UI fraud rates. When $p_i^\ast$ responds endogenously to the change in $\tau$ implied
Figure 7: Each graph plots the response of a particular moment in response to a change in unemployed flow utility (UI collectors), $b$. The upper left figure plots the response of the Take-up Rate and the upper right figure the response of the unemployment rate. The lower left figure plots the Improper Denial Rate and the lower right figure the UI Fraud Rate.
Figure 8: Each graph plots the response of a particular moment in response to a change in unemployed flow utility (UI collectors), $b$, for the model with endogenous UI collections costs (solid lines) relative to model with fixed collection costs (dashed lines). The upper-left figure compares the response of the Take-up Rate and the upper-right figure the response of the Unemployment Rate. The lower-left and lower-right graphs display the different responses of the Improper Denial and UI Fraud Rates, respectively.
by the increase in $b$, they are both increasing (i.e. for $i = B, N$). This has two effects. First it works to decrease $\chi^*_i$, dampening the increase caused by the increase in $b$; as a result, the number of workers applying for UI benefits does not increase as fast. Second, it implies that more workers are denied benefits. This puts upward pressure on the improper denial rate and downward pressure on the UI fraud rate. Comparing Figures 7(c) and 8(c) and Figures 7(d) and 8(d) illustrates how these competing effects influence the improper denial and UI fraud rates.

To further underscore the importance of allowing for endogenous UI collection costs consider the response of the take-up and unemployment rates to an increase in UI benefits in the experiments mentioned above. Specifically, compare the response with endogenous UI collection costs in Figures 7(a) and 7(b) to the experiment where we hold UI collection costs constant (by keeping $p^*_i$ fixed). Figures 8(a) and 8(b) compare the responses. In Figure 8(a) the take-up rate is more concave when UI collection costs are endogenous; therefore, the response of the take-up rate is smaller as UI benefits continue to increase, relative to the case of fixed collection costs. Intuitively, as $b$ increases, the associated increase in $\tau$ increases the costs of collecting UI benefits causing the net gain of collecting to rise slower.

Changes in the UI take-up rate have an important impact on the response of the unemployment rate to changes in UI benefits. As a result, the unemployment rate responds “slower” to changes in UI benefits when UI collection costs are endogenous, relative to the case of fixed UI collection costs. Figure 8(b) shows this effect. Again, this occurs because of the “slower” change in the take-up rate, implying the impact of UI benefits on the unemployment rate is reduced, as more generous benefits have little impact if no one collects them.

Overall, this comparative statics exercise has highlighted the importance of allowing for an endogenous UI take-up rate and of incorporating endogenous costs of collecting UI benefits. In the next section we explore the welfare implications of the model and equilibrium.

8 Welfare

The relationships between experience rating, improper denials, and the take-up rate have important implications for welfare. Several questions arise when considering this link. If the take-up rate is driven by the utility costs associated with collecting benefits, how significant are these costs? Given the implied costs of collecting, what are the optimal take-up, improper denial, and UI fraud
rates?

We conduct several welfare experiments to answer these questions. To begin, we compare the equilibrium outcomes to those of a social planner. The social planner solves the following problem:

$$\max_{(\theta, p_B, p_N)} (1-u)y + \lambda \left\{ n_B^E F(\chi_B^*) \left[ \tau(p_B s_B - 1) - c(p_B) - p_B E(\chi|\chi \leq \chi_B^*) \right] + n_N^E F(\chi_N^*) \left[ \tau(p_N s_N - 1) - c(p_N) - p_N E(\chi|\chi \leq \chi_N^*) \right] + n_U^B b + n_U^N d - \theta u \gamma \right\}$$

s.t. $\chi_i^* = \frac{(1 - p_i s_i)(b - d)}{p_i [r + \theta q(\theta)]}, i \in \{B, N\}$

$$n_B^E, n_N^E, n_U^B, n_U^N \quad \text{follow equilibrium flow equations in (21) – (24)}$$

Equation (29) represents a standard welfare function of output net of vacancy costs, adjusted for the costs and benefits associated with eligibility verifications. Specifically, for each group of employed workers, $i = B, N$, the welfare function includes the net gain (to the firm or a social planner) of utilizing the eligibility verification technology minus the expected utility cost to those workers actually verified. Notice the planner maximizes welfare subject to equilibrium conditions, namely the determination of $\chi_i^*, i = B, N$ and the equilibrium flow equations.

Table 6 presents the welfare results. Consider first the baseline equilibrium compared to the planner’s problem in Equation (29), which is presented in the second column of Table 6. The planner’s allocation implies a welfare gain of 1.79%, with a lower unemployment rate and average duration of unemployment. Moreover, the planner prefers to have no one collect UI benefits, with a take-up rate of essentially 0. This is accomplished by setting $p_i = 1, i = B, N$, discouraging workers from applying for UI benefits. Note, the take-up rate, improper denial rate, and UI fraud rate are all positive because of the $p_i \leq 1$ restriction. Combined with $s_i < 1$, this implies that the planner cannot guarantee a worker’s UI application is rejected. Although the planner sets $p_i = 1$, there still exist some workers with very low $\chi$ who decide to apply for UI benefits, and some of them are successful. This implies a positive, but very low take-up rate, and positive improper denial and UI fraud rates.

The reasons for the planner’s preference for a take-up rate of effectively 0 are twofold. First, utilizing the UI system is costly. It implies a “tax,” $\tau$, and the expected costs of utilizing the verification technologies. Second, the alternative to the UI system is quite appealing in the baseline parameterization. This is true because $d = 0.678$ implies the flow utility of a non-collector remains
This table breaks down the welfare gains of the social planner’s allocation and the full-commitment equilibrium, both relatively close to the baseline equilibrium.

<table>
<thead>
<tr>
<th></th>
<th>Social Planner</th>
<th>Full-commitment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline $d$</td>
<td>$d = 0$</td>
</tr>
<tr>
<td>% Welfare Gain</td>
<td>1.79%</td>
<td>2.15%</td>
</tr>
<tr>
<td>Unemp. Rate</td>
<td>5.70%</td>
<td>7.24%</td>
</tr>
<tr>
<td>Duration</td>
<td>4.32</td>
<td>5.58</td>
</tr>
<tr>
<td>Take-up Rate</td>
<td>0.01</td>
<td>1.00</td>
</tr>
<tr>
<td>Improper Denial Rate</td>
<td>0.20</td>
<td>0.00</td>
</tr>
<tr>
<td>Fraud Rate</td>
<td>0.24</td>
<td>1.00</td>
</tr>
</tbody>
</table>

To provide some insight on this dimension, we consider an alternative parameterization with $d = 0$, which is presented in the third column of Table 6. Here we re-calibrate $\mu_\chi$ so that the equilibrium take-up rate matches the value in the baseline parameterization (0.74). This implies $\mu_\chi = 459$. Under this parameterization we obtain the opposite result: the planner prefers to have everyone use the UI system with a 100% take-up rate. In this case, however, the planner sets $p_i = 0, i = B, N$. Thus, the planner prefers to have everyone utilize the UI system and is willing to allow all ineligible workers to collect in order to forgo the costs associated with using the verification technology. Under this allocation, the unemployment rate and average duration of unemployment are higher than the baseline levels. This is due to the increase in the take-up rate which raises the average flow utility for unemployed workers, decreasing $\theta$ as in Pissarides (2000). Moreover, the

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16It is “free” in that it is not financed at all by taxes, nor is it financed by workers via precautionary savings, which would similarly reduce consumption while employed.
improper denial rate goes to zero while the fraud rate is 100%, since the verification technology is never used.

Next, we consider an economy with full-commitment. With full-commitment, all ineligible workers commit to never applying for UI benefits, and the firm commits to never challenging the claims of the UI-eligible. In this economy, the take-up rate is 100% as all eligible unemployed collect, and the verification technologies are never used. The last two columns of Table 6 present the results from this experiment. For the baseline level of $d$, the full-commitment equilibrium actually reduces welfare slightly. The unemployment rate is higher under full commitment, as all workers collect UI benefits. The reduction in welfare happens under $d = 0.678$ for similar reasons as discussed above in the planner’s problem with the baseline $d$: while UI benefits are financed (partially) by a tax, the non-collector consumption is not. Given this, the net gain from having all eligible workers collect is negative. The high utility offered by $d$ is essentially free (relative to the alternative, $b$).

In the last column of Table 6, we consider the full-commitment equilibrium under the alternative parameterization with $d = 0$. In this case, the welfare gain of full commitment is relatively large at 3.03% (this is relative to the baseline equilibrium). This emphasizes the costs imposed by the information asymmetries regarding eligibility. These asymmetries combined with experienced rated taxes lead firms to challenge eligibility of both UI-eligible and ineligible workers. This imposes two costs to the workers: (i) some UI-eligible workers prefer not to collect benefits and (ii) those deciding to collect directly pay a utility cost when eligibility is verified. The full-commitment equilibrium has a much higher unemployment rate and average unemployment duration at 8.93% and 7.01 months, respectively, relative to the baseline economy (6.54% and 5.00 months).

9 Conclusion

In this paper we explored the micro-foundations of the costs of collecting UI benefits. Using data across U.S. states, we characterize the relationship of the UI take-up rate with several key variables. There exists evidence that the likelihood of having eligibility verified and benefits improperly denied has a significant negative impact on the take-up rate. Based on these results, we develop a search model with matching frictions that incorporates UI eligibility and endogenous UI collection costs. UI benefits are financed by an experience rated tax levied on firms. This tax gives firms incentives to
challenge claims for both UI eligible and ineligible workers applying for UI benefits. In equilibrium there exist both improper denials and UI fraud. The costs of collecting UI benefits are those associated with an eligibility verification.

Analytically we show that the model qualitatively captures the correlations identified in our empirical analysis. We then calibrate the model to U.S. data and explore the model’s quantitative implications. The results suggest important implications from endogenous UI collection costs. The take-up rate and unemployment rate respond slower to changes in UI benefits when collection costs are endogenous, relative to a model with fixed collection costs. In addition, the link with experience rating provides a possible explanation for the negative relationship between the unemployment rate and take-up rate across U.S. states found in the empirical analysis.

The unclaimed benefits imply an externality, as firms do not consider the welfare cost to workers when optimally deciding how often to use the verification technology. A welfare analysis shows that the costs of this externality are significant, with welfare costs ranging from 1.79%–3.03%, depending on the comparison economy. A social planner prefers to never use the verification technology, and welfare increases when full-commitment similarly renders the verification technology unnecessary.

Overall our results show that considering both unclaimed benefits and endogenous costs of collecting UI have important implications for labor markets and UI policy. The link with experience rating represents an important dimension of our analysis. While we assumed exogenous separations, allowing for endogenous separations represents an interesting direction for future research. This would allow one to explore the joint determination of firm decisions about separation and whether or not to initiate a UI eligibility review, helping to illuminate the full extent to which experience rating affects labor market outcomes.
References


A Robustness Checks

This section displays the same results presented in Tables 1 and 2, with the exception that we use the “unadjusted” take-up rate as the dependent variable. That is, we do not adjust the number of insured unemployed to include those improperly denied.

From Tables 7 and 8 the results presented in Section 2.7 still hold. As expected, improper denials have a slightly lower impact on the take-up rate in Tables 1 and 2 relative to Tables 7 and 8. This is expected given the relationship between improper denials and the un-adjusted take-up rate.

State Fixed Effects Regression using Raw (Un-Adjusted) Take-up Rate

<table>
<thead>
<tr>
<th>Specification with Fraud</th>
<th>Specification without Fraud</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coef.</td>
<td>s.e.</td>
</tr>
<tr>
<td>RR</td>
<td>0.58</td>
</tr>
<tr>
<td>UR</td>
<td>-0.64</td>
</tr>
<tr>
<td>ID</td>
<td>-0.30</td>
</tr>
<tr>
<td>Fraud</td>
<td>1.57</td>
</tr>
<tr>
<td>Constant</td>
<td>0.54</td>
</tr>
<tr>
<td>n = 510</td>
<td></td>
</tr>
<tr>
<td>F(50, 455) = 34.61</td>
<td></td>
</tr>
<tr>
<td>Prob &gt; F = 0.0000</td>
<td></td>
</tr>
</tbody>
</table>

B Proofs

This section presents the proofs of the results presented above. We begin with the proof of Proposition 1.

Proof of Proposition 1: The worker’s decision to collect UI benefits or not is given by Equa-
tion (13). Define $\Gamma(\chi)$ as,

$$
\Gamma(\chi) = p_i \left[ -\chi + s_i (N - U) \right] + \left[ U - E_i(\chi) \right] - \left( N - E_i(\chi) \right)
$$

$$
= -p_i \chi + N(p_i s_i - 1) + U[p_i(1 - s_i) + (1 - p_i)]
$$

$$
= -p_i \chi + (1 - p_i s_i) \left( U - N \right)
$$

(31)

From Equations (1) and (2) the difference $U(\chi) - N(\chi)$ is given by:

$$
U(\chi) - N(\chi) = \frac{b - d}{r + \theta q(\theta)}
$$

(32)

Plugging this expression into Equation (31) gives,

$$
\Gamma(\chi) = -p_i \chi + (1 - p_i s_i) \left( \frac{b - d}{r + \theta q(\theta)} \right)
$$

(33)

Now, $\Gamma(0) > 0$ given $p_i > 0$, $s_i > 0$, and $b > d$. Moreover, $\lim_{\chi \to \infty} \Gamma(\chi) = -\infty$. Given that $\Gamma'(\chi) = -p_i < 0$, $\Gamma(\chi)$ is strictly decreasing in $\chi$. The strict monotonicity combined with the fact that $\Gamma(\chi)$ starts positive and is eventually negative implies there exists a unique $\chi_i^*$ such that $\Gamma(\chi_i^*) = 0$, $\Gamma(\chi) < 0$, $\chi > \chi_i^*$, and $\Gamma(\chi) \geq 0$, $\chi \leq \chi_i^*$.

Finally, solving for the zero in Equation (33) gives:

$$
\chi_i^* = \frac{(1 - p_i s_i)(b - d)}{p_i r + \theta q(\theta)}
$$

Proof of Proposition 2: Define the function on the LHS of Equation (9) as $g(p)$. Existence of a solution to $g(p) = 0$ is shown as follows. First, notice that $\lim_{p \to 0} = \infty$, and $\lim_{p \to \infty} < 0$. Therefore, we can define a closed and bounded interval, $[a, b]$, $a > 0$, $b < 0$, such that $g(a) > 0$ and $g(b) < 0$. The intermediate value theorem establishes the existence of a $p^*$ such that $g(p^*) = 0$. Uniqueness of this $p^*$ is established by the monotonicity of the function $g(p)$.

Next, when $c(p) = c p_i^\nu$, the FOC in Equation (9) is:

$$
s_i \tau = \nu c p_i^{\nu - 1}
$$

$$
\Rightarrow p_i = \left[ \frac{s_i \tau}{\nu c} \right]^{\frac{1}{\nu - 1}}
$$

Proof of Lemma 1: In this case, the optimal choice of $p_i^*$ is given by the solution to Equation (9).

Let $g(p) \equiv c'(p)$ and define the inverse of $g(p)$ by $h \equiv g^{-1}$. Thus, $p_i^* = h(s_i \tau)$. Differentiating this with respect to $\tau$ yields:

$$
\frac{\partial p_i^*}{\partial \tau} = h'(s_i \tau) = \frac{s_i}{f'(p_i^*)} = \frac{s_i}{c''(p_i^*)} > 0
$$
where the inequality follows from the properties of \( c(p) \) (strictly convex). Partially differentiating with respect to \( s_i \) follows similarly. ■

**Proof of Lemma 2:** We begin with the partial derivative with respect to \( b \). Differentiating the expression for \( \chi_i^* \) in Equation (14) with respect to \( b \) we have:

\[
\frac{\partial \chi_i^*}{\partial b} = \frac{1 - p_i s_i}{p_i [r + \theta q(\theta)]} > 0
\]  

(34)

Next, differentiating with respect to \( \tau \) yields

\[
\frac{\partial \chi_i^*}{\partial \tau} = \frac{\partial \chi_i^*}{\partial p_i^*} \frac{\partial p_i^*}{\partial \tau} = - \left[ \frac{b - d}{(p_i^*)^2 [r + \theta q(\theta)]} \right] \frac{\partial p_i^*}{\partial \tau} < 0
\]  

(35)

where the inequality derives from \( b > d, p_i^* > 0, \) and \( \frac{\partial p_i^*}{\partial \tau} > 0 \) (from Lemma 1). Then, differentiating with respect to \( s_i \) gives,

\[
\frac{\partial \chi_i^*}{s_i} = \left[ \frac{(b - d)(p_i + s_i \frac{\partial p_i}{\partial s_i})[p_i(r + \theta q(\theta)) + (1 - p_i s_i)(b - d)[r + \theta q(\theta)] \frac{\partial p_i}{\partial s_i}}{p_i^2 [r + \theta q(\theta)]^2} \right] < 0
\]  

(36)

where the inequality stems from \( b > d, p_i^* > 0, \) and \( \frac{\partial p_i^*}{\partial s_i} > 0 \) (from Lemma 1). Finally, differentiating with respect to \( \theta \) yields,

\[
\frac{\partial \chi_i^*}{\partial \theta} = \frac{-(1 - p_i s_i)(b - d)}{p_i^2 [r + \theta q(\theta)]^2} \frac{\partial(\theta q(\theta))}{\partial \theta} < 0
\]  

(37)

where again the inequality arises from \( b > d, p_i^* > 0, \) and \( \frac{\partial(\theta q(\theta))}{\partial \theta} > 0 \) (given the properties of the matching function). ■

**Proof of Proposition 3:** We show this result by partially differentiating the expression for the take-up rate in Equation (26) with respect to the appropriate parameter. For a general parameter \( z \), we have:

\[
\frac{\partial TUR}{\partial z} = \frac{\lambda}{\sigma} \left[ (1 - p_N s_N) f(\chi_N^*) \frac{\partial \chi_N^*}{\partial z} - F(\chi_N^*) s_N \frac{\partial p_N}{\partial z} \right] + \left[ (1 - p_B s_B) f(\chi_B^*) \frac{\partial \chi_B^*}{\partial z} - F(\chi_B^*) s_B \frac{\partial p_B}{\partial z} \right]
\]  

(38)

Using Lemmas 1 and 2 to plug in the appropriate derivatives yields the results. ■

**B.1 Derivation of Piecewise Wage Function**

In each case, recall that wages are determined by the Nash F.O.C. in Equation (11), which again is:

\[
(1 - \beta) [E_N(\chi) - N] = \beta [J_N(\chi) - V]
\]
The key aspect of deriving the wage in each range of \( \chi \) is finding an expression for \( E_N(\chi) - N(\chi) \).

It is convenient to define the following terms:

\[
C_i(\chi) = -\chi + \frac{s_i(d-b)}{r + \theta q(\theta)}, \quad i = N, B
\]

(39)

For an employed worker, this term represents the utility change from a challenged UI claim. The worker incurs the flow utility cost \(-\chi\) and with probability \( s_i \) the challenge is successful and the worker transitions from \( U \) to \( N \). On the firm side we also define the following:

\[
\Pi_i = \tau(p_i s_i - 1) - c(p_i), \quad i = N, B
\]

(40)

We begin with the case where \( \chi \leq \chi^*_N \). For this range of \( \chi \), all workers prefer to file a UI claim if separated, regardless of eligibility status. The expression for \( E_N - N \) in this case is derived as follows. From Equation (2), substituting \( U = \frac{b + \theta q(\theta)E_N}{r + \theta q(\theta)} \) from Equation (1) gives

\[
rE_N = w_1 + \lambda \left\{ p_N C_N(\chi) + \left[ U - E_N \right] + \sigma \left[ E_B - E_N \right] \right\}
\]

\[
\Rightarrow rE_N = w_1 + \lambda \left\{ p_N C_N(\chi) + \frac{b}{r + \theta q(\theta)} - \frac{rE_N}{r + \theta q(\theta)} \right\} + \sigma \left[ E_B - E_N \right]
\]

\[
\Rightarrow rE_N = \left[ \frac{r + \theta q(\theta)}{r + \lambda + \theta q(\theta)} \right] \left[ w_1 + \lambda \left\{ p_N C_N(\chi) + \frac{b}{r + \theta q(\theta)} \right\} + \sigma \left[ E_B - E_N \right] \right]
\]

(41)

Similarly, using Equation (3) an expression for \( rE_B \) is derived as:

\[
rE_B = w_1 + \lambda \left\{ p_B C_B(\chi) + \left[ U - E_B \right] \right\} = w_1 + \lambda \left\{ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} + \frac{\theta q(\theta)E_N}{r + \theta q(\theta)} - E_B \right\}
\]

\[
\Rightarrow rE_B = \left[ 1 + \frac{\lambda}{r + \theta q(\theta)} \right] = w_1 + \lambda \left\{ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} - \frac{\theta q(\theta)[E_B - E_N]}{r + \theta q(\theta)} \right\}
\]

\[
\Rightarrow rE_B = \left[ \frac{r + \lambda + \theta q(\theta)}{r + \theta q(\theta)} \right] \left[ w_1 + \lambda \left\{ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} - \frac{\theta q(\theta)[E_B - E_N]}{r + \theta q(\theta)} \right\} \right]
\]

(42)

Next, using Equation (41) and Equation (42) we derive an expression for \( E_B - E_N \). Specifically,

\[
rE_B - rE_N = \left[ \frac{r + \lambda + \theta q(\theta)}{r + \theta q(\theta)} \right] \left\{ \lambda \left[ p_B C_B(\chi) - p_N C_N(\chi) \right] - \frac{\lambda \theta q(\theta)}{r + \theta q(\theta)} \left[ E_B - E_N \right] - \sigma \left[ E_B - E_N \right] \right\}
\]

\[
\Rightarrow (E_B - E_N) \left[ r + \frac{\lambda \theta q(\theta)}{r + \lambda + \theta q(\theta)} + \frac{\sigma [r + \theta q(\theta)]}{r + \lambda + \theta q(\theta)} \right] = \left[ \frac{r + \theta q(\theta)}{r + \lambda + \theta q(\theta)} \right] \left\{ \lambda \left[ p_B C_B(\chi) - p_N C_N(\chi) \right] \right\}
\]

\[
\Rightarrow E_B - E_N = \left[ \frac{r + \theta q(\theta)}{r [r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta) + \sigma [r + \theta q(\theta)]} \right] \left( \lambda \left[ p_B C_B(\chi) - p_N C_N(\chi) \right] \right)
\]

(43)
Plugging Equation (43) into Equation (41) we then have,

\[ rE_N = \left( \frac{r + \theta q(\theta)}{r + \lambda + \theta q(\theta)} \right) \left\{ w_1 + \lambda \left[ p_N C_N(\chi) + \frac{b}{r + \theta q(\theta)} \right] + \right. \\
\left. \frac{\sigma[r + \theta q(\theta)]}{r[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta) + \sigma[r + \theta q(\theta)]} \right) \left( \lambda [p_B C_B(\chi) - p_N C_N(\chi)] \right) \right\} \]

Then, the expression for \( E_N - N \) is derived by subtracting \( rN = d + \theta q(\theta)[E_N - N] \) from the expression for \( rE_N \) above to get:

\[ E_N - N = \frac{1}{r + \lambda + \theta q(\theta)} \left\{ w_1 + \lambda \left[ p_N C_N(\chi) + \frac{b}{r + \theta q(\theta)} \right] + \right. \\
\left. \frac{\sigma[r + \theta q(\theta)]}{r[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta) + \sigma[r + \theta q(\theta)]} \right) \left( \lambda [p_B C_B(\chi) - p_N C_N(\chi)] \right) \right\} - \frac{d}{r + \theta q(\theta)} \tag{44} \]

For the firm side, in the case of \( \chi \leq \chi^*_N \), all separated workers decide to file a UI claim; as a result, the expression for \( J_N \) in this case is:

\[ rJ_N = y - w_1 + \lambda \left[ \Pi_N - J_N \right] + \sigma \left[ J_B - J_N \right] \]

\[ J_N = \frac{1}{r + \lambda + \sigma} \left\{ y - w_1 + \lambda \Pi_N + \frac{\sigma}{r + \lambda} \left( y - w_1 + \lambda \Pi_B \right) \right\} = \left\{ \frac{y - w_1}{r + \lambda + \sigma} \right\} \left[ \frac{r + \lambda + \sigma}{r + \lambda} \right] + \left\{ \frac{\lambda}{r + \lambda + \sigma} \right\} \left[ \Pi_N + \frac{\sigma}{r + \lambda} \Pi_B \right] \tag{45} \]

Then, to derive the expression for \( w_1 \), we plug Equation (44) and Equation (45) into the Nash F.O.C. in Equation (11):

\[ \frac{\beta[r + \lambda + \theta q(\theta)]}{(1 - \beta)[r + \lambda + \sigma]} \left\{ (y - w_1) \left( \frac{r + \lambda + \sigma}{r + \lambda} \right) + \lambda \left[ \Pi_N + \frac{\sigma}{r + \lambda} \Pi_B \right] \right\} = w_1 + \lambda \left[ p_N C_N(\chi) + \frac{b}{r + \theta q(\theta)} \right] \]

\[ + \left[ \frac{\sigma[r + \theta q(\theta)]}{r[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta) + \sigma[r + \theta q(\theta)]} \right) \left( \lambda [p_B C_B(\chi) - p_N C_N(\chi)] \right) \right\} - \frac{d}{r + \theta q(\theta)} \]

Simplifying this yields:

\[ w_1 = \frac{\beta[r + \lambda + \theta q(\theta)]}{(1 - \beta)(r + \lambda)} \left\{ (y) \left( \frac{r + \lambda + \sigma}{r + \lambda} \right) + \lambda \left[ \Pi_N + \frac{\sigma}{r + \lambda} \Pi_B \right] \right\} - \lambda \left[ p_N C_N(\chi) + \frac{b}{r + \theta q(\theta)} \right] \]

\[ - \left[ \frac{\sigma[r + \theta q(\theta)]}{r[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta) + \sigma[r + \theta q(\theta)]} \right) \left( \lambda [p_B C_B(\chi) - p_N C_N(\chi)] \right) + \frac{d}{r + \theta q(\theta)} \tag{46} \]

Next consider the case where \( \chi^*_N < \chi \leq \chi^*_B \). In this range, a UI-ineligible worker does not file a claim if separated; the UI-eligible worker does, however. Again, using the Nash F.O.C. in
Equation (11), $E_N(\chi) - N(\chi)$ represents the key term to derive. Towards this end, combining Equation (1) and Equation (2) gives

$$N(\chi) - U(\chi) = \frac{d - b}{r + \theta q(\theta)}$$ (47)

Next, simplifying Equation (3) using $U = \frac{b + \theta q(\theta)E_N}{r + \theta q(\theta)}$ gives

$$rE_N = w_2 + \lambda \left\{ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} - \frac{\theta q(\theta)}{r + \theta q(\theta)} [E_B - E_N] - \frac{rE_B}{r + \theta q(\theta)} \right\}$$

$$\Rightarrow rE_B = \left[ \frac{r + \theta q(\theta)}{r + \lambda + \theta q(\theta)} \right] \left[ w_2 + \lambda \left\{ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} - \frac{\theta q(\theta)}{r + \theta q(\theta)} [E_B - E_N] \right\} \right]$$ (48)

Next, consider the difference $E_B - E_N$. Using the expression for $E_B$ above in Equation (48), we have:

$$rE_B - rE_N = \left( \frac{r + \theta q(\theta)}{r + \lambda + \theta q(\theta)} \right) \left[ w_2 + \lambda \left\{ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} \right\} \right] -$$

$$w_2 - \lambda [N - E_N] - (\sigma + \frac{\lambda \theta q(\theta)}{r + \lambda + \theta q(\theta)})[E_B - E_N]$$

$$\Rightarrow (E_B - E_N) = \left[ \frac{r + \theta q(\theta)}{r + \lambda + \theta q(\theta)} \right] \left[ w_2 + \lambda \left\{ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} \right\} \right] - w_2 - \lambda [N - E_N]$$

$$\Rightarrow E_B - E_N = \left[ \frac{1}{(r + \sigma)(r + \lambda + \theta q(\theta)) + \lambda \theta q(\theta)} \right] \left[ -\lambda w_\lambda (r + \theta q(\theta)) \left( p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} \right) \right] -$$

$$\left[ \frac{\lambda (r + \lambda + \theta q(\theta))}{(r + \sigma)(r + \lambda + \theta q(\theta)) + \lambda \theta q(\theta)} \right] [N - E_N]$$ (49)

Plugging Equation (49) into Equation (4) then gives:

$$rE_N = w_2 \left[ 1 - \frac{\lambda \sigma}{(r + \sigma)(r + \lambda + \theta q(\theta)) + \lambda \theta q(\theta)} \right] - \lambda [E_N - N] \left[ 1 - \frac{\lambda \sigma [r + \lambda + \theta q(\theta)]}{(r + \sigma)(r + \lambda + \theta q(\theta)) + \lambda \theta q(\theta)} \right] +$$

$$\left[ \frac{\lambda \sigma [r + \theta q(\theta)]}{(r + \sigma)(r + \lambda + \theta q(\theta)) + \lambda \theta q(\theta)} \right] \left[ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} \right]$$ (50)

Finally, combining Equation (50) with Equation (2) yields an expression for $E_N - N$ in this
Then, plugging Equation (51) and Equation (52) into the Nash F.O.C. and simplifying we have:

\[ r[E_N - N] = \left[ \frac{(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta) - \lambda \sigma}{(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta)} \right] w_2 + \]

\[ \lambda [E_N - N] \left[ \frac{\lambda \sigma[r + \theta q(\theta)]}{(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta)} \right] \left[ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} \right] - \]

\[ \lambda [E_N - N] \left[ \frac{\lambda \sigma[r + \theta q(\theta)]}{(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta)} \right] \left[ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} \right] - d - \theta q(\theta)[E_N - N] \]

\[ \Rightarrow [r + \theta q(\theta)][(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta)] [E_N - N] = [(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta) - \lambda \sigma] w_2 + \]

\[ \lambda [\sigma[r + \theta q(\theta)] \left[ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} \right] + \lambda [(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta) - \lambda \sigma] w_2 + \]

\[ \lambda [\sigma[r + \theta q(\theta)] \left[ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} \right] - \frac{d}{[(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta)]]} \]

\[ \Rightarrow [r + \lambda + \theta q(\theta)][(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta) - \lambda^2 \sigma] [E_N - N] = [(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta) - \lambda \sigma] w_2 + \]

\[ \lambda [\sigma[r + \theta q(\theta)] \left[ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} \right] - \frac{d}{[(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta)]]} \]

Defining \( \Gamma \) as:

\[ \Gamma \equiv [r + \lambda + \theta q(\theta)][(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta) - \lambda^2 \sigma] \]

we then have:

\[ E_N - N = \frac{1}{\Gamma} \left\{ [(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta) - \lambda \sigma] w_2 + [\lambda [\sigma[r + \theta q(\theta)] \left[ p_B C_B(\chi) + \frac{b}{r + \theta q(\theta)} \right] - \frac{d}{[(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta)]} \right] \right\} \]

(51)

Next, we need an expression for \( J_N(\chi) \) for \( \chi_N < \chi \leq \chi^*_B \). From Equation (6), in this case:

\[ J_N = \frac{y - w_2 + \sigma J_B}{r + \lambda + \sigma} \]

where from Equation (7) in this case we have:

\[ J_B = \frac{y - w_2 + \lambda \Pi_B}{r + \lambda} \]

where recall \( \Pi_B \) is defined in Equation (40) above. Combining these two equations yields:

\[ J_N = \frac{1}{r + \lambda + \sigma} \left( y - w_2 \right) \left( 1 + \frac{\sigma}{r + \lambda} \right) + \frac{\lambda \sigma \Pi_B}{r + \lambda} \]

(52)

Then, plugging Equation (51) and Equation (52) into the Nash F.O.C. and simplifying we have:

\[ w_2 \left\{ \frac{\beta \Gamma}{(1 - \beta)(r + \lambda)} + [(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda(\theta q(\theta) - \sigma)] \right\} = \frac{\beta \Gamma}{(1 - \beta)(r + \lambda)\left[ y + \frac{\lambda \sigma \Pi_B}{r + \lambda + \sigma} \right] - \frac{d}{[(r + \sigma)[r + \lambda + \theta q(\theta)] + \lambda \theta q(\theta)]} \right\} \]

(53)
Now for the case where $\chi > \chi_B^*$. For this range of $\chi$, no workers find it desirable to apply for UI; as a result, the value functions are simply:

$$
    rE_N = w_3 + \lambda [N - E_N] \\
    rN = d + \theta q(\theta) [E_N - N]
$$

which implies that $E_N - N = \frac{w_3 - d}{r + \lambda + \theta q(\theta)}$. Then, when no workers apply for benefits if separated, the value of a filled vacancy is given by $rJ_N = y - w_3 + \lambda [V - J_N]$, which under the free-entry assumption $V = 0$ yields $J_N = \frac{y - w_3}{r + \lambda}$. Using these expressions for $E_N - N$ and $J_N$ and plugging into Equation (11) we have:

$$
    \frac{\beta}{r + \lambda} [y - w_3] = \frac{1 - \beta}{r + \lambda + \theta q(\theta)} [w_3 - d] \\
    \Rightarrow \frac{\beta [r + \lambda + \theta q(\theta)]}{(1 - \beta) [r + \lambda]} y = w_3 \left[ 1 + \frac{\beta (r + \lambda + \theta q(\theta))}{(1 - \beta) [r + \lambda]} \right] \\
    \Rightarrow w_3 = \frac{\beta [r + \lambda + \theta q(\theta)] y}{r + \lambda + \theta q(\theta)}
$$